Physics 6303 Homework Set 7 October 10, 2018

Due: Wednesday, October 24, 2018

1. A stretched string of length L is tied at one end (x = 0) so that it cannot move and at the other end (x = L), it is held by a massless ring that is free to slide up and down on a rod without friction. (a) Show that the boundary condition at x = L is given by $\frac{\partial y(L)}{\partial x} = 0$. (You may use Newton's 2nd law to do this.) (b) Use the one-dimensional wave equation, $\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$, and separation of variables with separation constant $-k^2$ to obtain a general solution for the motion of the string. Specifically, find the eigenfunctions and the eigenvalues.

2. A right circular cylinder having length L and radius R has its top end and curved side held at zero potential (grounded). The bottom end is held at a constant potential V_o. Determine the potential everywhere inside the cylinder.

3. Evaluate $\int_{-\infty}^{+\infty} e^{|x|}\,\delta(x^2 + 2x - 3)dx$. You should be able to obtain a numerical result. Please note that you have here a Dirac delta function of a function.

4. (a) A uniform disk having radius R and mass M lies in the x-y plane. Express its mass density in terms of Cartesian coordinates. (b) Current I exists in a circular loop of wire having radius R in the x-y- plane with its center at the origin. Write an expression for the current density in spherical coordinates. (c) Show that your current density in part (b) integrates to the correct current I.

5. Consider the boundary value problem $\frac{d^2 y}{dx^2} = f(x)$ with $y(0) = 0$ and $\frac{dy(1)}{dx} = 0$. Determine the Green function for this problem and, using this Green function, determine the solution when $f(x) = x^2$.

6. (a) Find the Green function for the one-dimensional Poisson equation $\frac{d^2 V}{dx^2} = -\frac{\rho(x)}{\epsilon_o}$, subject to the boundary conditions $V(x) = 0$ at $x = 0$ and $x = a$. (b) Use the Green function to calculate $V(x)$ when $\rho(x) = \sin\frac{\pi x}{a}$.