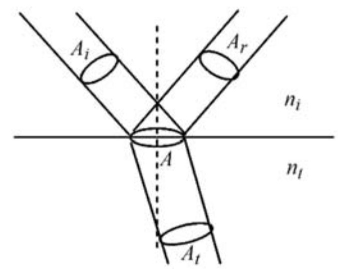
Lecture 11 was examination 1.

LAST TIME: $I = \frac{c^2\epsilon_o}{2}\frac{E_o^2}{c} = \frac{c\epsilon_o}{2}E_o^2$ in free space, $I = \epsilon v\langle E^2\rangle_T$; $v = \frac{c}{n} = \lambda_m \nu_m$; $\lambda_m = \frac{\lambda_{vac}}{n}$
Boundary conditions, Fresnel equations (coefficients)

Recall that using the electromagnetic approach, we recovered Snell's law, but we also now know how much light is reflected and how much is transmitted as a function of the angles and indices of refraction. The polarizing angle for the electric field parallel to the plane of incidence was also something new because geometrical optics cannot consider polarization.

Now we want to extend our discussion to include not only the amplitude coefficients but also the energy transmitted across the boundary because that is what our detectors measure. Our equations are written first for the case of perfect dielectrics – no absorption. We are interested in the energy flow per unit time across a unit area. The following figure shows a two-dimensional version of how things look at a boundary. The area A is the area at the surface, and each of the areas shown are given by



$$A_i = A\cos\theta_i\,; \quad A_r = A\cos\theta_r\,; \quad A_t = A\cos\theta_t.$$

Therefore, the power in each of the beams is given by

$$P_i = I_i A\cos\theta_i\,; \quad P_r = I_r A\cos\theta_r\,; \quad P_t = I_t A\cos\theta_t.$$

The reflectance is defined as the ratio of the reflected power to the incident power, so

$$R \equiv \frac{I_r A\cos\theta_r}{I_i A\cos\theta_i} = \frac{I_r}{I_i} = \frac{\frac{v_r\epsilon_r E_{or}^2}{2}}{\frac{v_i\epsilon_i E_{oi}^2}{2}} = \left(\frac{E_{or}}{E_{oi}}\right)^2 = r^2.$$

The transmittance, however, looks a bit different because the two waves are not in the same medium and do not have the same area. The result for the transmittance is given by

$$T = \frac{n_t\cos\theta_t}{n_i\cos\theta_i}\left(\frac{E_{ot}}{E_{oi}}\right)^2 = \frac{n_t\cos\theta_t}{n_i\cos\theta_i}t^2.$$

Now we can understand why the transmittance is not just equal to the transmission coefficient squared. The indices of refraction are present because they are indicators of the speed at which energy is transmitted. The cosines are present because the areas of the incident beam and the transmitted beam are different. With no absorption, the energy flow into and out of the surface must be equal. This conservation of energy results in

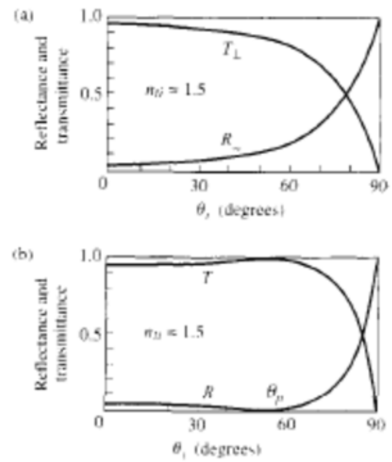
$$I_i A \cos\theta_i = I_r A \cos\theta_r + I_t A \cos\theta_t.$$

We divide through by the left-hand side to obtain

$$1 = R + T.$$

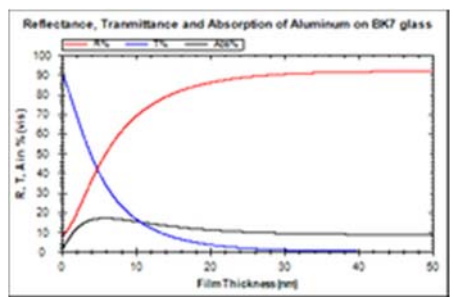
This is the conservation of energy when no absorption is present. When absorption is present, we would expect to find

$$R + T + A = 1.$$



One of the common ways to measure absorption is to measure the transmittance and reflectance and use $A = 1 - (R + T)$. Here are some graphs that show transmittance and reflectance for both the electric field perpendicular and parallel to the plane of incidence. These are for external reflections. You can see the effect at the polarizing angle where the reflectance becomes zero and the transmittance becomes one. Because we know the general expressions for the reflectance and transmittance, we may use either $r_\perp$, $t_\perp$, $r_\parallel$, *or* $t_\parallel$ to get the corresponding expression for T or R. For normal incidence on glass with the index of refraction 1.5, we see that the reflectance is given by

$$R = r^2 = \left(\frac{n_i - n_t}{n_i + n_t}\right)^2 = 0.04.$$



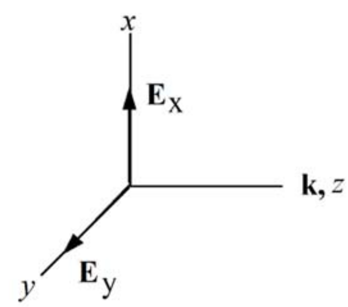
The one-way glass you sometimes see is accomplished by coating a thin layer of metal on glass to create a situation where the reflectance and transmittance are about equal. One side is kept darker than the other side, so the persons on the dark side can see the persons on the bright side, but the persons on the bright side cannot see the persons on the dark side because their reflection is much brighter than the transmitted light from the dark side. The figure to the right shows the reflectance, transmittance, and absorbance as a function of the aluminum film thickness on BK7 glass.

Polarization

An electromagnetic wave is said to be polarized if the electric field vector has a well-defined and unchanging path. The easiest way to visual polarized light is to consider the behavior of the electric field when we imagine we look at how the electric field behaves in time at a fixed location in space. Under these conditions, the electric field vector should trace out a repeatable path, usually a line, circle, or an ellipse depending on how the wave is polarized. Because the electric field is a vector, it is possible to create the total electric field by superposing (adding) two electric field vectors with the appropriate amplitude and phase.

Before we start a discussion of general polarization, we should comment on what is meant by light that is not polarized. Light that does not satisfy our definition of polarized light is said to be unpolarized, randomly polarized, or simply natural light. The electric field in this type of light is randomly distributed. You could not trace out a predictable path of the electric field if you looked at its time dependence in a specific plane.

Let's look at the superposition of two electromagnetic waves whose amplitudes and phase difference can be varied. We take the propagation direction to be along the positive z direction, so our fields will lie in the x-y plane as shown. It is important to understand that the two fields may not reach their maximum values at the same time, so there is a phase difference between the two fields. The components of the two fields are given by



$$E_x = E_{ox}\cos(kz - \omega t) \text{ and } E_y = E_{oy}\cos(kz - \omega t + \epsilon),$$

where ϵ is the phase difference between the two fields. In vector notation, the total field is given by

$$\mathbf{E}(z,t) = E_x(z,t)\hat{\mathbf{x}} + E_y(z,t)\hat{\mathbf{y}}.$$

We will eliminate the t from our equations so we can see what the spatial path looks like. To do that, we expand the cosine function in E_y to obtain

$$E_y = E_{oy}[\cos(kz - \omega t)\cos\epsilon - \sin(kz - \omega t)\sin\epsilon]$$

$$= E_{oy}\left[\frac{E_x}{E_{ox}}\cos\epsilon - \left(1 - \frac{{E_x}^2}{{E_{ox}}^2}\right)^{1/2}\sin\epsilon\right]$$

Therefore,

$$\left(\frac{E_y}{E_{oy}} - \frac{E_x}{E_{ox}}\cos\epsilon\right)^2 = \left(1 - \frac{{E_x}^2}{{E_{ox}}^2}\right)\sin^2\epsilon.$$

Finally,

$$\left(\frac{E_y}{E_{oy}}\right)^2 - 2\frac{E_xE_y}{E_{ox}E_{oy}}\cos\epsilon + \left(\frac{E_x}{E_{ox}}\right)^2 = \sin^2\epsilon.$$

You may remember that the general equation for a rotated and translated conic section (circle, ellipse, or hyperbola) is given by

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

Here, E_x and E_y play the role of x and y, and $D = E = 0$. We wish to investigate the path the electric field traces out in a section where z = constant when we allow $E_{ox}, E_{oy},$ and ϵ to vary.

Case 1: $\epsilon = 0, \pm 2\pi, \pm 4\pi, etc.$ E_{ox} and E_{oy} arbitrary.

$$\left(\frac{E_y}{E_{oy}} - \frac{E_x}{E_{ox}}\right)^2 = 0 \implies E_y = \left(\frac{E_{oy}}{E_{ox}}\right) E_x$$

This case results in linearly polarized light with the electric field making an angle of

$$\theta = \tan^{-1}\frac{E_{oy}}{E_{ox}}.$$

Should E_{ox} or E_{oy} be zero, then the LP state is along one of the coordinate axis.

$\epsilon = \pm\pi, \pm 3\pi, etc.$ E_{ox} and E_{oy} arbitrary.

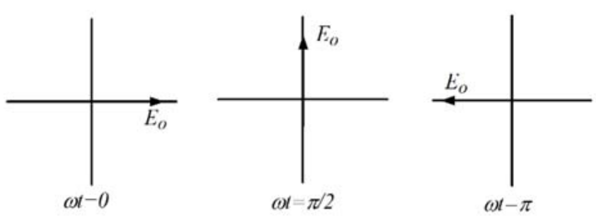
$$\left(\frac{E_y}{E_{oy}} + \frac{E_x}{E_{ox}}\right)^2 = 0 \implies E_y = -\left(\frac{E_{oy}}{E_{ox}}\right) E_x$$

This is still an LP state with **E** anti parallel to the earlier state.

Case 2: $\epsilon = \pm\pi/2$ with $E_{ox} = E_{oy} = E_o$

$$\left(\frac{E_y}{E_{oy}}\right)^2 + \left(\frac{E_x}{E_{ox}}\right)^2 = 1 \implies E_x^2 + E_y^2 = E_0^2.$$

We have the equation of a circle in either case, but there is a difference that we can see if we make a plot in time of how the electric field behaves. At z = 0, how does the electric field rotate? Here are three figures at different *ωt* times to see what happens. $E_x = E_o\cos(-\omega t)$ and $E_y = E_o\cos(-\omega t + \pi/2)$.



With $\epsilon = +\pi/2$, the electric field rotates ccw at a fixed position in space. This is said to be left circularly polarized light (LCP). You can also show that if $\epsilon = -\frac{\pi}{2}$, the electric field rotates cw at a fixed position in space. This case is said to be right circularly polarized light (RCP). In optics, we agree to look back toward the source to determine the handedness. This is not true in all disciplines.

Case 3: $\epsilon = +\frac{\pi}{2}$ with $E_{ox} \neq E_{oy} \neq 0$
This results in our equation becoming

$$\left(\frac{E_y}{E_{oy}}\right)^2 + \left(\frac{E_x}{E_{ox}}\right)^2 = 1,$$

which is the equation of an ellipse with the major and minor axes aligned with the coordinate axes. Finally, if everything is arbitrary but not zero, we get the most general equation for an ellipse with the major and minor axes rotated with respect to the coordinate axes. The equation is given by

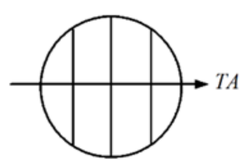
$$\left(\frac{E_y}{E_{oy}}\right)^2 - 2\frac{E_x E_y}{E_{ox}E_{oy}}\cos\epsilon + \left(\frac{E_x}{E_{ox}}\right)^2 = \sin^2\epsilon,$$

which is our most general equation. The angle the ellipse axes make with respect ot the coordinate axes is given by

$$2\alpha = \tan^{-1}\frac{2E_{ox}E_{oy}\cos\epsilon}{E_{ox}^2 - E_{oy}^2}.$$

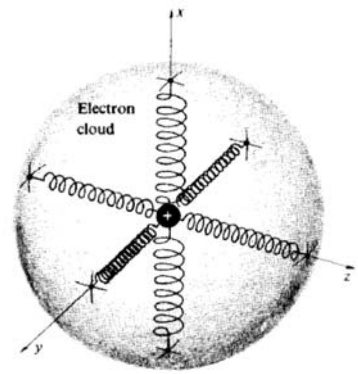
There is an entire field called ellipsometry where the detailed study of how materials change the polarization state can be used to determine the absorption and index of refraction of materials as well as the thickness of thin films.

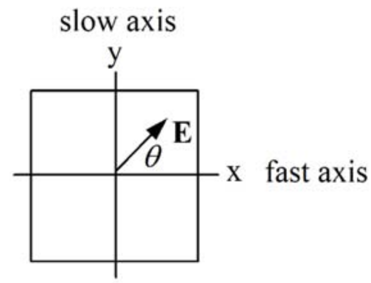
For now, however, how do we produce these effects. The easiest to understand is the linear polarizer. Recall that metals are good conductors of electricity, so they are generally good reflectors of light because the electric field is mostly reflected. A wire grid polarizer consists of very closely spaced wires streched between rigid holders. When the electric field is parallel to the wire grids, the electric field is mostly reflected or absorbed because of currents that are set up in the wires, but when the field is perpendicular to the wires, no currents can be set up, so the field transmits through the polarizer. You might note that this arrangement is opposite to the way you are shown when transverse waves on a string are discussed. For visible light, it is hard to make free standing wire grid polarizers, so long chains of conductors with iodine are used, but the effect is the same. Your text explains the details of these so-called H sheets. A linear polarizer is essentially a highly anisotropic material that behaves like a conductor for component of fields along one direction and like a dielectric (nonconductor) for components of fields in the other perpendicular direction. Here is a figure that shows the idea.



Creating the phase shifts is done in a different way. Phase shifts are created using birefringent materials. In these materials, waves with electric fields along one of the axes travel at a different speed than waves with the electric field along one of the other axes. One axis in the material is called the slow axis, and the other axis is called the fast axis. The figure on the following page shows the basic idea of these anisotropic materials. We will say more later about the actual structure of the materials that cause these effects.

You may recall that you spent considerable time in both your introductory and intermediate mechanics course studying harmonic oscillators. As you were probably told, the reason is not that we study harmonic oscillators for their own sake, but because they are a good model for how anisotropic materials behave. As you might guess, the effective spring constant determines how the material responds to an electric field. For this model, both the direction of propagation of the wave and the polarization of the electric field determine this behavior. Typically, these materials are crystalline so this anisotropic behavior is observable. For the figure shown, the electric field given by E_x will travel slightly faster through the material than the E_y component. We know that the difference in speed is related to the difference between n_f and n_s. The optical path difference between the two waves is given by





$$OPD = |n_f - n_s| d,$$

where d is the thickness of the material. How do we relate the *OPD* to the phase difference between the waves, what we have called ϵ? When the wave travels one wavelength λ, the phase has changed by 2π. Therefore, we might expect that the relationship between phase difference and *OPD* is given by

$$\frac{OPD}{\lambda} = \frac{\epsilon}{2\pi}.$$

Suppose we wanted to create a phase difference of $\pi/2$. That means that

$$\frac{OPD}{\lambda} = \frac{\pi/2}{2\pi} = \frac{1}{4} \text{ so } OPD = \frac{\lambda}{4} = |n_f - n_s| d.$$

For a specific wavelength of light, we can adject the thickness of the material to give a quarter wave plate is we know the values of n_f and n_s.

NEXT TIME: Birefringence, optic axes, ordinary and extraordinary indices of refraction, and examples, Jones vectors and Jones matrices description of polarization.