Physics 6303 | Homework Set 1 | August 20, 2018

Due: Wednesday, August 29, 2018

1. In class, we did not consider the successive application of the scalar product and the vector product. (a) Show that $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ may be written as a determinant with the components of each vector forming a row of the determinant. (b) What is the geometrical interpretation of this product? This product is the triple scalar product.

2. Suppose that a surface is defined by $\varphi(x, y, z) = c$, where c is a constant. Show that $\boldsymbol{\nabla}\varphi(x, y, z)$ is perpendicular to the surface.

3. A point charge Q is at rest at the origin of a coordinate system. A square surface, $2a \times 2a$, in the y-z plane is located a distance a from the origin and is centered on the x-axis. (a) Calculate the electric field flux through the square surface by working out the surface integral. (b) Based on your result, state a far simpler way to determine the flux of the electric field through this surface.

4. (a) Calculate the line integral $\oint \mathbf{F} \cdot d\boldsymbol{\ell}$, with $\mathbf{F} = x^3 y^2 (\hat{\mathbf{x}} + \hat{\mathbf{y}})$, around a square of side a centered at the origin in the x-y plane. (b) Calculate $\iint (\boldsymbol{\nabla} \times \mathbf{F}) \cdot \mathbf{n} dS$ to verify Stokes' theorem for this case. (c) If $\mathbf{F}$ is a force, is it conservative?

5. Consider the vector field $\mathbf{u} = k\mathbf{r}$, where $\mathbf{r}$ is the usual position vector in Cartesian coordinates. Verify that the divergence theorem holds for this field by calculating $\iiint (\boldsymbol{\nabla} \cdot \mathbf{u}) dV$ and $\iint \mathbf{u} \cdot \mathbf{n} dS$, where S is the surface bounded by a cube of side a centered at the origin.