NAME ______________________ LAB PARTNERS ______________________

Instructor ______________________ ______________________

Volume Measurement and Uncertainty *Experiment* 1

INTRODUCTION

The laws of physics are based on measurements of physical quantities such as length, time, temperature, charge, and current. You should understand two important points as you perform experiments. First, all measurements have some uncertainty in them, and second, it is necessary to know how uncertainties in the basic measurements affect uncertainties in the calculated quantities. In this experiment, you will measure the volume of a plate and the volume of a hollow cylinder with a bottom (a metal cup). You should read the section ***Introduction to Error Analysis*** before you start the experiment.

THEORY

The volume V of a plate is given by $V = LWH$, where L is the length of the plate, W is its width, and H is its thickness. If the uncertainty in each of the measurements of L, W, and H is given by ΔL, ΔW, and ΔH, respectively, then we may write $V + \Delta V = (L + \Delta L)(W + \Delta W)(H + \Delta H)$. If we expand the right-hand side and neglect terms that are multiples of two or more Δs, we find that

$$V + \Delta V = LWH + LW\Delta H + WH\Delta L + LH\Delta W. \qquad (1)$$

We subtract V from each side to obtain

$$\Delta V = LW\Delta H + WH\Delta L + LH\Delta W. \qquad (2)$$

From these results, we obtain the fractional uncertainty in V to be

$$\frac{\Delta V}{V} = \frac{\Delta L}{L} + \frac{\Delta W}{W} + \frac{\Delta H}{H}. \qquad (3)$$

If L, W, and H are measured only one time, then the uncertainty must be estimated in some reasonable way. One general rule frequently used is to estimate the result to one-tenth of the smallest scale division on the measuring device. For a ruler marked off in 1 mm divisions, we could estimate our reading as 1.1 mm. What is meant by this is that we believe our measurement is 1.1 mm ± 0.1 mm. If multiple measurements of a quantity are made, then we may use the standard deviation of the mean as discussed in the section ***Introduction to Error Analysis***.

When one of the quantities measured is raised to a power, then the error analysis is somewhat more complicated. Consider a cylinder having radius R and length L. The volume V is given by $V = \pi R^2 L$. For this case,

$$V + \Delta V = \pi(R + \Delta R)^2(L + \Delta L). \tag{4}$$

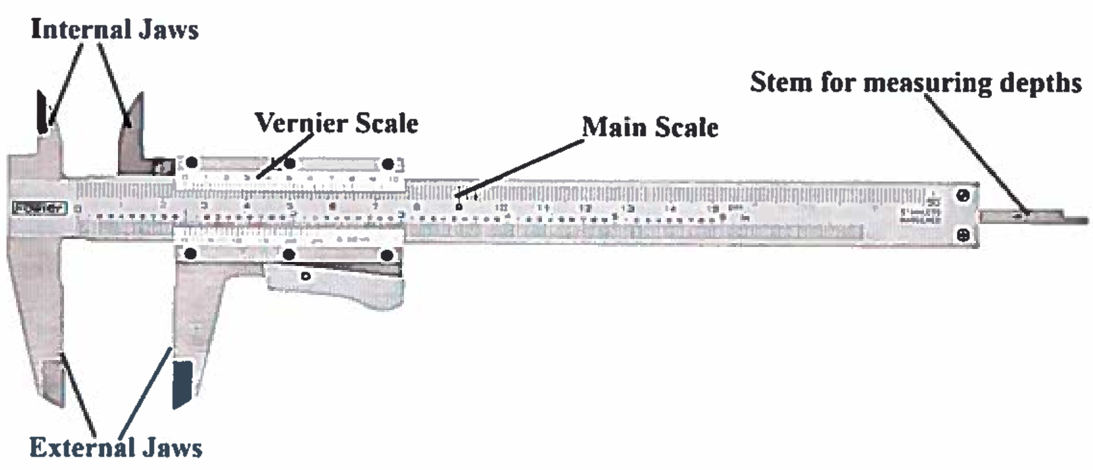
We may follow the procedure above to calculate the ***fractional uncertainty*** in the volume of the cylinder as

$$\frac{\Delta V}{V} = 2\frac{\Delta R}{R} + \frac{\Delta L}{L}. \tag{5}$$

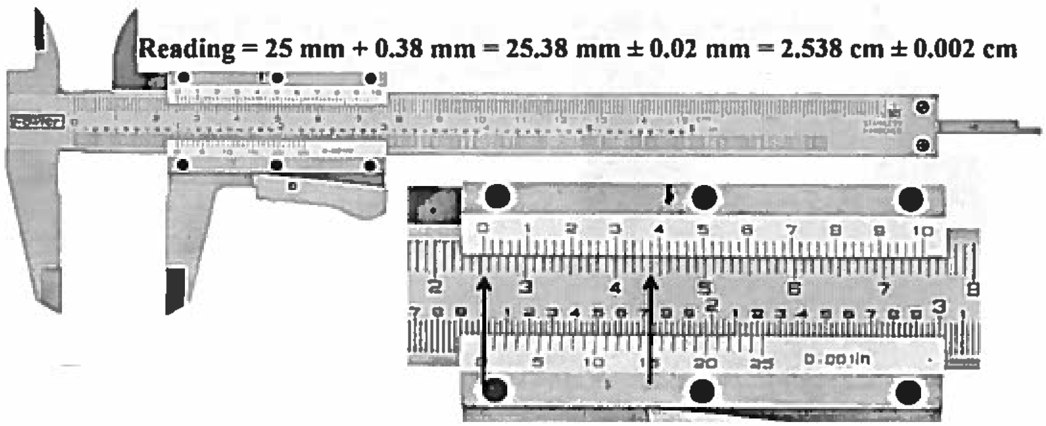
Notice now that the power of R multiplies the fractional uncertainty in R, so extra care must be taken to measure the radius. As a general rule, the power of the quantity multiplies the fractional uncertainty in that quantity. The higher that power, the more carefully that quantity must be measured to minimize the uncertainty in the final result.

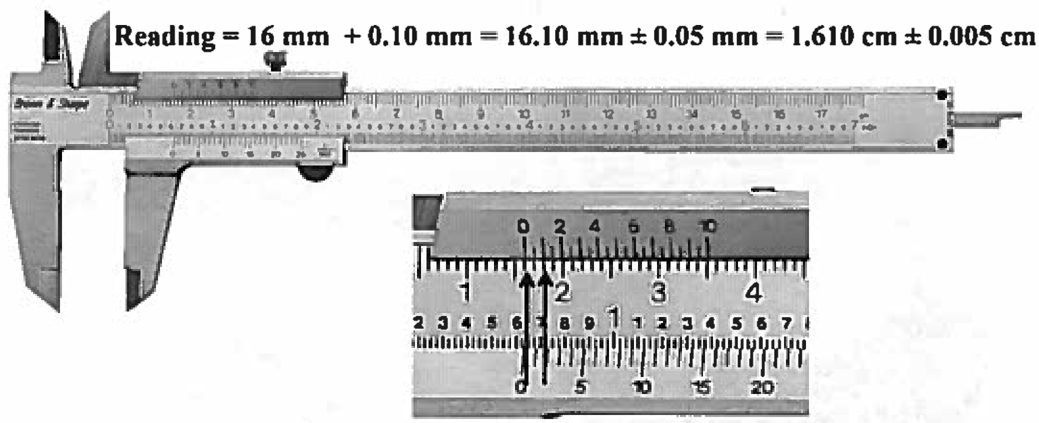
EXPERIMENT NO. 1

1. Before starting the measurements with the Vernier caliper and the micrometer caliper, it is useful to understand better how each of them works. The figure below shows the basic parts of a Vernier caliper similar to the one you use.

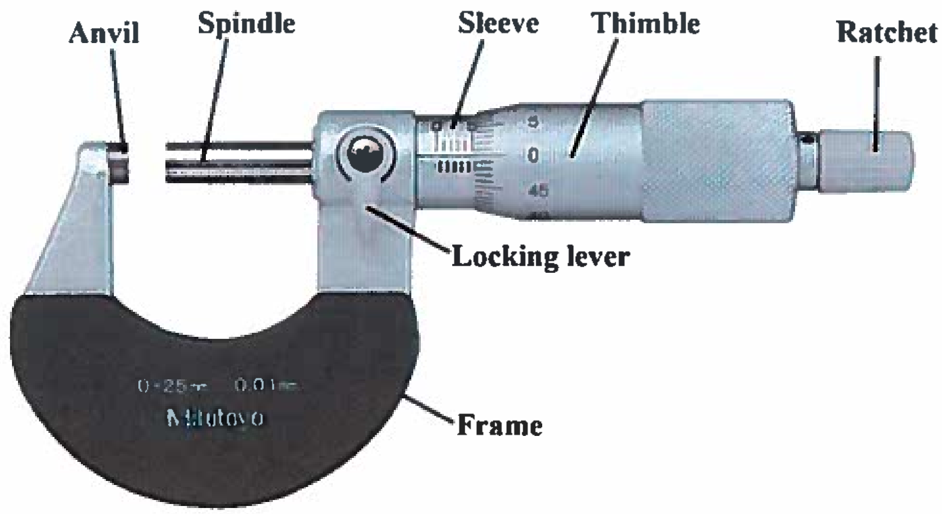


The basic principle of the Vernier caliper is the relationship between the sizes of the divisions on the Vernier scale to those on the main scale. The Vernier scale divides $(N - 1)$ main scale divisions into N divisions on the Vernier scale. The Vernier caliper shown above divides 49 mm on the main scale into 50 divisions on the Vernier scale. As the Vernier scale slides along the main scale, each successive mark will line up with a mark on the main scale after it moves $1/50$ of a mm, or 0.02 mm. Therefore, the Vernier caliper shown above may be read to the nearest 0.02 mm. If the Vernier scale divided 19 mm on the main scale into 20 divisions, then the most precise reading would be 1/20 mm or 0.05 mm. The figures on the next page show how to read a Vernier caliper whose resolution is 0.02 or 0.05 mm.

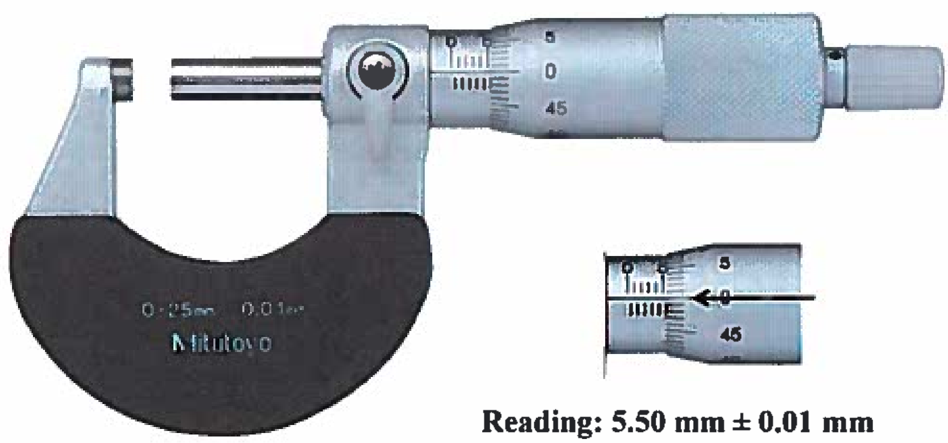




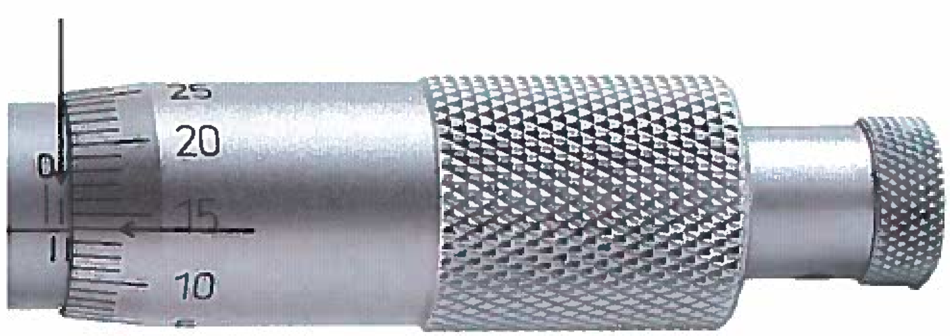
2. The micrometer operates on a slightly different principle. The figure on the next page shows a typical micrometer with the various parts labeled. A typical metric micrometer has a sleeve (barrel) marked off in 0.500 mm steps. The spindle has 50 segments around it and advances 0.500 mm when it is rotated once. This means that each mark on the spindle corresponds to 0.01 mm, so the micrometer may be read to the nearest 0.01 mm. Some micrometers also have an additional Vernier scale, but we will not consider those here.



The figures below show an example of how to read a metric micrometer with a resolution of 0.01 mm.



Reading: 5.50 mm ± 0.01 mm



Reading = 1 mm + 0.5 mm + 0.14 mm = 1.64 mm ± 0.01 mm

3. Each measurement that follows should be made five times and recorded in Table 1.1. Measure the length of the metal plate using the side of the meter stick divided into 10 parts, measure its width using the side of the meter stick with the centimeter scale, measure its thickness using the micrometer, measure its length using the side of the meter stick with the millimeter scale, and measure its width using the Vernier caliper.

Table 1.1: Metal Plate

	Length using meter divided into 10 parts	Width using meter with ½ cm divisions	Thickness using micrometer	Length using meter scale with mm divisions	Width using Vernier caliper
1					
2					
3					
4					
5					
Mean					
Standard Deviation of the mean					
Uncertainty					
Use these columns for	Case 1	Case 1	Case 1		
Use these columns for			Case 2	Case 2	Case 2

4. Calculate the volumes and the uncertainties in the volumes of the metal plate for each of the two cases shown in the table. Note that each of the cases uses different devices for the measurements, so you should expect different uncertainties for the two cases. Show your calculations in the space on the next page.

$Case\ 1\ Volume\ =$ ______________ $Uncertainty\ in\ volume\ =$ ______________

$Case\ 2\ Volume\ =$ ______________ $Uncertainty\ in\ volume\ =$ ______________

How well did your method of measure compensate for the larger uncertainties associated with the instruments specified for *Case* 1? Justify your answer.

5. Measure the outer diameter, inner diameter, outer length, and inner length of the metal cup. Record five measurements for each value in Table 1.2. ***Use the measuring devices that will give the best precision in the volume of the metal cup and indicate which device you used for each measurement in the table.*** Calculate the inner and outer radii and record their values in the table.

Table 1.2: Metal Cylinder

	Outer Diameter	Outer Radius	Outer Length	Inner Length	Inner Diameter	Inner Radius
1						
2						
3						
4						
5						
Mean						
Standard Deviation of the Mean						
Uncertainty						
Instrument Used						

Show all Table 1.2 calculations in the space below.

6. Calculate the outer volume, inner volume, and volume of the metal, showing your calculations in the space below.

$Outer\ Volume$ = ________________ $Uncertainty$ = ________________

$Inner\ Volume$ = ________________ $Uncertainty$ = ________________

$Volume\ of\ Metal$ = ________________ $Uncertainty$ = ________________

QUESTIONS

1. Suppose you tried to measure the thickness of the metal cup directly with the Vernier caliper or the micrometer; *i.e.*, you do not measure the inner and outer radii separately. What problem(s) might arise with such a thickness measurement?

2. Suppose you measured the volume of the metal plate using a device with 0.001 mm precision for all dimensions. What precision in the volume would be achieved?