Physics 6303 | Homework Set 4 | September 12, 2018

Due: Wednesday, September 19, 2018

1. If A^{ij} is symmetric, show that $A_{ij} = A_{ji}$.

2. The contravariant form of Newton's second law in a general coordinate system is given by

$$F^i = m\left(\ddot{x}^i + \Gamma^i_{kj}\dot{x}^k\dot{x}^j\right),$$

where Γ^i_{kj} is the Christoffel symbol of the second kind. (a) Use the metric for plane polar coordinates and its connection to the Christoffel symbols to write Newton's second law in plane polar coordinates. (b) Explain the physical meaning of the two components.

3. Show that the product of a matrix $A = \begin{bmatrix} a & b \\ \frac{-a^2}{b} & -a \end{bmatrix}$ with itself is zero. What conclusions do you draw from this observation?

4. Recall that we defined the cross product of two vectors by $(\mathbf{U} \times \mathbf{V})_i = \epsilon_{ijk}U_jV_k$. (a) Show that the tensor given by $T_{ij} = U_iV_j - V_iU_j$ can represent the components of the cross product. (b) Write the tensor in matrix form.

5. Use the expressions for $A^i{}_j = \frac{\partial \bar{x}^i}{\partial x^j}$ and $B^j{}_i = \frac{\partial x^j}{\partial \bar{x}^i}$ to show that matrix $\mathbb{B}^T = \mathbb{A}^{-1}$ in general. Note that the index that appears first is the row index, and the second index is the column. This convention allows us to continue to use the same notation that we have used when all indices are up or down. Do not make this hard – it is not.

6. In class, I showed you how to transform the electric field from a long, straight wire using the transformation for a contravariant electric field vector. Use the value I gave you for transforming the field as if it were a covariant vector to carry out the same calculation. Comment on your results.