

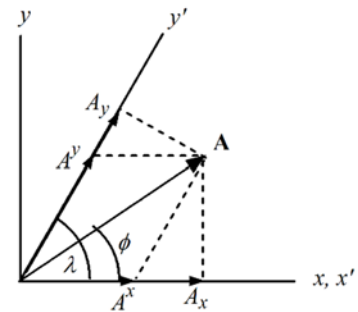
Due: Wednesday, September 5, 2018

1. Consider the linear transformation of vector components given by $\begin{bmatrix} A'_x \\ A'_y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} A_x \\ A_y \end{bmatrix}$. By the way, such 2-D transformations are called *affine transformations* and generally stretch, skew, reflect, or invert the objects being transformed. They are useful in computer graphics and modeling. (a) For what conditions on a , b , c , and d is this a distance preserving transformation? (b) Does the rotation matrix satisfy these conditions? (c) Calculate the inverse of the rotation matrix. (d) Calculate the determinant of the rotation matrix.

2. A particle moves in a circle of radius R at a constant speed v . Without writing components of vectors, (a) prove that the velocity is perpendicular to the position vector, (b) prove that the velocity is perpendicular to the acceleration, and (c) prove that the acceleration is oppositely directed to the position vector and has magnitude $\frac{v^2}{R}$.

3. (a) Calculate $\nabla \cdot (\mathbf{E} \times \mathbf{B})$ in terms of curl \mathbf{E} and curl \mathbf{B} . (b) What is the physical significance of the term $(\mathbf{E} \times \mathbf{B})$? You will use this identity frequently in electrodynamics.

4. Consider once more the nonorthogonal coordinate system with vector \mathbf{A} represented as before. Using only algebra, geometry, and trigonometry, show that $A^2 = A_x A^x + A_y A^y$.



5. A solid object rotates with angular velocity $\boldsymbol{\omega}$. (a) Using cylindrical coordinates with the z -axis along the rotation axis, find the components of the velocity vector \mathbf{v} at an arbitrary point within the body. (b) Use the expression for the curl in cylindrical coordinates to evaluate $\nabla \times \mathbf{v}$. Comment on your answer.

6. Following the procedure we used to derive the general expression for the divergence in curvilinear coordinates, derive the general expression for the curl in curvilinear coordinates.

7. Consider the following matrix given by

$$\begin{pmatrix} \cos \theta & \sin \theta & 0 \\ \sin \theta & -\cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Does this matrix represent a rotation of the coordinate axes? If not, what transformation does it represent?