Physics 3312 Lecture 4 January 28, 2019

LAST TIME: Types of lenses, sign conventions, magnification, multiple lenses, ray tracing

We should note that the determinant of each of these matrices is 1. $\det\mathcal{R} = \det\begin{bmatrix}1 & -D\\ 0 & 1\end{bmatrix} = 1$ $\det\mathcal{T} = \det\begin{bmatrix}1 & 0\\ \frac{d}{n} & 1\end{bmatrix} = 1$. These two conditions imply that det $\boldsymbol{a}$ =1 as well. Do you remember why?

We need to consider one last issue before we look at some other topics in chapter 5. Where do off-axis rays focus? Here is how the matrices look.

$$\begin{bmatrix}\alpha_i\\ y_i\end{bmatrix} = \begin{bmatrix}1 & 0\\ s_i & 1\end{bmatrix}\begin{bmatrix}1 & -\frac{1}{f}\\ 0 & 1\end{bmatrix}\begin{bmatrix}\alpha_o\\ y_o\end{bmatrix} = \begin{bmatrix}1 & -\frac{1}{f}\\ s_i & -\frac{s_i}{f}+1\end{bmatrix}\begin{bmatrix}\alpha_o\\ y_o\end{bmatrix} = \begin{bmatrix}\alpha_o - \frac{y_o}{f}\\ \alpha_o s_i + y_o\left(1-\frac{s_i}{f}\right)\end{bmatrix}$$

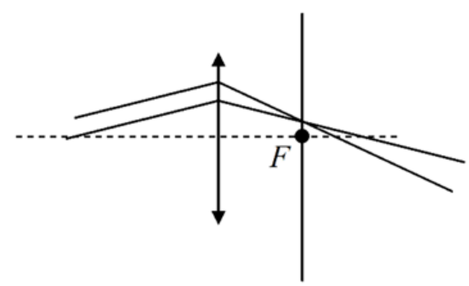
If $y_o = 0, \alpha_i = \alpha_o$ and $y_i = \alpha_o s_i$. For any other $y_o, \alpha_i' = \alpha_o - \frac{y_o}{f}$ and $y_i' = \alpha_o s_i + y_o - \frac{y_o s_i}{f}$.

We want to know the value of s_i such that $y_i' = y_i$. (The two parallel rays match.)

$$y_i' - y_o = \alpha_o s_i - \frac{y_o s_i}{f} = s_i\left(\alpha_o - \frac{y_o}{f}\right)$$

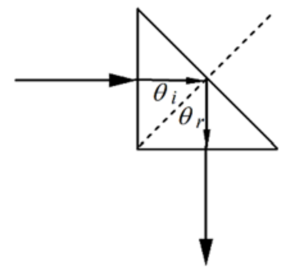
$$\alpha_o s_i - y_o = s_i\alpha_o - s_i\frac{y_o}{f} \Longrightarrow s_i = f.$$

Here is a figure to illustrate what we have proved for small angles. The two parallel rays come together at the focal plane even if they are not parallel to the optical axis.

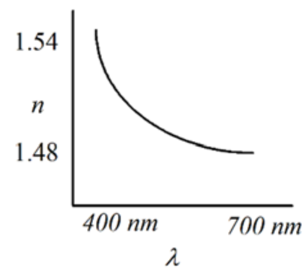


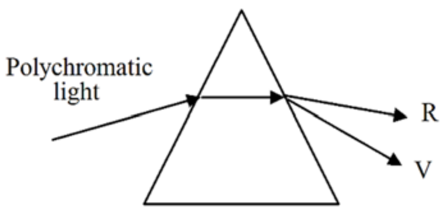
Prisms usually are considered as either reflecting or dispersing. We have already discussed reflecting prisms to some degree and have seen that they are based on total internal reflection, but what is dispersion? Dispersion is the dependence of the index of refraction n on the wavelength λ (color). In other words, $n = n(\lambda)$.

Consider reflecting prisms first. Recall our earlier discussion where the critical angle for total internal reflection occurs when we go from a material has an index of refraction larger than the material into which it transmits. We found that $\theta_c = \sin^{-1}\frac{n_t}{n_i}$ with $n_t < n_i$. θ_c for a glass –air interface is about 41.8°. There prisms are usually used whenever we want to redirect light without much loss in intensity. See text for more information.



Dispersing prisms use the dependence of the refractive index on wavelength for their operation. Here is a rough sketch of how that dependence looks in the visible region (400 nm to 700 nm). Rainbows depend on this and reflection. Here is a figure that shows how polychromatic light is dispersed on an equilateral triangular prism.





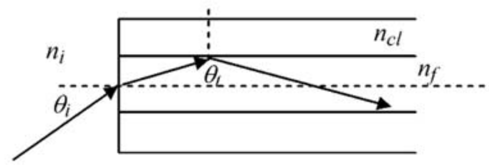
We may give an empirical equation, Cauchy equation, to describe this dependence by the equation

$$n = C_1 + \frac{C_2}{\lambda^2} + \frac{C_3}{\lambda^4} + \cdots$$

The larger the variation in the index of refractive index, the more the dispersion. Highly dispersive prisms are used in prism spectrometers, devices to study the wavelength dependence of the refractive index in materials. Such studies are useful in characterizing materials. We will say more about these topics when we get to the EM theory.

Optical fibers

Here is the ray optic description of who they work.



$$n_i \sin\theta_i = n_t \sin\theta_t$$

$$n_i \sin\theta_i = n_f \sin\theta_t$$

At the core-cladding boundary, we apply Snell's law again.

$$n_f \sin(90^o - \theta_t) = n_{cl}\sin\theta_{tcl} \Rightarrow n_f\cos\theta_t = n_{cl}\; if\; \theta_{tcl} = 90^0$$

We wish to know the maximum θ_i for TIR at the core-cladding interface. Therefore,

$$n_i \sin\theta_{\max} = n_f \sin\theta_t \;\text{ with}\; \frac{n_{cl}}{n_f} = \cos\theta_t$$

$$(n_i \sin\theta_{\max})^2 = \left(n_f \sin\theta_t\right)^2 \Rightarrow n_i^2 \sin^2\theta_{\max} = n_f^2(1-\cos^2\theta_t).$$

Finally, $n_i^2\sin^2\theta_{\max} = n_f^2\left(1-\left(\frac{n_{cl}}{n_f}\right)^2\right) = n_f^2 - n_{cl}^2$ and $n_i\sin\theta_{\max} = \left[n_f^2 - n_{cl}^2\right]^{\frac{1}{2}}$.

$n_i \sin\theta_{\max}$ is usually called the numerical aperture NA of the fiber and is the maximum angle at which light can enter and still remain in the fiber by TIR.

A second important parameter of the optical fiber is the attenuation per length or energy loss per length as the light transmits along the fiber.

$$\frac{P_0}{P_i} = 10^{-\alpha L/10}$$

so that

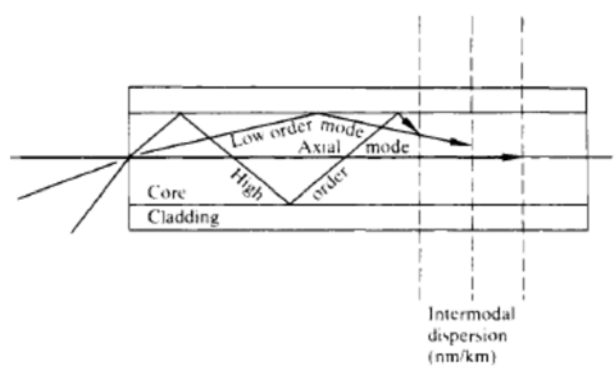
$$\log\frac{P_0}{P_i} = \frac{-\alpha L}{10} \text{ and } \alpha = -\frac{10}{L}\log\frac{P_0}{P_i}.$$

If $\frac{P_0}{P_i} = 0.5$ in 1 kilometer,

$$\alpha = -\frac{10}{1\text{ km}}\log 0.5 = -10(-0.301) = 3\frac{\text{dB}}{\text{km}}.$$

The first optical fibers that achieved an attenuation of less than 20 dB/km were made back in the early 1970s and that really started the communications revolution. The two primary mechanisms for energy loss are scattering and absorption. Rayleigh scattering is proportional to $1/\lambda^4$, so the push initially was to create communication systems with longer wavelengths. Absorption was primarily caused by the OH ion, so eliminating impurities in the fibers was critical. Present fibers can be made with attenuation of 0.1 dB/km.

The last major consideration in making an optical fiber is dispersion, but this dispersion is primarily what is known as intermodal dispersion. In the ray picture, each ray propagating at a different angle in the fiber really represents a mode in the fiber in the wave picture. Rays that travel along the center of the fiber reach the end of the fiber before those that travel at larger angles in the fiber as shown in the figure. Your text shows that this dispersion measured in time is approximately given by



$$\Delta t = \frac{Ln_f}{c}\left(\frac{n_f}{n_{cl}} - 1\right)$$

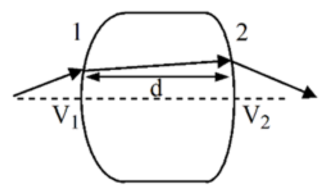
The need for a cladding around the fiber is clear when you consider what happens if the core is surrounded by air. Then,

$$\Delta t = \frac{1(1.5)}{3\times10^8}\left(\frac{1.5}{1} - 1\right) = \frac{(2\times10^{-9})\text{s}}{\text{m}} = \frac{(2\times10^{-6})\text{s}}{\text{km}}.$$

This means that a pulse will spread out in time 2 microseconds every kilometer it travels. In 2 microseconds, light travels a distance $2\times10^{-6}(3\times10^8) = 600$ m. You cannot propogate any decent signal speed with this arrangement.

This means we must make the cladding and core have indices of refraction quite close. This limits the numerical aperture but improves the rate at which digital signals can be sent along the fiber without overlapping one another. See your text on pages 199 – 200 for more information. Elaborate a bit on this idea.

Thick lenses – The figure to the right shows our situation with a thick lens. We no longer set $d = 0$ in our transfer equation, so here is the matrix multiplication to get the system matrix for the thick lens case with the lens assumed to be in air.



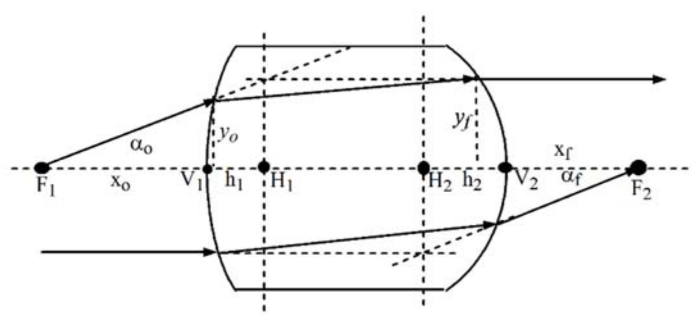
$$\mathcal{A}_{21} = \mathcal{R}_2\, \mathcal{T}_{21}\, \mathcal{R}_1 = \begin{bmatrix} 1 & -D_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{d_l}{n_l} & 1 \end{bmatrix} \begin{bmatrix} 1 & -D_1 \\ 0 & 1 \end{bmatrix}$$

$$D = \frac{n_t - n_i}{R} \Rightarrow D_1 = \frac{n_l - 1}{R_1} \text{ and } D_2 = \frac{1 - n_l}{R_2}$$

For this case, ths system matrix becomes

$$\begin{bmatrix} 1 - \dfrac{D_2 d_l}{n_l} & D_1 D_2 \dfrac{d_l}{n_l} - D_1 - D_2 \\ \dfrac{d_l}{n_l} & 1 - \dfrac{D_1 d_l}{n_l} \end{bmatrix}$$

We could create the object-image matrix and apply the imaging condition, but this approach does not yield any simple results. There is a much better way to deal with thick lenses. As you recall, with thin lenses, we assumed that the refraction was concentrated at the middle of the lens. This is not possible to do with thick lenses, but it is possible to find two points where the refraction at each surface seems to take place Here is a figure to illustrate this idea. Let's identify F_o and F_i and F_1 and F_2 since everything else is in terms of 1 and 2. Furthermore, make the following assignments.



$V_1H_1 = h_1 \quad F_1H_1 = f_1 \quad V_2H_2 = h_2 \quad F_2H_2 = f_2 \quad x_o = F_1V_1 \quad x_f = F_2V_2.$

Therefore, $f_1 = x_o + h_1$ and $f_2 = x_f + h_2$. Let's start a ray from F_1 at angle α_o and see what happens to it using our matrix approach. Transfer the ray to V_1.

$$r_o = \begin{bmatrix} \alpha_o \\ 0 \end{bmatrix}$$

$$r_o' = \begin{bmatrix} 1 & 0 \\ x_o & 1 \end{bmatrix} \begin{bmatrix} \alpha_o \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha_o \\ \alpha_o x_o \end{bmatrix} = \begin{bmatrix} \alpha_o \\ y_o \end{bmatrix} \Longrightarrow y_o = \alpha_o x_o$$

Now consider the final ray that is given by

$$r_f = \begin{bmatrix} 0 \\ y_f \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \alpha_o \\ y_o \end{bmatrix} = \begin{bmatrix} a_{11}\alpha_o + a_{12}y_o \\ a_{21}\alpha_o + a_{22}y_o \end{bmatrix}.$$

Therefore, $0 = a_{11}\alpha_o + a_{12}y_o$ and $y_f = a_{21}\alpha_o + a_{22}y_o$.

But $f_1 = \frac{y_f}{\alpha_o} = \frac{a_{21}\alpha_o + a_{22}y_o}{\alpha_o}$ and $y_o = -\frac{a_{11}\alpha_o}{a_{12}}$ so $f_1 = \frac{a_{21}\alpha_o}{\alpha_o} - \frac{a_{22}a_{11}\alpha_o}{a_{12}\alpha_o}$

Finally, $f_1 = -\frac{a_{11}a_{22} - a_{21}a_{12}}{a_{12}} = -\frac{1}{a_{12}}$

Now notice that we may also obtain h_1 and h_2 in terms of the elements of the system matrix.

$$h_1 = f_1 - x_o = -\frac{1}{a_{12}} - \frac{y_o}{\alpha_o} = -\frac{1}{a_{12}} - -\frac{a_{11}}{a_{12}} = \frac{1 - a_{11}}{-a_{12}}.$$

Similarly,

$$h_2 = \frac{a_{22} - 1}{-a_{12}}$$

NEXT TIME: What all this means and examples.