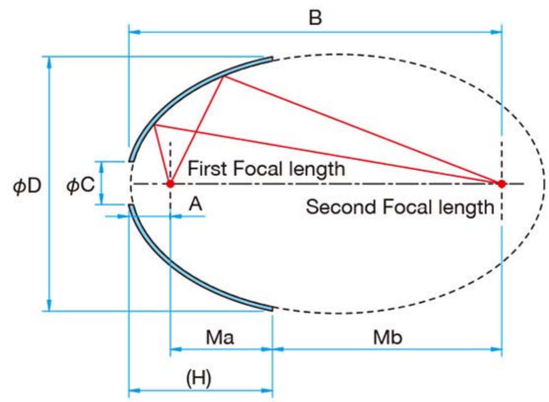
Physics 3312 Lecture 8 February 11, 2019

LAST TIME: Aberration function, reducing spherical aberration, Seidel aberrations, introduction to waves

Before we continue our study of waves, I told you we would have a look at some of the aberrations and how they relate to the equations that we wrote. We have also discussed the role of mirrors in optical systems, so before we start to look at some of the aberrations, I want to show you some interesting ways of using mirrors.

Do you recall some of the advantages and disadvantages of mirrors in optical systems? Suppose we want to collect as much light as we can from a source. Using an ordinary lens, we will intercept only a small portion of the light that is available. Consider the following ellipsoidal mirror. Notice how much more light we collect using this mirror. Here is a short video that shows this great efficiency on a smaller scale. Show video disc 22 ch 28 videos 03 and 04.

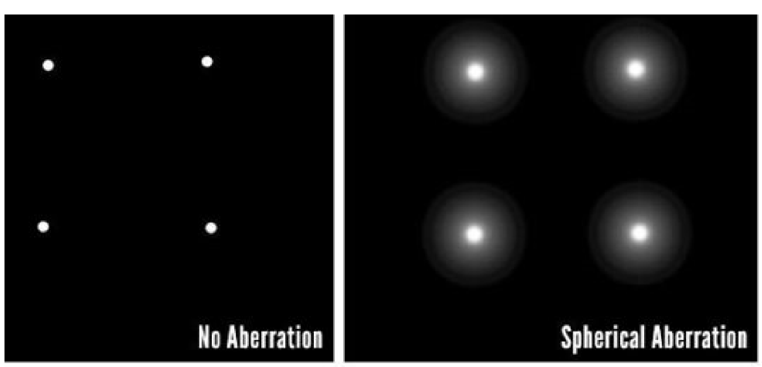


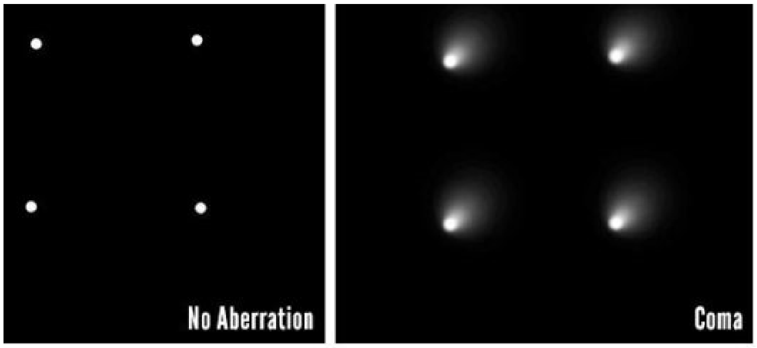

Now let's return to our Seidel aberrations and look at some of the actual aberrations to see how their behavior compares with what we had. Recall the Seidel aberrations are given by the following equation.

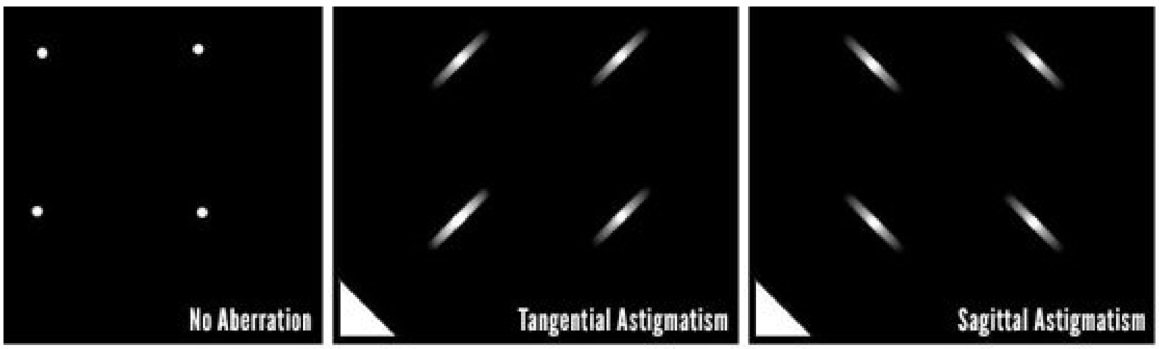
$$a(Q) = \;_0C_{40}r^4 + \;_1C_{31}h'\,r^3\cos\theta + \;_2C_{22}h'^2r^2\cos^2\theta + \;_2C_{20}h'^2r^2 + \;_3C_{11}h'^3r\cos\theta.$$

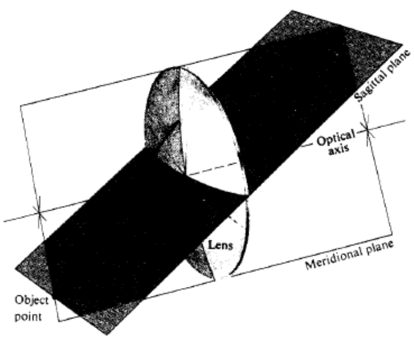
First term is spherical aberration – no angular dependence
Second term is coma
Third term is astigmatism
Fourth term is curvature of field
Fifth term is distortion

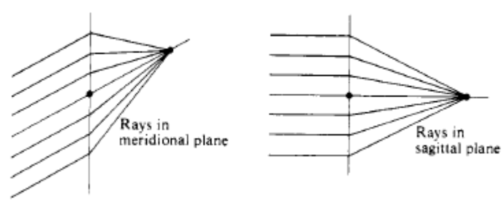
Here are some figures that show some of these aberrations.

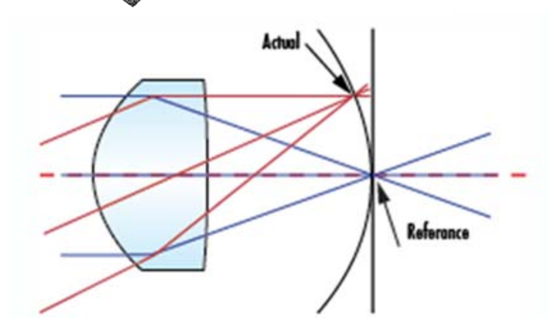


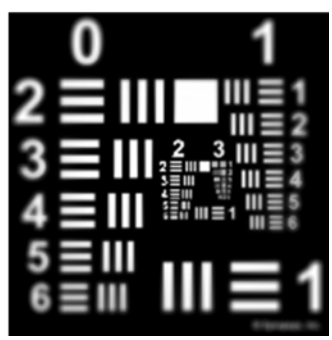


Curvature of field
Show disc 22 chap 60 lenses.

There are also some good figures in your textbook that might be useful.

Let's resume our study of waves. We saw that a traveling wave in one dimension could be represented by a wave function of the form given by

$$\psi(x,t) = f(x \pm vt).$$

You will finish as homework the calculation I started and show that the one-dimensional wave equation is given by

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = 0.$$

This equation is know as the one-dimensional wave equation without energy loss. This equation is a second order, linear partial differential equation. Second order refers to the highest order derivative occuring in the equation, and linear means that no cross terms of the function with itself or its derivatives occurs. For equations that are linear, the sum of any solutions is also a solution to the equation. Note that this is a homogeneous partial differential equation; *i.e*., there is no source term on the right hand side. The most useful form of a wave to consider for our purposes in this course is the harmonic wave given by

$$\psi(x,0) = A \sin kx = f(x).$$

This means that our traveling wave has the form $\psi(x,t) = A \sin k(x \pm vt) = f(x \pm vt)$. Because sine functions are repetitive, we know that there is some distance over which the wave repeats itself. Let's see what that distance is.

$$\psi(x+\lambda,t) = A \sin[k(x+\lambda) \pm kvt] = A \sin(k(x \pm vt) + k\lambda).$$

We know that the sine function is periodic $2\pi$, so $k\lambda = 2\pi$ and $k = \frac{2\pi}{\lambda}$. $k$ is called the propagation number and $\lambda$ is the spatial period or wavelength. There is also a temporal period and a frequency to go along with it.

$$\psi(x,t) = \psi(x,t+\tau) \implies \sin k(x - vt) = \sin k[x - v(t \pm \tau)]$$

Therefore, $\sin(kx - kvt) = \sin(kx - kvt \mp kv\tau)$. This means that $kv\tau = 2\pi = \frac{2\pi}{\lambda}v\tau$, so $v = \frac{\lambda}{\tau} = \lambda\nu$, where $\nu$ is the temporal frequency measured in Hertz. It is customary to define an angular frequency $\omega$ given by $\omega = 2\pi\nu$, measured in radians per second. We also find it useful to define $\kappa = \frac{1}{\lambda}$ = wave number or spatial frequency. It has units of inverse length. Finally, we usually write our periodic wave function as $\psi(x,t) = A\sin(kx \mp \omega t)$. $(kx \mp \omega t)$ is called the phase of the wave. When k, w, and t are constant, x = constant implies a place parallel to y-z. This is called a plane wave because the surface of constant phase are planes. Notice that x and t may change with the phase remaining constant. Then we find $k\,dx \mp \omega dt = 0$. $\frac{dx}{dt} = v_p = v = \frac{\omega}{k}$. For a typical light wave in the middle of the visible spectrum, let's get some idea of the size of some of these numbers. Set $\lambda = 550$ nm and use $v = c = 3 \times 10^8 \frac{\text{m}}{\text{s}}$. Therefore, $\nu = \frac{3\times10^8}{550\times10^{-9}} = 5.45 \times 10^{14}$ Hz. This gives a period of $1.83 \times 10^{-15}$ s. $\omega = 3.42 \times 10^{15}$ rad/s.

$k = 1.14 \times 10^7$ m$^{-1}$. $\kappa = 1.80 \times 10^6$ m$^{-1}$

Recall that because the wave equation is linear, we may add two solutions to get another solution.

$$\psi(x,t) = \psi_1(x,t) + \psi_2(x,t).$$

This is the superposition principle and forms the basis for much of what we are able to do in physics. Interference, diffraction, and polarization all rely on this principle. However, rather than try to add sines and cosines, it is much easier to work with the complex form of a wave. We use the representation given by

$$e^{i\theta} = \cos\theta + i\sin\theta \Longrightarrow \cos\theta = Re\ e^{i\theta}.$$

We may then write $\psi(x,t) = Re\left[Ae^{i(\omega t - kx + \epsilon)}\right]$ or just $\psi(x,t) = Ae^{i(\omega t - kx+\epsilon)}$. Here $\omega t - kx + \epsilon$ is the phase of the wave, and $\epsilon$ is the phase constant. Reminder

$$\bar{z} = re^{i\theta} = r\cos\theta + i\,r\sin\theta = x + iy$$

The complex conjugate of $\bar{z}$ is designated as $\bar{z}^* = (x+iy)^* = x - iy$. Therefore

$$\bar{z} + \bar{z}^* = 2x = 2\,Re\,z \ \text{ and } \bar{z} - \bar{z}^* = 2iy = 2i\,Im\,\bar{z}.$$

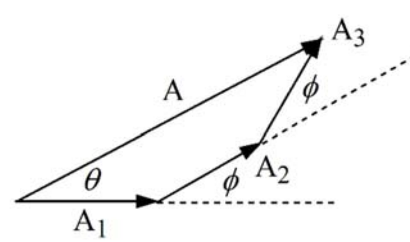
Recall further that

$$r = |\bar{z}| = \sqrt{(\bar{z}\bar{z}^*)},$$

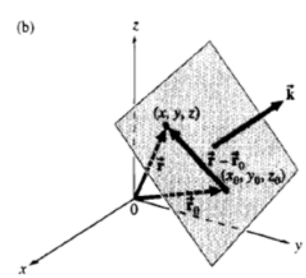
but

$$\bar{z}^2 = (x+iy)(x+iy) = x^2 - y^2 + 2ixy.$$

Adding waves is mostly about keeping track of phase. An easy way to represent addition of waves is by the use of phasors. A phasor is just a vector whose angle represents the phase of the wave relative to another wave. For example, if we wish to represent the addition of three waves, here is how it works. The use of phasors will become more evident when we deal with interference and diffraction.




So far, we have considered only waves whose direction of propagation was along a coordinate axis. How do we deal with waves that propagate om am arbitrary direction? Here is a figure from your text that illustrates how this is done. Remember that a plane may be represented by a vector whose direction is perpendicular to the plane.




$$(\mathbf{r} - \mathbf{r}_o)\cdot\mathbf{k} = 0 \Longrightarrow \mathbf{k}\cdot\mathbf{r} = \text{const}$$

We may then write

$$\psi(x,t) = Ae^{i(\mathbf{k}\cdot\mathbf{r} \mp \omega t)}$$

NEXT TIME: Spherical waves, cylindrical waves, and Maxwell's equations