

## **GENERAL EQUILIBRIUM OF SPATIAL PRODUCT AND LABOR MARKETS\***

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**ABSTRACT.** A simple general equilibrium model relates spatial product markets and spatial labor markets. The firm is treated as being a spatial monopolist or as a Lösschian competitor in the output market and as a spatial monopsonist in the labor market. Derived free spatial demand and free regional labor supply are defined, and their properties examined. The model provides the framework for analyzing the impact of a technological improvement in labor productivity on the structure of the spatial markets. The impact of entry on spatial labor supply is an important determinant of whether or not entry lowers wages and raises output prices. Unlike the spaceless competitive paradigm, zero-profit long-run equilibrium can occur in a space economy under conditions of increasing returns to scale.

### 1. INTRODUCTION

Based on the pioneering work of Lössch (1954), traditional models of spatial competition concentrate on spatial aspects of the product market and, for the most part, neglect spatial aspects of the related labor market. Some recent work does stress the spatial labor market, but does so in a partial equilibrium framework that does not account for the dynamic interrelations between product and labor markets (Nakagome, 1986). Our paper takes a first step toward explicitly linking the two markets in a simple general equilibrium framework in economic space. Utility-maximizing consumers work for and consume the products of profit-maximizing firms in spatially differentiated labor and product markets. The friction of distance is found to curtail the areas of both input and output markets. As a result, the structure of the spatial labor markets surrounding each firm is shown to have a profound effect on the decision making of firms.

Not only is the structure of the spatial labor market important, but the organizational structure of the output market and the technology of the production process are major components of the analysis. The long-run general equilibrium consequences of spatial monopoly are contrasted with Lösschian spatial competition under conditions of increasing returns to scale. We find that the entry of more firms does not guarantee lower product prices or higher wages; in fact the opposite is more likely to occur. Thus, our work confirms earlier findings (Beckmann and Thisse, 1987) of the price and wage effects of entry in economic space. Moreover,

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unlike the case of spaceless competition, we find that zero-profit long-run equilibrium can occur under conditions of increasing returns to scale.

The rest of the paper is organized as follows: In Section 2, we set up the model of general equilibrium in economic space. Fundamental properties of spatial labor supply and product demand are derived. Spatial monopoly is explored in Section 3 by examining properties of free spatial labor supply and derived free spatial demand. In Section 4, we examine the impacts of spatial competition on the endogenous variables under conditions of long-run zero economic profits. In Section 5, we reveal how technological innovation impacts the spatial system. And Section 6 concludes the paper, by providing a synthesis of our findings on the spatially differentiated product and labor markets.

## 2. A MODEL OF SPATIAL GENERAL EQUILIBRIUM

### *Assumptions*

Assume the following:

(a) Consumers are distributed uniformly along a one-dimensional circular market.

(b) Consumers have identical tastes reflected by a utility function with leisure time and output as arguments.

(c) All consumers are price takers with identical fixed factor endowments and maximize their utility by commuting, supplying labor, and purchasing a single output (in exchange for either coupon money or labor directly).

(d) Firms have identical production functions with labor as the only variable input. A conventional hill-shaped single-peaked average product curve is assumed.

(e) Identical firms are distributed uniformly and discretely along the circle. Each firm acts as a local monopsonist in the local labor market. Firms sell output f.o.b. mill (in the case of coupon transactions) as a local monopolists, but can be subject to Löschian spatial competition in both the labor market and the product market.<sup>1</sup> The f.o.b. mill price is normalized to unity.

(f) Commuting and freight transport rates (per unit distance) are constant; the commuting rate is normalized to unity and the freight transport rate  $t$  is sometimes normalized to one.<sup>2</sup>

### *The Related Markets*

The proposed model of spatial general equilibrium integrates consumer and firm behavior. The regional labor supply that each firm faces differs by the degree

<sup>1</sup>Löschian competition is said to take place when rival firms react hypersensitively to a given firm's change in price or wage rate. The product/labor market area of each firm remains the same regardless of its price/wage policy.

<sup>2</sup>The costs of distance may be interpreted to include the cost of time forgone while commuting and the cost of deterioration on shipment. However, for analytical simplicity direct transportation costs alone will be assumed in this paper.

of spatial competition that each firm encounters. In the long run, the firm must choose both market radius and local employment.

*The Consumer.* Each consumer is endowed with  $H$  hours of time (per day, month or year) consequent to Assumption (c). If the wage rate per hour is  $w$  and commuting cost per unit of the time endowment is  $x$ , then  $(w - x)H$  is the net value of the endowment in terms of output. The consumer purchases leisure  $h_x$  and output  $q_x$  consumed at an individual's residential location  $x$  distance units (in terms of costs) from the firm, the commuting rate being unity pursuant to Assumption (f). Thus

$$(1) \quad (w - x)H = (w - x)h_x + (1 + tx)q_x$$

where  $(1 + tx)$  represents the delivered c.i.f. price of output, given that the f.o.b. mill price is unity and  $t$  is the freight rate per unit of distance. The freight rate may be assumed without loss of generality to be either one or zero. When  $t = 0$ , work trips and shopping trips coincide and commuters pay for their own transportation to work, but not for the product transportation to their homes. When  $t = 1$ , work trips and shopping trips are separate and consumers must pay both commuting costs and product distribution costs.

The goal of the individual at  $x$  is to maximize his/her utility  $U$  which is a function of leisure  $h_x$  and consumption of output  $q_x$  [Assumption (b)]. Constrained optimization subject to (1) requires

$$(2) \quad \frac{U_1}{w - x} = \frac{U_2}{1 + tx}$$

where  $U_1$  represents the marginal utility of leisure and  $U_2$  the marginal utility of the product. Derived from (2) and (1) is the basic individual labor supply  $L_x (= H - h_x)$  as a function of  $(w - x)/(1 + tx)$ . Thus

$$(3) \quad L_x = f\left(\frac{w - x}{1 + tx}\right)$$

Three related notes are warranted here. First, the basic individual labor supply is a function  $f$  of the *relative net wage rate*, i.e., net of commuting cost, and *relative to the delivered (or c.i.f.) price of output*. The effective wage rate is not just the gross wage paid at the mill. The gross wage,  $w$ , must be discounted twice: once for the consumer's personal commuting expenses and again for freight transport charges. Thus, the distance variable  $x$  can be viewed as an accelerating factor of discount in the individual supply of labor.

Second, while the functional form of the labor supply,  $f$ , depends on the form of the underlying utility function, the labor supply is assumed to be an increasing function of the effective wage, e.g.,  $f' > 0$ . The following utility function yields a simple linear basic labor supply:

$$(4) \quad U = q_x^{1/2} + \frac{h_x}{2}$$

Combining (1), (2), and (4) yields the linear function  $f$

$$(3) \quad L_x = f\left(\frac{w-x}{1+tx}\right) = \frac{w-x}{1+tx}$$

Third, also derived from (1) and (3) is the basic demand for output

$$(5) \quad q_x = \frac{w-x}{1+tx} f\left(\frac{w-x}{1+tx}\right) = \phi(w, x) \quad (\phi_1 > 0, \phi_2 < 0)$$

where  $\phi_1 = \partial\phi/\partial w > 0$  and  $\phi_2 = \partial\phi/\partial x < 0$  show respectively that the demand for output is an increasing function of  $w$ , while it is a decreasing function of  $x$ .

But where is the price of the product? So far, the product price has been expressed in terms of the product itself. The product price thus defined is unity. However, if the product price is to be expressed in terms of labor demanded and if the factor demanded (denoted by  $w$ ) is itself to be expressed in terms of the product, the product price at the mill,  $m$ , must be  $m = 1/w$ .<sup>3</sup> Equation (5) then may be rewritten as

$$(5') \quad q_x = \phi(1/m, x)$$

It is Equation (5') that may be called the derived basic individual demand function, which is decreasing in mill price,  $m$ .

*The Regional Labor Supply Function.* Each local (monopsonist) firm faces the regional supply function as an aggregate of individual labor supplies relevant to the firm.

Case A below defines the regional labor supply when work trips and shopping trips coincide. Wages are paid directly in kind and commuters carry output home at no extra cost. Thus, based on (3) with  $t = 0$

Case A:  $t = 0$

$$(6a) \quad L = \begin{cases} 2 \int_0^{w-f^{-1}(0)} f(w-x) dx = 2 \int_{f^{-1}(0)}^w f(x) dx & [w < f^{-1}(0) + x_o] \\ 2 \int_0^{x_o} f(w-x) dx = 2 \int_{w-x_o}^w f(x) dx & [w \geq f^{-1}(0) + x_o] \end{cases}$$

where  $x_o$  represents the fixed market radii of the firm in the case of Löschian competition. Note that as long as wage rate,  $w$ , remains below the minimum wage rate,  $f^{-1}(0)$ , plus this critical distance cost,  $x_o$ , i.e.,  $w < f^{-1}(0) + x_o$ , the firm is a spatial monopsonist. Löschian competition becomes effective only at wage rates above this critical sum.

Case B in turn is based on the assumption that wages are paid in coupon money which workers use to purchase the firm's product in their leisure time

<sup>3</sup>See Ohta and Kataoka (1982) and Ohta (1988) for further details.

(distinct from their commuting time). For simplicity,  $t$  in (3) is now assumed to be unity. Thus

Case B:  $t = 1$

$$(6b) \quad L = \begin{cases} 2(1 + w) \int_{f^{-1}(0)}^w \gamma(u) du & [w < f^{-1}(0) + [1 + f^{-1}(0)]x_0] \\ 2(1 + w) \int_{(w-x_0)/(1+x_0)}^w \gamma(u) du & [w \geq f^{-1}(0) + [1 + f^{-1}(0)]x_0] \end{cases}$$

where  $u = (w - x)/(1 + x)$  and  $\gamma(u) = f(u)/(1 + u)^2$ .

Figure 1 illustrates several inverse regional labor-supply curves under conditions of spatial monopoly and Lösschian spatial competition for the case  $t = 0$ . The free regional labor supply,  $w(L)$ , is experienced by a spatial monopolist and is analogous to the concept of free spatial demand in the output market. Competitive regional labor supply,  $w(L, x_0)$ , is experienced by Lösschian competitors if the wage is greater than  $f^{-1}(0) + x_0$ . Thus the competitive and free regional labor supply meet at this critical minimum wage, the sum of the reservation wage at zero hours plus the maximum commute cost ( $x_0$ ) to the market boundary. Similarly in the

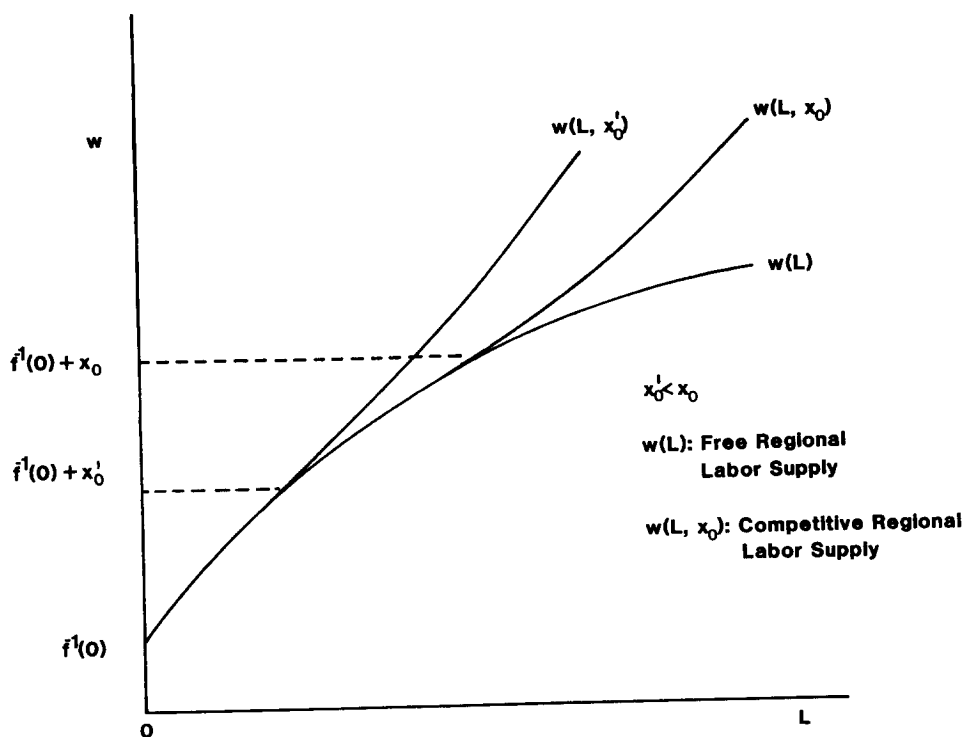


FIGURE 1: Regional Labor Supply under Alternative Market Radii [Based On Equation (6a)].

traditional Löschian output model free and competitive spatial demand meet at a critical maximum mill price, defined as the maximum the consumer is willing to pay for the output minus the maximum delivery cost to the fixed market boundary. As Löschian entry occurs and a firm's locally monopsonized labor-market radius ( $x$ ) decreases, the competitive regional labor supply curve twists inward and upward. Again this is analogous to the impact of entry on competitive spatial demand where Löschian entry causes the spatial demand to twist inward and downward (Greenhut, Hwang, and Ohta, 1975; Ohta, 1988, p. 217). See footnote 9 for further discussion about the relation between free and regional labor supply and product demand in our expanded model.

*The Firm.* A local firm that faces the regional labor supply function also faces the technological input-output relation given by the production function. Pursuant to Assumption (d) the production technology may be given in terms of the average product  $AP_L$  function. Thus

$$(7) \quad AP_L \equiv Q/L = g(L) \quad (g' \geq 0 \text{ for } L \leq L_o > 0)$$

where  $Q$  is the total output produced by the local firm with the input of labor  $L$  which is available subject to its supply function (6). In Equation (7),  $AP_L$  reaches its maximum at  $L = L_o$ .

For later analytical expediency the following transformation is introduced here

$$(8) \quad \ell = L/2x_o$$

where  $x_o$  is radius of the (labor) market that the firm monopolizes locally under conditions of Löschian competition. The  $\ell$  term therefore stands for average employment per unit area. This  $\ell$ , multiplied by the fixed total area, yields the aggregate employment, since Löschian competition divides the total space into equal pieces for individual firms to monopolize.

The firm's goal is to maximize profit. Profit  $\pi$  may be defined as *either* the residual amount of output demanded after in-kind payments of wages and fixed costs,  $F_o$ , or as the residual amount of labor demanded. Thus

$$(9a) \quad \pi_1 = Q - wL - F_o$$

$$(9b) \quad \pi_2 = mQ - mwL - mF_o$$

The two alternative definitions of profit,  $\pi_1$  and  $\pi_2$ , above are not strictly equivalent, however. This is because maximization of  $\pi_1$  may not coincide with maximization of  $\pi_2$  unless the mill price,  $m$ , is treated as a fixed parameter in the labor market. Nevertheless, under conditions of long-run zero-profit equilibrium the two maximization conditions do become equivalent.<sup>4</sup>

Henceforth (9a) will be used as the definition of profit,  $\pi$  (deleting the

<sup>4</sup>To see this note that  $\pi_2 = m\pi_1$ . Thus,  $d\pi_2 = dm\pi_1 + m d\pi_1$ ,  $m > 0$ ; therefore  $d\pi_2 = 0$  iff  $d\pi_1 = 0$  when  $\pi_1 = 0$ .

subscript). Profit maximization requires optimization with respect to employment  $L$ . Thus

$$\frac{d\pi}{dL} = \frac{dQ}{dL} - w - \frac{dw}{dL} L = 0$$

By using (7) and (8), this can be rewritten further as

$$(10) \quad g(2x_o\ell) + 2g'(2x_o\ell)x_o\ell = w(2x_o\ell, x_o) + 2w_1(2x_o\ell, x_o)x_o\ell$$

where the left-hand side may be interpreted as the marginal-revenue product (marginal revenue is unity) while the right-hand side is the marginal expenditure on labor. Note in this connection that the term  $w(2x_o\ell, x_o)$  represents the inverse labor-supply function as a function of regional labor,  $2x_o\ell$ , as well as the market radius,  $x_o$ . The term  $w_1(2x_o\ell, x_o)$  is defined to be the partial derivative of  $w$  with respect to the first argument, i.e.,  $w_1 = \partial w / \partial (2x_o\ell)$ .

*The Long-run Equilibrium Condition.* As long as profit remains positive, new firms are enticed to enter the market. Each entry subject to Assumption (e) requires all the incumbent firms to relocate. As a result, all firms face identical but smaller market radii and lower profits than those they had before entry and relocation.

In the long run, the individual firm's profits, defined as in (9a), must vanish when entry also comes to a halt.<sup>5</sup> Thus

$$(11) \quad g(2x_o\ell) = w(2x_o\ell, x_o) + F_o/2x_o\ell$$

Contained in this long-run equilibrium condition are two endogenous variables: long-run market radius,  $x_o$ , and average regional employment per unit area,  $\ell$ . The optimization condition (10) also contains the same two variables. Since no other endogenous variables are contained in these two equations, they may readily be solved for the two long-run equilibrium values ( $x_o^*, \ell^*$ ).

A diagrammatic solution for  $x_o^*$  and  $\ell^*$  [via  $L^*$  and (8)] is presented in Figure 2 and is represented by the tangency point E. The hill-shaped curve,  $AP_L$ , represents the average product curve,  $g$ , and the U-shaped  $ATC$  curve represents the vertical sum of the inverse regional labor-supply function,  $w(L, x_o)$  (average variable labor cost), and the average fixed cost,  $AFC$ . It should be noted that while both  $AP_L$  and  $AFC$  curves are fixed under the conditions of a given technology, part of the regional labor supply twists inward and upward upon each rival entry at a distance as illustrated in Figure 1. The U-shaped average total cost curve tends to roll up accordingly—until it becomes tangent to the fixed  $AP_L$  curve. Note that this tangency point E satisfies conditions of Equations (10) and (11) simultaneously.<sup>6</sup> Underlying E therefore are the equilibrium values ( $x_o^*, \ell^*$ ).

<sup>5</sup>We assume a zero-profit equilibrium. Although beyond the scope of our present paper, future expansions could model the possibility of positive profits in an entry deterring equilibrium. Two papers that allow for the possibility of positive profits are Eaton (1976) and Eswaran and Ware (1986).

<sup>6</sup>Tangency E guarantees the zero-profit equilibrium conditions of (11) directly. Moreover, tangency requires the derivatives (with respect to  $L = 2x_o\ell$ ) of the right-hand side and the left-hand side of (11) to be equal. Thus,  $g'(2x_o\ell) = w_1(2x_o\ell, x_o) - F_o/(2x_o\ell)^2$ . Combining this with (11) readily yields (10).

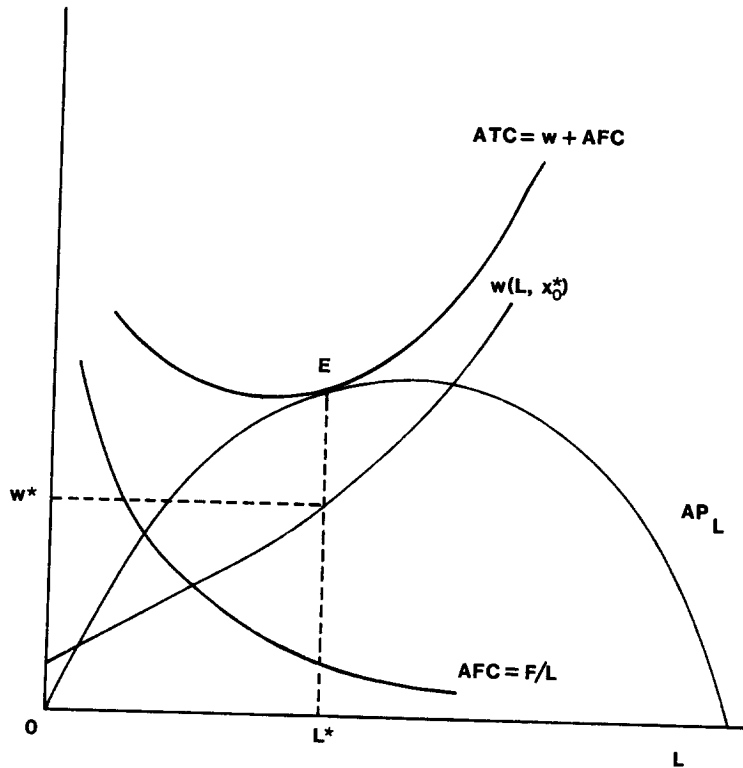


FIGURE 2: Spatial General Equilibrium in the Long Run.

Related to these are the long-run equilibrium values of regional employment,  $L^* \equiv 2x_0^*$ ; wage rate,  $w^*$  [given by the lower line(s) of Equation (6)]; individual labor supply,  $L_x^*$  at location  $x$  [Equation (3)]; individual product-demand quantity,  $q_x^*$ , at location  $x$  [Equation (5)]; and individual leisure consumption,  $h_x^*$ , at location  $x$  via  $h_x^* \equiv H - L_x^*$ .

### 3. SPATIAL MONOPOLY

We are now in position to analyze parts of the proposed model more deeply to better appreciate its characteristics. To begin, closer observation of the regional labor supply function derived above as Equation (6) yields the following results, summarized as Proposition 1.

*Proposition 1a:* Under conditions of pure monopoly, the regional labor supply function is strictly convex in  $w$  regardless of the form of the basic labor supply given that the basic labor supply is a monotonically increasing function of net wage rate, net of commuting and freight costs.

*Proposition 1b:* Under conditions of Löschian spatial competition the form of the regional labor supply function reflects the form of the basic individual labor-supply function.

Proof of this proposition is deferred to the Appendix.



The market conditions for parts a and b of Proposition 1 are not defined independently because they each depend on the wage rate. When the wage rate is set sufficiently low such that  $w < x_0$  [assuming  $f^{-1}(0) = 0$  for simplicity], no effective spatial competition takes place. Part a applies accordingly. Part b applies when the wage rate is set sufficiently greater than  $w = x_0$  to provoke rival reactions.

Proposition 1 is a dual counterpart to similar propositions on the free spatial demand function established elsewhere.<sup>7</sup> However, previous results on the functional form of the free spatial demand are based on a strictly partial equilibrium model of the spatial product market, with no related labor market. A question therefore arises whether the same generally convex free spatial demand function [typical of spatial product markets given linear transport costs and uniform density as is assumed here (Batten, 1988)] is derivable from the present model of related spatial markets.

To answer this question, remember that the basic (individual) demand for output is given by Equation (5). The free spatial demand is obtainable by simply integrating this equation over the relevant market radius. Thus, assuming  $t = 0$  (though not needed) for simplicity

$$(12) \quad Q = 2 \int_0^{w-f^{-1}(0)} (w-x)f(w-x) dx = 2 \int_{f^{-1}(0)}^w xf(x) dx \quad (w = 1/m)$$

Differentiating this twice with respect to the mill price,  $m$ , readily yields the result that the free spatial demand slopes downward and is convex toward the origin.<sup>8</sup>

When Lösschian competition comes about in the labor market with wage rates

<sup>7</sup>See Greenhut and Ohta (1975), Ohta (1980, 1981), and Batten (1988) for more detail.

<sup>8</sup>Differentiating (12) with respect to  $m$  yields

$$\begin{aligned} dQ/dm &= (dQ/dW)(dw/dm) = -2f(w)m^{-3} < 0 \\ d^2Q/dm^2 &= 6f(w)m^{-4} + f'(w)m^{-5} > 0 \end{aligned}$$

This proves Proposition 2a under the assumption that  $t = 0$ .

When  $t \neq 0$ , the free spatial demand is given by

$$Q = 2 \int_{f^{-1}(0)}^w \frac{(1+tw)uf(u)}{(1+tu)^2} du \quad \left( u = \frac{w-x}{1+tx} \right)$$

Therefore

$$\frac{dQ}{dw} \frac{dw}{dm} = \frac{2wf(w)}{1+tw} (-m^{-2}) < 0 \quad (w = 1/m)$$

and

$$\frac{d^2Q}{dm^2} = \frac{2}{m+t} \left( \frac{2f}{m} + \frac{f}{m+t} + \frac{f'}{m^2} \right) m^{-2} > 0$$

Thus Proposition 2a follows even when  $t > 0$ .

This definitive result on the shape of the free spatial demand under related market conditions is based on our uniformity assumptions regarding worker/consumer distribution and the transport cost structure. See Batten (1988) where these conventional assumptions are relaxed in partial equilibrium models to reveal certain special conditions under which our convexity result may not follow.

$w$  exceeding  $f^{-1}(0) + x_o$  [as per (6a)] and with fixed market radius,  $x_o$ , the dual Löschian conditions apply to the product market with mill price below  $1/[f^{-1}(0) + x_o]$  since  $m \equiv 1/w$ .<sup>9</sup> These observations yield the following proposition as a dual to Proposition 1.

*Proposition 2a:* Provided that the basic labor supply is a monotonically increasing function of net wage rates, net of commuting and freight costs, the free spatial demand in the product market is strictly convex in mill price.

*Proposition 2b:* The form of the spatial market demand reflects the form of the individual derived demand, derived from the basic labor supply function under conditions of Löschian competition.

#### 4. LÖSCHIAN SPATIAL COMPETITION

We now turn our attention to spatial competition. Considering the zero-profit long-run equilibrium condition, note that the tangency equilibrium E can occur either on the increasing part or the decreasing part of the  $AP_L$  curve depending upon the fixed-cost level,  $F_o$ .<sup>10</sup> When the fixed cost is sufficiently high, no tangency is feasible, much less on the increasing part of the  $AP_L$ . However, if the fixed cost is sufficiently low, then the long-run tangency equilibrium is likely to occur on the

<sup>9</sup>Insofar as Löschian competition is considered to arise in the labor market when  $w > f^{-1}(0) + x_o$ , the derived market demand for the product [via (5) with  $t = 0$ ] must be given by

$$Q = 2 \int_0^{x_o} (1/m - x) f(1/m - x) dx = 2 \int_0^{x_o} g(m, x) dx \quad \left[ m < \frac{1}{f^{-1}(0) + x_o} \right]$$

where  $g(m, x) \equiv (1/m - x) f(1/m - x)$ . Differentiating with respect to  $m$  yields

$$\frac{dQ}{dm} = 2 \int_0^{x_o} g_m(m, x) dx < 0$$

$$\frac{d^2Q}{dm^2} = 2 \int_0^{x_o} g_{mm}(m, x) dx \geq 0 \quad (\text{if } g_{mm} \geq 0)$$

This proves Proposition 2b.

In our model of interrelated markets, demand and labor supply are mutually related, hence so are free and competitive product demand and labor supply. Löschian product demand (or Löschian labor supply) and the free spatial demand (or free labor supply) meet at  $m = 1/[f^{-1}(0) + x_o]$  [or  $w = f^{-1}(0) + x_o$ ] as defined above [or (6a) and Figure 1 in the text]. However, note that the Löschian "kink" at this point is not really a sharp kink. Instead the slope of the monopoly curve and the slope of Löschian curve coincide at this point. Evaluating the derivative of (6a) upperline at  $w = f^{-1}(0) + x_o$  yields  $dL/dw = 2f(w) = 2f[f^{-1}(0) + x_o]$  while the lower line evaluation at the same point yields  $dL/dw = 2f(w) - 2f(w - x_o) = 2f(w) - 2f[f^{-1}(0)] = 2f(w) = 2f[f^{-1}(0) + x_o]$  since  $f[f^{-1}(0)] = 0$ . Gannon (1971) established similar properties for the competitive spatial demand curve in the traditional Löschian model.

<sup>10</sup>Recall in this connection that while the curve  $AP_L$  is fixed, the  $ATC$  curve is subject to variation as it depends upon the regional labor supply which in turn depends upon the firm's market radius,  $x_o$ . With greater entry of new firms the market radius,  $x_o$ , tends to be squeezed: the curve is twisted upward and leftward upon each rival encroachment. This shrinkage yields a concomitant twist in the  $ATC$ .

increasing part of the  $AP_L$ . The underlying  $MP_L$  is also likely to slope upward in the neighborhood of equilibrium. These observations may be rephrased as Proposition 3.

*Proposition 3.* Under conditions of Löschian spatial competition in the long run and of a production function which is hill shaped in input  $L$ , firms in industries with low fixed costs tend to experience economies of scale while firms in industries with high fixed costs tend to experience diseconomies of scale.

Alternatively stated, the marginal-revenue product in equilibrium may decrease (increase) if fixed costs are sufficiently low (high) under conditions of spatial competition. It then follows that insofar as the regional labor supply becomes less elastic due to spatial competition (as is the case for linear individual labor supply), the so-called “perverse” effect of spatial competition to lower the wage rate (Nakagome, 1986) is reinforced if the fixed costs are sufficiently low. Low fixed costs induce substantial entry and greatly curtail individual firms’ outputs.<sup>11</sup> However, the perverse effect of spatial competition is weakened or possibly even reversed when fixed costs are sufficiently large. Summarizing these observations yields Proposition 4.

*Proposition 4.* Provided that the production function is hill shaped in input and the elasticity of the individual labor supply increases with distance from the firm, spatial competition may yield a higher wage rate (lower mill price) initially, but further entry tends to reverse this trend eventually, lowering the wage rate and raising the mill price, unless inhibited by higher fixed costs.

To appreciate this proposition more fully, recall Equation (10) which can be rewritten as

$$(10') \quad MP_L = w(1 + 1/\eta)$$

where  $MP_L$  is the marginal product of labor and  $\eta$  is the elasticity of labor supply. Note that if  $MP_L$  decreases concomitantly with  $\eta$  due to spatial competition, then  $w$  must also decline. Even if  $MP_L$  increases when output is decreased, it is possible that wage rates decline if  $\eta$  declines sufficiently. The perverse effect of spatial competition to lower the wage and to raise the mill price is thus confirmed under conditions more general than those specified in prior models.

<sup>11</sup>The elasticity  $\eta$  of free regional labor supply is given via (6) as

$$\eta = \frac{dL}{dw} \frac{w}{L} = \frac{f(w)w}{\int_{f^{-1}(0)}^w f(x) dx}$$

Thus  $\eta$  is dependent upon the form  $f$  of the basic individual labor supply function. It can be shown more specifically that  $\eta$  becomes more or less elastic (i.e., higher or lower) than the elasticity of basic labor supply according as the latter curve is relatively less or more convex than a certain standard curve. See Ohta (1981) and Greenhut, Norman, and Hung (1987) for the counterpart demand relationship.

## 5. IMPACT OF INNOVATION

Our final subject of interest is the impact of innovation and subsequent spatial competition upon long-run equilibrium levels of employment and the related endogenous variables. When technical progress shifts the production function upward, profit increases and regional employment may increase in the short run. But profit induces new entry, yielding a new tangency equilibrium on a new  $AP_L$  curve. It is possible that the new tangency equilibrium after innovation may occur to the left of the original tangency point E. Thus, innovation *may* even decrease regional employment in the long run. However, easier entry means a greater number of regions monopolized by individual firms. The aggregate employment therefore may increase if regional employment decreases are followed by further spatial entry. This question may be examined more fully by reconsidering the long-run equilibrium conditions (10) and (11) respecified as

$$(10') \quad g(2x_o\ell) + \alpha + 2g'(2x_o\ell)x_o\ell - w(2x_o\ell, x_o) - 2w_1(2x_o\ell, x_o)x_o\ell = 0$$

$$(11') \quad g(2x_o\ell) + \alpha - w(2x_o\ell, x_o) - F_o/2x_o\ell = 0$$

where  $\alpha$ , a nonnegative shift parameter, reflects the level of production technology. In what follows, the implicit function theorem is applied to a comparative static analysis of a change in  $\alpha$ . Based on the foregoing analysis note initially that the functional forms  $g$  and  $w$  have the following characteristics over the relevant domains:

$$g' \geq 0 \quad (\text{according as } 2x_o\ell \leq L_o), \quad g'' < 0, \quad w_1 > 0, \quad w_2 < 0, \quad \text{and } w_{12} \text{ (or } w_{21}) < 0$$

where  $g'$  and  $g''$  represent the first and the second derivatives of  $g$  while  $w_i$  is the partial derivative of  $w$  with respect to the  $i$ th argument and  $w_{ij}$  is the partial of  $w_i$  with respect to the  $j$ th argument. For simplicity, we hereafter assume further that the basic individual labor supply function is linear so that  $w_{11} = 0$  under conditions of Lösschian competition.

Totally differentiating (10') and (11') with respect to  $\alpha$  and applying Cramer's Rule yields

$$\frac{dx_o}{d\alpha} = \frac{\begin{vmatrix} -1 & \mathbf{D}_2 \\ -1 & \mathbf{D} \end{vmatrix}}{|\mathbf{D}|}$$

$$\frac{d\ell}{d\alpha} = \frac{\begin{vmatrix} \mathbf{D}_1 & -1 \\ -1 & \mathbf{D} \end{vmatrix}}{|\mathbf{D}|}$$

where

$$\mathbf{D} = 4 \begin{bmatrix} 2(g' - w_1)\ell + 2(g''\ell - w_{12}/2)x_o\ell - w_2/2 & 2(g' - w_1)x_o + 2g''x_o^2\ell \\ (g' - w_1)\ell + \frac{F_o\ell}{(2x_o\ell)^2} - w_2/2 & (g' - w_1)x_o + \frac{F_o x_o}{(2x_o\ell)^2} \end{bmatrix}$$

and  $D_i = i$ th column of  $D$ . Further evaluating the above equations yields

$$(13) \quad \frac{dx_o}{d\alpha} = \Gamma$$

$$(14) \quad \frac{d\ell}{d\alpha} = \epsilon/2x_o\psi - \ell\Gamma/x_o$$

where  $\Gamma \equiv [(g' - w_1) + 2g''x_o\ell - F_o/(2x_o\ell)^2]/w_2\psi$ ,  $\epsilon \equiv \partial w_2/\partial(2x_o\ell) \cdot 2x_o\ell/w_2$  ( $= 2w_{21}x_o\ell/w_2$ ), and  $\psi \equiv (1 - \epsilon)[(g' - w_1) + 2g''x_o\ell - (1 + \epsilon/2)F_o/2x_o\ell]^2$ .

Note that  $\epsilon$  is the elasticity of the “entry impact on the wage (rate)” with respect to regional employment and is positive since  $w_{21} < 0$  and  $w_2 < 0$ . Moreover, the elasticity can be shown to be greater than one in the case of linear individual labor supply.<sup>12</sup> It then follows that the sign of  $\psi$  becomes negative when spatial competition squeezes regional employment sufficiently, turning the sign of  $g' - w_1$  nonnegative.<sup>13</sup> Even when  $\psi$  is negative, it may appear that the sign of  $\Gamma$  could be positive if both fixed cost  $F_o$  and the absolute value of  $g''$  are sufficiently small. This proviso, however, is invalid because a positive  $\Gamma$  requires  $dx_o/d\alpha$  of (14) to be positive, i.e., innovation to increase the firm’s market radius, which is an impossible result. Innovation defined as an increase in  $\alpha$  brings about a short-run positive profit which in turn calls forth new entry, thereby reducing each firm’s market radius. Thus,  $\Gamma$  must necessarily be negative! The sign of  $d\ell/d\alpha$  in (14) is now seen to depend upon the relative magnitudes of  $\epsilon$  and  $\Gamma$ , among other factors. Thus, for example, the higher is  $\epsilon$  (which is positive under *spatial* competition), the smaller  $\Gamma$  (in absolute value) and the smaller  $\psi$  (in absolute value), the more likely is the sign of  $d\ell/d\alpha$  to become negative (and vice versa). This result may be restated as Proposition 5.

*Proposition 5.* When economic space matters in the sense that the regional labor-supply function is twisted upward ( $w_{12} < 0$ ) and leftward ( $w_2 < 0$ ) with each rival encroachment (causing  $\epsilon$  to become positive), it is possible for aggregate employment ( $=\ell$  multiplied by the fixed total market area) to decline after innovation.

Innovation is more likely to decrease aggregate employment the smaller is the fixed

<sup>12</sup>When  $L_x = w - x$ , the (inverse) regional labor supply function under Löschian competition is specifiable as  $w = (L + x_o^2)/2x_o$ . It then follows that

$$w_2 = (x_o - 2\ell)/2x_o \quad (\ell > x_o/2, w_{21} = -1/2x_o^2)$$

and therefore

$$\epsilon = 2w_{21}x_o\ell/w_2 = 1/(1 - x_o/2\ell) > 1$$

The range for  $\ell > x_o/2$  or  $L > x_o^2$  is derived from the constrained domain for  $w > x_o$ .

<sup>13</sup>Under conditions of  $g'' < 0$  the value of  $g'$  is less the greater is the regional employment;  $g'$  can even be negative, when  $g' - w_1$  becomes negative. It is then possible, though not necessary, that  $\psi$  becomes positive.

cost, implying not only  $\psi$  to be smaller, but also  $g' - w_1$  to be positive and larger in the neighborhood of equilibrium.

Related to a change in aggregate employment and market radius is the amount of regional employment,  $L$ . Insofar as both  $\ell$  and  $x_o$  decline, regional employment must also decline after innovation. What other general relations can be set forth? What if  $\ell$  increases after innovation? In particular, can regional employment increase accordingly? To answer the last question recall from Equation (8) that  $L = 2x_o\ell$ . It therefore follows that

$$(15) \quad \frac{dL}{d\alpha} \frac{1}{L} = \frac{d\ell}{d\alpha} \frac{1}{\ell} + \frac{dx_o}{d\alpha} \frac{1}{x_o} = \epsilon/2x_o\ell\psi$$

where  $\epsilon$  is shown above to be positive under conditions of spatial competition. The sign of (15) therefore depends solely upon the sign of  $\psi$ . The sign of  $\psi$  can be deduced from the required negativity of  $\Gamma$ . Note that  $\psi$  appears in the denominator of  $\Gamma$ . Given the negative  $w_2$  also in the denominator of (17), the numerator as a whole must have the same sign as the denominator term  $\psi$ . These two terms must have the same and negative sign only. This is because the numerator can be positive only if  $(g' - w_1)$  is positive, implying  $\psi$  strictly negative, and causing a violation of the required sign compatibility with the numerator of  $\Gamma$ . This means that even if  $(g' - w_1)$  happened to be positive, as is the case when fixed cost is small, its magnitude may not exceed the absolute value of the sum of the remaining negative terms.<sup>14</sup> In any case,  $\psi$  must be negative as is the numerator of  $\Gamma$ . Thus, the sign of Equation (15) must be always negative; it cannot be positive even when  $d\ell/d\alpha$  happens to be positive.<sup>15</sup> This establishes Proposition 6.

*Proposition 6.* Under conditions of Löschian spatial competition, regional employment always declines after innovation, even if aggregate employment is increased as it may when the fixed cost (of entry) is sufficiently large.

Underlying Propositions 5 and 6 are those assumptions listed in Section 2 of the paper and the additional simplifying assumption that the basic individual labor supply is a linear function of the wage rate.<sup>16</sup>

<sup>14</sup>This requirement is well founded in consideration of the negative term  $F_o/(2x_o\ell)^2$  whose absolute value becomes increasingly large as spatial competition squeezes regional employment ( $2x_o\ell$ ) substantially after innovation.

<sup>15</sup>Equation (14) can be rewritten as

$$d\ell/d\alpha = -\ell[(g' - w_1) + 2g''x_o\ell - w_{12}x_o - F_o/(2x_o\ell)^2]/x_o[(g' - w_1)(w_2 - 2w_{12}x_o\ell) + 2w_2g''x_o\ell - (w_2 + w_{12}x_o\ell)F_o/(2x_o\ell)^2]$$

This term can be seen to be positive if  $F_o$  is sufficiently large so that both its numerator and denominator become positive.

<sup>16</sup>The linearity assumption of the basic labor supply is not crucial. As long as the basic marginal expenditure curve,  $ME_L$ , intersects the concave  $MP_L$  curve from below these propositions are likely to remain valid.

## 6. CONCLUSION

Synthesizing the foregoing analyses reveals the importance of the distance variable and its nontrivial impact upon orthodox nonspatial theory of general equilibrium. Using the simplest form of general equilibrium with one product and one factor, we have extended previous results and established new properties of integrated spatial economic systems. Spatial competition has been shown not only to raise mill prices but also to lower wage rates under conditions more general than those of prior models.

Important findings are the general *convexity* of both the labor supply function and the derived product-demand function faced by the regional monopolist firm (Propositions 1 and 2). The property of convexity in turn relates to the effect of spatial competition on the shift in the labor supply curve: each rival encroachment causes the inverse regional labor supply curve to fall back on itself. The relevant part (right segment) of the labor supply curve is not only shifted leftward, but also becomes increasingly steep upon each additional rival entry. The regional labor supply tends to become less and less elastic as entry continues.

Not only do the functional forms of spatial labor supply and product demand reinforce that price rises and wage lowers with entry (Proposition 4), but so do technological characteristics such as economies of scale and low fixed cost. Moreover, the same spatial competitive effect is shown to play a major role in causing the negative effect of innovation upon aggregate and regional employment when the fixed cost of entry is sufficiently small and the production technology is characterized by a hill-shaped production function (Propositions 5 and 6).

While in the present paper we deliberately seek analytical simplicity in order to show some interesting complexities of spatial competition, the model could productively be extended to a two-good two-factor model. One of the two goods may be treated as a commodity money which is neither perishable nor producible and is fixed in aggregate supply. Or a second factor that is a location-specific good, such as lot size, may be introduced. Extensions along these lines will be valuable to the further understanding of spatial economic systems.

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### APPENDIX: A PROOF OF PROPOSITION 1

Taking the first and the second derivatives respectively of the upper line of (6a) with respect to  $w$  yields

$$(A-1) \quad \frac{dL}{dw} = 2f(w) > 0 \quad [f^{-1}(0) + x_o > w > f^{-1}(0)]$$

$$(A-2) \quad \frac{d^2L}{dw^2} = 2f'(w) > 0 \quad [f^{-1}(0) + x_o > w > f^{-1}(0)]$$

Similar derivatives applicable to (6b) are

$$(A-3) \quad \frac{dL}{dw} = \int_{f^{-1}(0)}^w \gamma(u) du + (1+w)\gamma(w) > 0$$

$$[f^{-1}(0) + [1 + f^{-1}(0)]x_o > w > f^{-1}(0)]$$

$$\frac{d^2L}{dw^2} = 2\gamma(w) + (1+w)\gamma'(w) \quad \left[ \gamma'(w) = \frac{f'(w)}{(1+w)^2} - \frac{2f'(w)}{(1+w)^3} \right]$$

$$(A-4) \quad = 2f'(w)/(1+w) > 0 \quad [f^{-1}(0) + [1 + f^{-1}(0)]x_o > w > f^{-1}(0)]$$

The positive signs of these derivatives proves part a of Proposition 1.

Part b of Proposition 1 in turn can readily be proved by taking derivatives of the lower line respectively of (6a) and (6b). Thus, from (6a)

$$(A-5) \quad \frac{dL}{dw} = 2 \int_0^{x_o} f'(w-x) dx > 0 \quad [w \geq f^{-1}(0) + x_o]$$

$$(A-6) \quad \frac{d^2L}{dw^2} = 2 \int_0^{x_o} f''(w-x) dx \geq 0 \quad [(f'' \geq 0), (w \geq f^{-1}(0) + x_o)]$$



Similarly from (6b)

$$(A-7) \quad \frac{dL}{dw} = 2 \int_0^{x_0} f' \left( \frac{w-x}{1+x} \right) \left( \frac{1}{1+x} \right) dx > 0 \quad [w \geq f^{-1}(0) + [1 + f^{-1}(0)]x_0]$$

$$(A-8) \quad \frac{d^2L}{dw^2} = 2 \int_0^{x_0} f'' \left( \frac{w-x}{1+x} \right) \left( \frac{1}{1+x} \right)^2 dx \underset{\leq 0}{\geq} 0 \quad [(f'' \underset{\leq 0}{\geq} 0), (w \geq f^{-1}(0) + [1 + f^{-1}(0)]x_0)]$$

For confirmation of this proof some specific form of  $f$  may be considered. The simplest form is linear, e.g.,  $f(u) = u$  as in (3'). Substituting this in (6b), which appears slightly more involved than (6a), yields

$$(A-9) \quad L = 2 \int_0^w \frac{w-x}{1+x} dx = (w+1) \ln(w+1) - w \quad (x_0 > w > 0)$$

$$(A-10) \quad L = 2 \int_0^{x_0} \frac{w-x}{1+x} dx = (w+1) \ln(1+x_0) - x_0 \quad (w \geq x_0)$$

Differentiating (A-9) with respect to  $w$  yields

$$(A-11) \quad \frac{dL}{dw} = 2 \ln(1+w) > 0 \quad (w > 0)$$

$$(A-12) \quad \frac{d^2L}{dw^2} = \frac{1}{1+w} > 0 \quad (w > 0)$$

Differentiating (A-10) correspondingly yields

$$(A-13) \quad \frac{dL}{dw} = 2 \ln(1+x_0) > 0 \quad (x_0 > 0)$$

$$(A-14) \quad \frac{d^2L}{dw^2} = 0$$

These results clearly confirm Proposition 1.