Physics 6303 | Homework Set 6 | October 3, 2018

Due: Wednesday, October 10, 2018

1. You know that one way of writing the solution to the ODE given by $\frac{d^2x}{dt^2} + \omega_o^2 x = 0$ is

$$x(t) = A \sin \omega_o t + B \cos \omega_o t.$$

Suppose, however, that you did not know anything about the elementary functions sine and cosine, and solve the equation using a series solution method. Show that your series matches the series representation for the sine and cosine functions.

2. Two concentric spheres have their centers at the origin. The inner sphere has radius a and is held at zero potential. The outer sphere has radius b and is held at potential $V(\theta) = V_o \sin^2\theta$. (a) Express $V(\theta)$ in terms of the Legendre polynomials $P_o(\cos\theta)$ and $P_2(\cos\theta)$. (b) Find the potential between the spheres.

3. A sphere having radius R is centered at the origin and has potential $V_o \sin^2\theta \cos 2\phi$ on its surface. (a) Show how to write the potential in terms of spherical harmonics. (b) Determine the potential inside and outside the sphere.

4. A long cylinder is cut in half so that the upper part ($0 < \phi < \pi$) can be held at a potential $-V_o \sin\theta$, and the lower part ($\pi < \phi < 2\pi$) can be held at a potential $+V_o$ as shown in the figure. Let R be the radius of the cylinder. Write the most general solution to Laplace's equation for this geometry. (b) Assuming that you are to determine the potential outside the cylinder, state which constants in the general equation must be zero and why. (c) Determine the potential outside the cylinder.

5. Rodrigues' formula for Legendre polynomials is given by $P_\ell(\mu) = \frac{1}{2^\ell \ell!}\frac{d^\ell}{d\mu^\ell}(\mu^2-1)^\ell$. Show that the first 3 Legendre polynomials, $\ell = 0$ to 2, are correctly given by this formula.