

Some practice using the Kronecker delta and the Levi-Civita symbol.

Notice that if \hat{e}_i and \hat{e}_j are unit vectors in an orthogonal coordinate system, then $\hat{e}_i \cdot \hat{e}_j = \delta_{ij}$.

If no repeated indices occur in the Levi-Civita symbol, then the following relationship holds.

$$\epsilon_{kil}\epsilon_{mpq} = \begin{vmatrix} \delta_{km} & \delta_{im} & \delta_{lm} \\ \delta_{kp} & \delta_{ip} & \delta_{lp} \\ \delta_{kq} & \delta_{iq} & \delta_{lq} \end{vmatrix}$$

$$\frac{\partial r_j}{\partial x_k} = \partial_k r_j = \delta_{kj}$$

Prove that $\delta_{kk} = 3$. The repeated index implies a summation, so in 3 dimensions,

$$\delta_{kk} = \delta_{11} + \delta_{22} + \delta_{33} = 3$$

Prove that $\delta_{ij}\epsilon_{ijk} = 0$. Note that $\delta_{ij}\epsilon_{ijk} = \epsilon_{iik} = 0$ since an index is repeated.

Prove that $\epsilon_{ijk}\epsilon_{ijk} = 6$. $\epsilon_{ijk}\epsilon_{ijk} = \delta_{jj}\delta_{kk} - \delta_{jk}\delta_{kj} = 9 - \delta_{kk} = 9 - 3 = 6$.