LAST TIME: More on Jones vectors and matrices, applications

Generalized Jones vector

$$\begin{bmatrix} E_{ox} \\ E_{oy}e^{i\epsilon} \end{bmatrix}$$

$$\begin{bmatrix} A \\ B+iC \end{bmatrix} = \begin{bmatrix} A \\ \left(\sqrt{B^2+C^2}\right)e^{i\epsilon} \end{bmatrix} = \begin{bmatrix} A \\ \left(\sqrt{B^2+C^2}\right)e^{i\tan^{-1}(C/B)} \end{bmatrix}.$$

Therefore,

$$E_{ox} = A;\ E_{oy} = \sqrt{B^2+C^2}; \text{and } \epsilon = \tan^{-1}(C/B)$$

LP states: $E_{ox}\ or\ E_{oy} = 0;\ \ \epsilon = \pm 2n\pi;\ \ \epsilon = \pm(2n+1)\pi$

CP states: $\epsilon = +\frac{\pi}{2};\ \ E_{ox} = E_{oy} = E_o \quad LCP\ ccw$

$\epsilon = -\frac{\pi}{2};\ \ E_{ox} = E_{oy} = E_o \quad RCP\ cw$

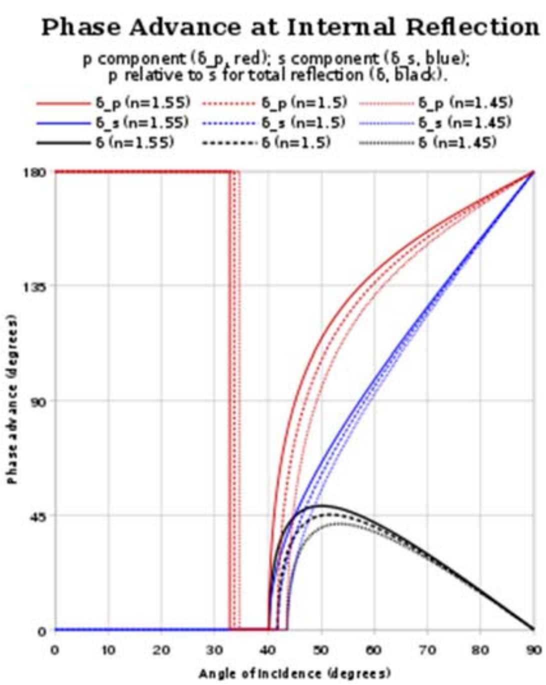
EP states: $\epsilon = +\frac{\pi}{2};\ \ E_{ox} \neq E_{oy} \neq 0 \quad LEP\ ccw$ axes $\parallel$ coordinate axes.

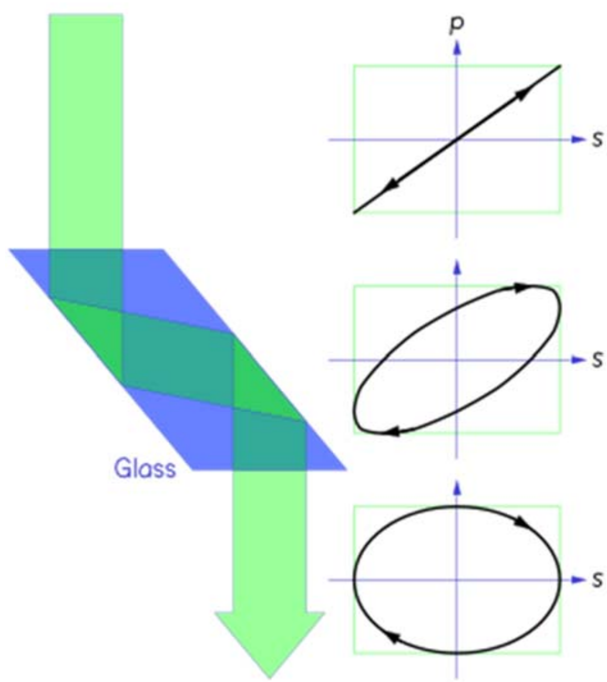
$\epsilon = -\frac{\pi}{2};\ \ E_{ox} \neq E_{oy} \neq 0 \quad REP\ cw$ axes $\parallel$ coordinate axes

ϵ arbitrary, E_{ox}& $E_{oy} \neq 0$; elliptical with axes not parallel to coordinate axes

Applications of some of the ideas we have discussed concerning polarization

Fresnel Rhomb





Let's check to see that if we have a phase shift of 45 degrees with $E_{ox} = E_{oy}$, the light is elliptically polarized with the major axis at 45 degrees.

$$\tan 2\alpha = \frac{2E_{ox}E_{oy}\cos\epsilon}{E_{ox}^2 - E_{oy}^2}.$$

Notice that no matter what the phase difference is, as long as $E_{ox} = E_{oy}$, the ellipse axes will be a 45 degrees. So what is the difference in how the ellipse looks for different phase differences? To see this, go back to the polarization ellipse with $\epsilon = 45$ degrees to obtain

$$\left(\frac{E_y}{E_{oy}}\right)^2 - 2\frac{E_xE_y}{E_{ox}E_{oy}}\cos\epsilon + \left(\frac{E_x}{E_{ox}}\right)^2 = \sin^2\epsilon.$$

Then

$$\left(\frac{E_y}{E_o}\right)^2 - 2\frac{E_xE_y}{E_o^2}\frac{\sqrt{2}}{2} + \left(\frac{E_x}{E_o}\right)^2 = \frac{1}{2}$$

and

$$E_x^2 - \sqrt{2}E_xE_y + E_y^2 = \frac{E_o^2}{2}.$$

Recall

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \text{ with } D = E = 0 \text{ here.}$$

It is tedious, but not terribly difficult, to show that in the rotated frame of the ellipse, we obtain

$$A'x'^2 + C'y'^2 + F' = 0$$

with

$$A' = A\cos^2\alpha + B\sin\alpha\cos\alpha + C\sin^2\alpha,$$

$$C' = A\sin^2\alpha - B\sin\alpha\cos\alpha + C\cos^2\alpha, \text{ and } F' = F.$$

For this example,

$$A' = \frac{1}{2} - \frac{\sqrt{2}}{2} + \frac{1}{2} = 1 - \frac{\sqrt{2}}{2} \text{ and } C' = \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{1}{2} = 1 + \frac{\sqrt{2}}{2}.$$

If we had left ϵ as a parameter, we see that it would be the determining factor in the values for A' and C' in the rotated coordinae system. I have shown this because it is important in determining what the rotated ellipse loos like. The more general polarization ellipse may be written as

$$\left(\frac{E_y}{E_{oy}}\right)^2 - 2\frac{E_xE_y}{E_{ox}E_{oy}}\cos\epsilon + \left(\frac{E_x}{E_{ox}}\right)^2 = \sin^2\epsilon.$$

so that

$$E_x^2 - 2E_xE_y\cos\epsilon + E_y^2 = E_o^2\sin^2\epsilon.$$

Here, $A = 1, C = 1, B = -2\cos\epsilon$, and $F = -E_o^2\sin^2\epsilon$. Use the transformations above to obtain

$$A' = \frac{1}{2} + (-2\cos\epsilon)\left(\frac{\sqrt{2}}{2}\right)^2 + \frac{1}{2} = 1 - \cos\epsilon$$

and

$$C' = \frac{1}{2} - (-2\cos\epsilon)\left(\frac{\sqrt{2}}{2}\right)^2 + \frac{1}{2} = 1 + \cos\epsilon.$$

Therefore, in the rotated coordinate system, the equation of the ellipse is given by

$$(1 + \cos\epsilon)E'^2_x + (1 - \cos\epsilon)E'^2_y = E_o^2 \sin^2\epsilon.$$

In a more standard form,

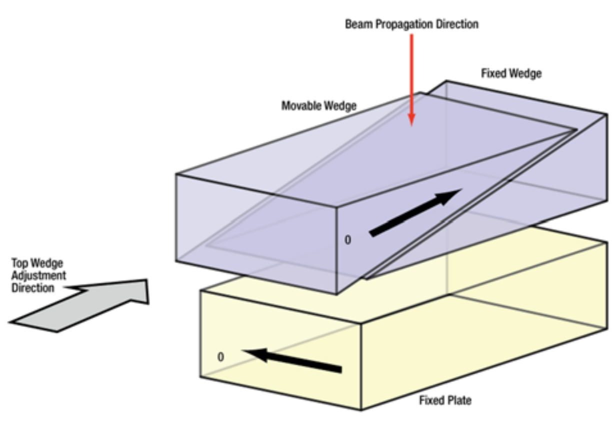
$$\frac{E'^2_x}{\left(\frac{E_o^2 \sin^2\epsilon}{1 + \cos\epsilon}\right)} + \frac{E'^2_y}{\left(\frac{E_o^2 \sin^2\epsilon}{1 - \cos\epsilon}\right)} = 1.$$

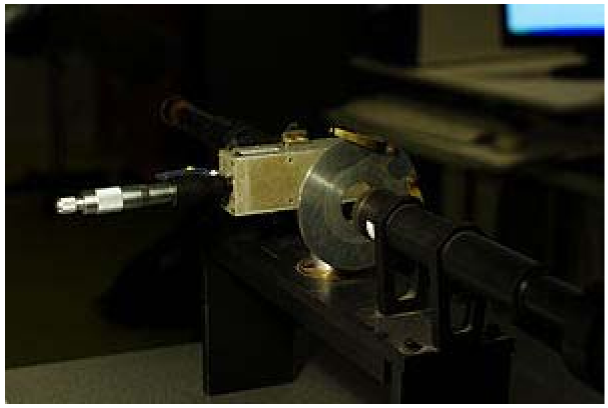
Finally,

$$a = \sqrt{\left(\frac{E_o^2 \sin^2\epsilon}{1 + \cos\epsilon}\right)} \text{ and } b = \sqrt{\left(\frac{E_o^2 \sin^2\epsilon}{1 - \cos\epsilon}\right)}$$

This all means that the phase difference between the two components determines the length of the major and minor axes in the rotated ellipse. Notice that when $\epsilon = 90$ degrees, a and b are equal, and we get a circle as expected.

Let's now have a look at another device that is very convenient in analyzing polarized light, specifically ellipsometry. The figures below show a Soleil-Babinet compensator.





The phase difference between the electric fields in the incident wave is given by

$$\epsilon = \frac{2\pi}{\lambda_o}(d_1 - d_2)|n_e - n_o|,$$

Where d_1 is the total thickness of the wedge and d_2 is the thickness of the fixed plate.

Analysis of Polarized Light

A. No Intensity Variation with Analyzer Alone

NOTE: QWP = Quarter Wave Plate

<table>
<tr><th>I. If with QWP in front of analyzer</th><th colspan="2">II. If with QWP in front of analyzer one finds a maximum, then</th></tr>
<tr><td>1. One has no intensity variation,

one has

natural unpolarized light</td><td>2. If one position of analyzer gives zero intensity,

one has

circularly polarized light</td><td>3. If no position of analyzer gives zero intensity,

one has

a mixture of circularly polarized light and unpolarized light</td></tr>
</table>

B. Intensity Variation with Analyzer Alone

<table>
<tr><th>I. If one position of the analyzer gives</th><th colspan="3">II. If no position of the analyzer gives zero intensity</th></tr>
<tr><td rowspan="3">1. Zero intensity,

one has

linearly polarized light</td><td colspan="3">2. Insert a QWP in front of analyzer with optic axis parallel to the position of maximum intensity</td></tr>
<tr><td rowspan="2">(a) If one gets zero intensity with analyzer,

one has

elliptically polarized light</td><td colspan="2">(b) If one gets no zero intensity,</td></tr>
<tr><td>(1) but the same analyzer setting as before gives the maximum intensity,

one has

mixture of linearly polarized light and unpolarized light</td><td>(2) but some other analyzer setting than before gives a maximum intensity,

one has

mixture of elliptically polarized light and linearly polarized light</td></tr>
</table>

Interference classification

Amplitude division | Wavefront division

Two – wave (Michelson interferometer) | Two wave (Young's 2-slit interference)
Multiwave (Fabry-Perot interferometer) | Multiwave (Multiple slit interference)

Two observe interference, the waves that are being added must have some degree of coherence. Waves are said to be coherent when they maintain a definite phase relationship between them. The distance over which this coherence is maintained is called the coherence length. As long as some degree of coherence is maintained, you should be able to observe interference. We can see this better if we think of light being produced by electronic transitions in the atom, each producing a wave packet having a length $\Delta L_c = c\,\Delta t = 3 \times 10^8 \times 10^{-9}$ m $\approx$ 30 cm. It is not hard to prove that, in terms of the wavelength, the coherence length is given by

$$\Delta L_c \cong \frac{\lambda_o^2}{\Delta\lambda}.$$

We begin by adding two waves having the same frequency with a relative phase of ϵ. The two waves are given by

$$\mathbf{E}_1 = \mathbf{E}_{o1} e^{i(kr_1 - \omega t)}$$

and

$$\mathbf{E}_2 = \mathbf{E}_{o2} e^{i(kr_1 - \omega t + \epsilon)}.$$

Therefore, the total field is given by

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \mathbf{E}_{o1} e^{i(kr_1 - \omega t)} + \mathbf{E}_{o2} e^{i(kr_1 - \omega t + \epsilon)}.$$

One very big advantage in working with the exponential form of the electric field is that iit is not hard to prove that the irradiance I is given by

$$I = \frac{1}{2} Re(\mathbf{E} \cdot \mathbf{E}^*), \text{which comes from } \langle \text{fg} \rangle = \frac{1}{2} Re(f g^*)$$

Therefore,

$$I = \frac{1}{2} Re\{\mathbf{E}_{o1} e^{i(kr_1 - \omega t)} \cdot \mathbf{E}_{o1} e^{-i(kr_1 - \omega t)} + \mathbf{E}_{o1} e^{i(kr_1 - \omega t)} \cdot \mathbf{E}_{o2} e^{-i(kr_1 - \omega t + \epsilon)} + \\ + \mathbf{E}_{o1} e^{-i(kr_1 - \omega t)} \cdot \mathbf{E}_{o2} e^{i(kr_1 - \omega t + \epsilon)} + \mathbf{E}_{o2} e^{i(kr_1 - \omega t + \epsilon)} \cdot \mathbf{E}_{o2} e^{i(kr_1 - \omega t + \epsilon)}\}$$

The first thing to notice is that the fields must have some parallel components or the cross terms will vanish leaving us with only

$$I = \frac{1}{2} Re(E_{o1}^2 + E_{o2}^2),$$

which is just $I_1 + I_2$. It is customary to assume that the two fields are parallel so we obtain

$$I = \frac{1}{2} Re\left(E_{o1}^2 + E_{o2}^2 + E_{o1}E_{o2} e^{ik(r_1 - r_2 + \epsilon)} + E_{o1}E_{o2} e^{-ik(r_1 - r_2 + \epsilon)}\right).$$

Finally,

$$I = \frac{1}{2} Re\{E_{o1}^2 + E_{o2}^2 + 2E_{o1}E_{o2}\cos[k(r_1 - r_2) + \epsilon]\}$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos[k(r_1 - r_2) + \epsilon].$$

The total phase difference breaks up neatly into two components, the phase difference due to the optical path difference $[k(r_1 - r_2)]$ and the remaining phase ϵ, which comes from the inherent phase difference + the phase differences due to reflections. I believe this is the most effective way to consider phase in interference problems. Usually, in optics, an inherent phase difference is difficult to build in, but in electronically controlled devices, it is relatively easy to do. We call the total phase difference $\delta = \delta_{OPD} + \delta_{in} + \delta_{ref}$. We note the following:

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2};\ \ \delta = 0, \pm 2\pi, \cdots \pm 2n\pi$$

and

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2};\ \ \delta = \pm\pi, \cdots \pm (2n+1)\pi$$

The fringe visibility is defined to be

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2},$$

so

$$V = 1$$

if

$$I_1 = I_2 = I_o.$$

After spring break, we look at examples.

NEXT TIME: Examples of interference and interferometers