ECE 6342 Fall 2016 HW 12 due 11/29 Problems from Discrete-time Signal Processing - Oppenheim , 2nd edition

Problem 1. 8.43

8.43. We want to implement the linear convolution of a 10,000-point sequence with an FIR impulse response that is 100 points long. The convolution is to be implemented by using DFTs and inverse DFTs of length 256.

(a) If the overlap–add method is used, what is the minimum number of 256-point DFTs and the minimum number of 256-point inverse DFTs needed to implement the convolution for the entire 10,000-point sequence? Justify your answer.

(b) If the overlap–save method is used, what is the minimum number of 256-point DFTs and the minimum number of 256-point inverse DFTs needed to implement the convolution for the entire 10,000-point sequence? Justify your answer.

(c) We will see in Chapter 9 that when N is a power of 2, an N-point DFT or inverse DFT requires $(N/2)\log_2 N$ complex multiplications and $N\log_2 N$ complex additions. For the same filter and impulse response length considered in Parts (a) and (b), compare the number of arithmetic operations (multiplications and additions) required in the overlap–add method, the overlap–save method, and direct convolution.

Problem 2. 9.2

9.2. Figure P9.2-1 shows the graph representation of a decimation-in-time FFT algorithm for $N = 8$. The heavy line shows a path from sample $x[7]$ to DFT sample $X[2]$.

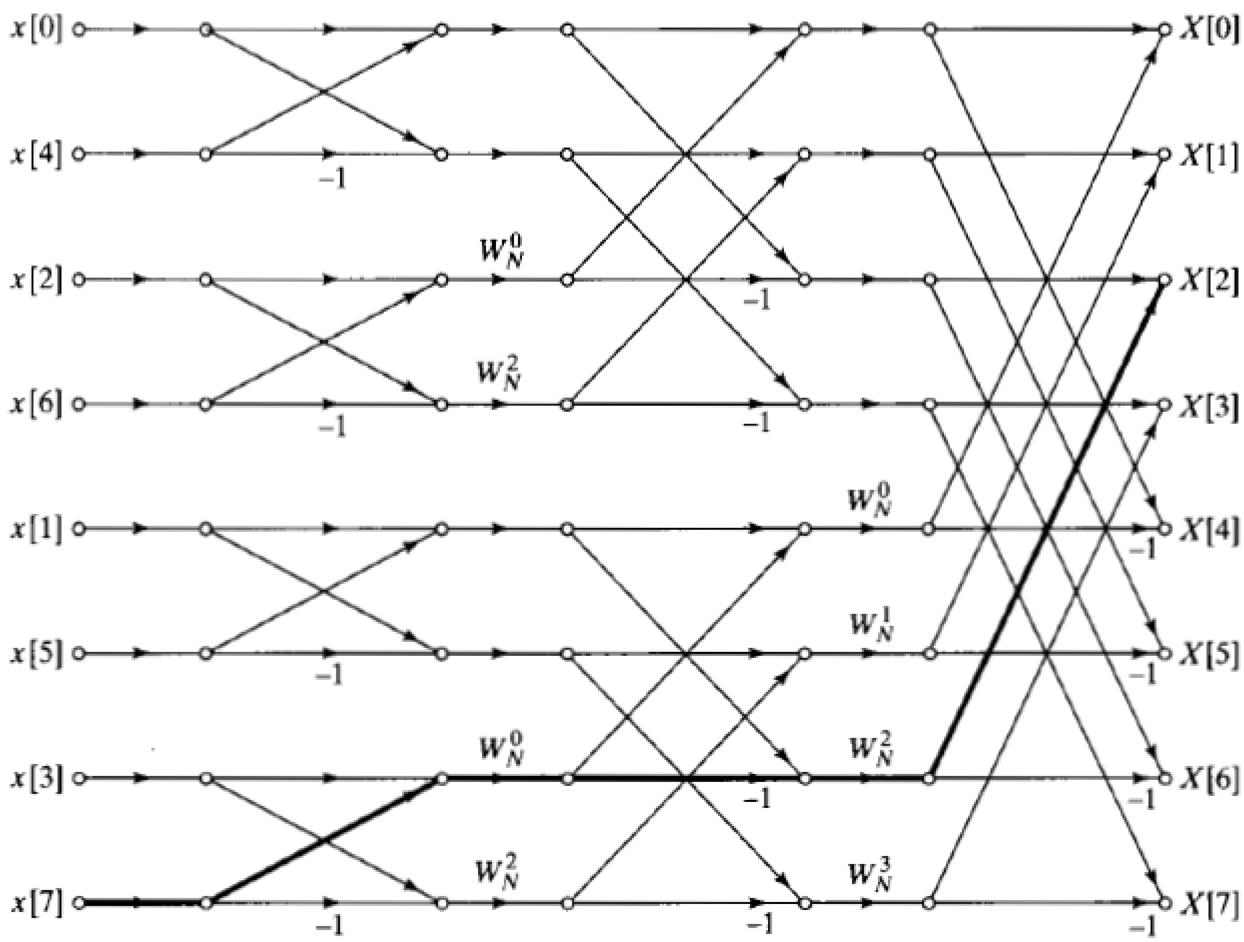

Figure P9.2-1

(a) What is the "gain" along the path that is emphasized in Figure P9.2-1?

(b) How many other paths in the flow graph begin at $x[7]$ and end at $X[2]$? Is this true in general? That is, how many paths are there between each input sample and each output sample?

(c) Now consider the DFT sample $X[2]$. By tracing paths in the flow graph of Figure P9.2-1, show that each input sample contributes the proper amount to the output DFT sample; i.e., verify that

$$X[2] = \sum_{n=0}^{N-1} x[n]e^{-j(2\pi/N)2n}.$$

<u>**Problem 3**</u> 10.1

10.1. A real continuous-time signal $x_c(t)$ is bandlimited to frequencies below 5 kHz; i.e., $X_c(j\Omega) = 0$ for $|\Omega| \geq 2\pi(5000)$. The signal $x_c(t)$ is sampled with a sampling rate of 10,000 samples per second (10 kHz) to produce a sequence $x[n] = x_c(nT)$ with $T = 10^{-4}$. Let $X[k]$ be the 1000-point DFT of $x[n]$.

(a) To what continuous-time frequency does the index $k = 150$ in $X[k]$ correspond?

(b) To what continuous-time frequency does the index $k = 800$ in $X[k]$ correspond?

<u>**Problem 4.**</u> 10.2

10.2. A continuous-time signal $x_c(t)$ is bandlimited to 5 kHz; i.e., $X_c(j\Omega) = 0$ for $|\Omega| \geq 2\pi(5000)$. $x_c(t)$ is sampled with period T, producing the sequence $x[n] = x_c(nT)$. To examine the spectral properties of the signal, we compute the N-point DFT of a segment of N samples of $x[n]$ using a computer program that requires $N = 2^v$, where v is an integer. Determine the *minimum* value for N and the range of sampling rates

$$F_{\min} < \frac{1}{T} < F_{\max}$$

such that aliasing is avoided and the effective spacing between DFT values is *less* than 5 Hz; i.e., the equivalent continuous-time frequencies at which the Fourier transform is evaluated are separated by less than 5 Hz.

<u>**Problem 5.**</u> 10.6

10.6. Let $x_c(t)$ be a real-valued, bandlimited signal whose Fourier transform $X_c(j\Omega)$ is zero for $|\Omega| \geq 2\pi(5000)$. The sequence $x[n]$ is obtained by sampling $x_c(t)$ at 10 kHz. Assume that the sequence $x[n]$ is zero for $n < 0$ and $n > 999$.

Let $X[k]$ denote the 1000-point DFT of $x[n]$. It is known that $X[900] = 1$ and $X[420] = 5$. Determine $X_c(j\Omega)$ for as many values of Ω as you can in the region $|\Omega| < 2\pi(5000)$.

<u>**Problem 6.**</u> 10.12

10.12. Assume that $x[n]$ is a 1000-point sequence obtained by sampling a continuous-time signal $x_c(t)$ at 8 kHz and that $X_c(j\Omega)$ is sufficiently bandlimited to avoid aliasing. What is the minimum DFT length N such that adjacent samples of $X[k]$ correspond to a frequency spacing of 5 Hz or less in the original continuous-time signal?

Problem 7. 10.21

10.21. Let $x[n] = \cos(2\pi n/5)$ and $v[n]$ be the sequence obtained by applying a 32-point rectangular window to $x[n]$ before computing $V(e^{j\omega})$. Sketch $|V(e^{j\omega})|$ for $-\pi \leq \omega \leq \pi$, labeling the frequencies of all peaks and the first nulls on either side of the peak. In addition, label the amplitudes of the peaks and the strongest side lobe of each peak.

Problem 8. 10.23

10.23. Consider a real time-limited continuous-time signal $x_c(t)$ whose duration is 100 ms. Assume that this signal has a bandlimited Fourier transform such that $X_c(j\Omega) = 0$ for $|\Omega| \geq 2\pi(10,000)$ rad/s; i.e., assume that aliasing is negligible. We want to compute samples of $X_c(j\Omega)$ with 5-Hz spacing over the interval $0 \leq \Omega \leq 2\pi(10,000)$. This can be done with a

4000-point DFT. Specifically, we want to obtain a 4000-point sequence $x[n]$ for which the 4000-point DFT is related to $X_c(j\Omega)$ by

$$X[k] = \alpha X_c(j2\pi \cdot 5 \cdot k), \qquad k = 0, 1, \ldots, 1999,$$

where α is a known scale factor. Three methods are proposed to obtain a 4000-point sequence whose DFT gives the desired samples of $X_c(j\Omega)$.

METHOD 1: $x_c(t)$ is sampled with a sampling period $T = 25\ \mu$s; i.e., we compute $X_1[k]$, the DFT of the sequence

$$x_1[n] = \begin{cases} x_c(nT), & n = 0, 1, \ldots, 3999, \\ 0, & \text{otherwise.} \end{cases}$$

Since $x_c(t)$ is time limited to 100 ms, $x_1[n]$ is a finite-length sequence of length 4000 (100 ms/25 μs).

METHOD 2: $x_c(t)$ is sampled with a sampling period of $T = 50\ \mu$s. Since $x_c(t)$ is time limited to 100 ms, the resulting sequence will have only 2000 (100 ms/50 μs) nonzero samples; i.e.,

$$x_2[n] = \begin{cases} x_c(nT), & n = 0, 1, \ldots, 1999, \\ 0, & \text{otherwise.} \end{cases}$$

In other words, the sequence is padded with zero-samples to create a 4000-point sequence for which the 4000-point DFT $X_2[k]$ is computed.

METHOD 3: $x_c(t)$ is sampled with a sampling period of $T = 50\ \mu$s, as in Method 2. The resulting 2000-point sequence is used to form the sequence $x_3[n]$ as follows:

$$x_3[n] = \begin{cases} x_c(nT), & 0 \leq n \leq 1999, \\ x_c((n - 2000)T), & 2000 \leq n \leq 3999, \\ 0, & \text{otherwise.} \end{cases}$$

The 4000-point DFT $X_3[k]$ of this sequence is computed.

For each of the three methods, determine how each 4000-point DFT is related to $X_c(j\Omega)$. Indicate this relationship in a sketch for a "typical" Fourier transform $X_c(j\Omega)$. State explicitly which method(s) provide the desired samples of $X_c(j\Omega)$.