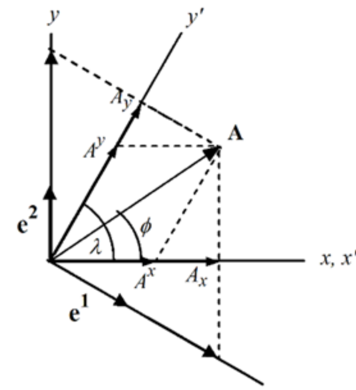
Physics 6303 Homework Set 3 September 5, 2018

Due: Wednesday, September 12, 2018

1. We will use the nonorthogonal coordinate system one more time. Here is the figure I showed in class when we developed the dual basis vector system. Remember that I wrote the basis vectors for the contravariant components and the dual basis vectors for the covariant components. You will find these in my notes on page 3 of Lecture 4. (a) Calculate the metric tensor $g_{ij} = \mathbf{e}_i \cdot \mathbf{e}_j$. (b) Calculate $g^{ij} = \mathbf{e}^i \cdot \mathbf{e}^j$. (c) Show that g^{ij} is the inverse of g_{ij}.



2. (a) From the figure, determine the covariant components of **A** using the angles given in the figure. (b) Using the expressions for g^{ij}, determine the values for the contravariant components of **A**. (c) Using the values determined here for A^i and A_i, obtain A_iA^i.

3. Let the following set of components be defined in a two-dimensional Cartesian space:

$$A_{ij} = \begin{bmatrix} -y^2 & xy \\ xy & -x^2 \end{bmatrix}$$

(a) Does this set of coordinates transform as a tensor? (b) Explain. *Hint*: Try to represent this expression in terms of objects in index form that you already know are tensors. Consider the following set of components, again in two-dimensional Cartesian space:

$$B_{ij} = \begin{bmatrix} x^2 & xy \\ xy & y^2 \end{bmatrix}$$

(c) Does this set of coordinates transform as a tensor? (d) Explain.

4. Recall that we wrote the electric quadrupole tensor as $Q_{ij} = \int (3\, x_i x_j - r^2 \delta_{ij}) \rho(\mathbf{x}) dV$. (a) From observation alone, how do you know this is a tensor? (b) Write Q_{xz} and Q_{zz}.

5. (a) Determine the eigenvalues of the matrix given by

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

(b) Determine the normalized eigenvectors and the angle between them. (c) Why is the angle not 90 degrees?

6. For the triangular mass whose moment of inertia tensor I_{ij} I found in class, determine the principal moments of inertia and the principal axis coordinate system. See Lecture 5B.