ECE 6342 Fall 2016 HW 11 due 11/22 Problems from Discrete-time Signal Processing - Oppenheim , 2nd ed

Problem 1. **8.6.** Consider the complex sequence

$$x[n] = \begin{cases} e^{j\omega_0 n}, & 0 \le n \le N-1, \\ 0, & \text{otherwise}. \end{cases}$$

(a) Find the Fourier transform $X(e^{j\omega})$ of $x[n]$.
(b) Find the N-point DFT $X[k]$ of the finite-length sequence $x[n]$.
(c) Find the DFT of $x[n]$ for the case $\omega_0 = 2\pi k_0/N$, where k_0 is an integer.

Problem 2. **8.8.** Let $X(e^{j\omega})$ denote the Fourier transform of the sequence $x[n] = \left(\frac{1}{2}\right)^n u[n]$. Let $y[n]$ denote a finite-duration sequence of length 10; i.e., $y[n] = 0, n < 0$, and $y[n] = 0, n \ge 10$. The 10-point DFT of $y[n]$, denoted by $Y[k]$, corresponds to 10 equally spaced samples of $X(e^{j\omega})$; i.e., $Y[k] = X(e^{j2\pi k/10})$. Determine $y[n]$.

Problem 3. **8.10.** The two eight-point sequences $x_1[n]$ and $x_2[n]$ shown in Figure P8.10-1 have DFTs $X_1[k]$ and $X_2[k]$, respectively. Determine the relationship between $X_1[k]$ and $X_2[k]$.

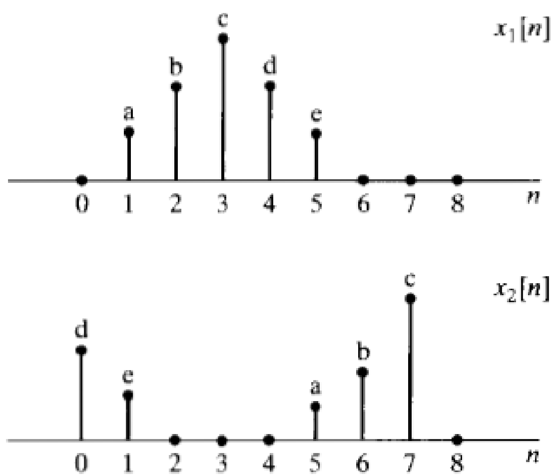

Figure P8.10-1

Problem 4. **8.14.** Two finite-length signals, $x_1[n]$ and $x_2[n]$, are sketched in Figure P8.14-1. Assume that $x_1[n]$ and $x_2[n]$ are zero outside of the region shown in the figure. Let $x_3[n]$ be the eight-point circular convolution of $x_1[n]$ with $x_2[n]$; i.e., $x_3[n] = x_1[n] \circledast_8 x_2[n]$. Determine $x_3[2]$.

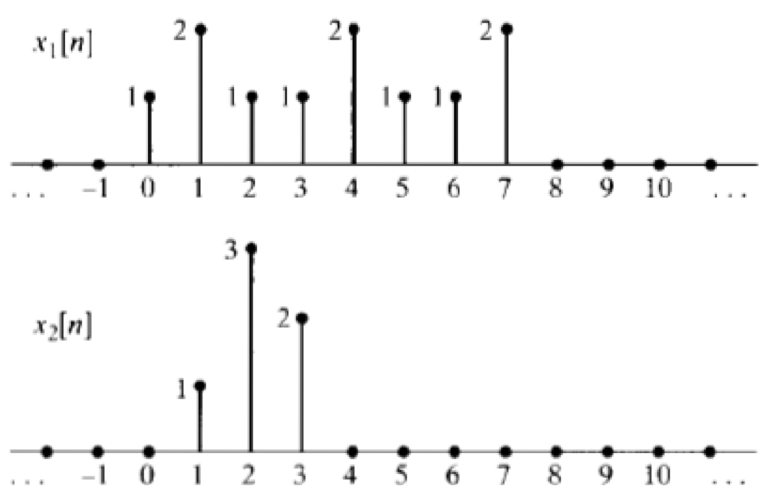

Figure P8.14-1

Problem 5. 8.15

8.15. Figure P8.15-1 shows two sequences $x_1[n]$ and $x_2[n]$. The value of $x_2[n]$ at time $n = 3$ is not known, but is shown as a variable a. Figure P8.15-2 shows $y[n]$, the four-point circular convolution of $x_1[n]$ and $x_2[n]$. Based on the graph of $y[n]$, can you specify a uniquely? If so, what is a? If not, give two possible values of a that would yield the sequence $y[n]$ as shown.

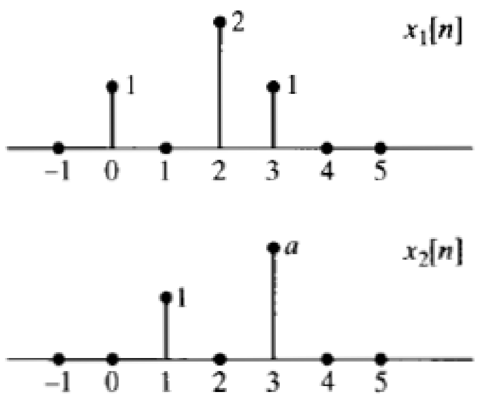

Figure P8.15-1

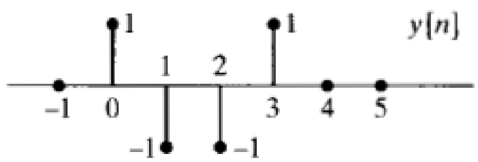

Figure P8.15-2

Problem 6. 8.20

8.20. Two finite-length sequences $x[n]$ and $x_1[n]$ are shown in Figure P8.20-1. The N-point DFTs of these sequences, $X[k]$ and $X_1[k]$, respectively, are related by the equation

$$X_1[k] = X[k]e^{j2\pi k2/N},$$

where N is an unknown constant. Can you determine a value of N consistent with Figure P8.20-1? Is your choice for N unique? If so, justify your answer. If not, find another choice of N consistent with the information given.

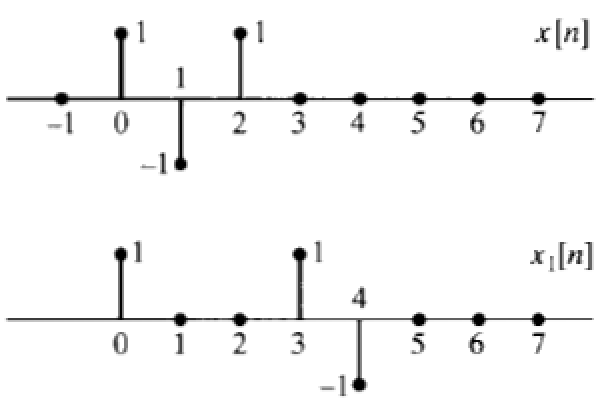

Figure P8.20-1

Problem 7. **8.23.** Consider a finite-duration sequence $x[n]$ of length P such that $x[n] = 0$ for $n < 0$ and $n \geq P$. We want to compute samples of the Fourier transform at the N equally spaced frequencies

$$\omega_k = \frac{2\pi k}{N}, \qquad k = 0, 1, \ldots, N-1.$$

Determine and justify procedures for computing the N samples of the Fourier transform using only one N-point DFT for the following two cases:

(a) $N > P$.

(b) $N < P$.

Problem 8.

8.31. The DFT of a finite-duration sequence corresponds to samples of its z-transform on the unit circle. For example, the DFT of a 10-point sequence $x[n]$ corresponds to samples of $X(z)$ at the 10 equally spaced points indicated in Figure P8.31-1. We wish to find the equally spaced samples of $X(z)$ on the contour shown in Figure P8.31-2; i.e., we wish to obtain

$$X(z)\Big|_{z=0.5e^{j[(2\pi k/10)+(\pi/10)]}}.$$

Show how to modify $x[n]$ to obtain a sequence $x_1[n]$ such that the DFT of $x_1[n]$ corresponds to the desired samples of $X(z)$.

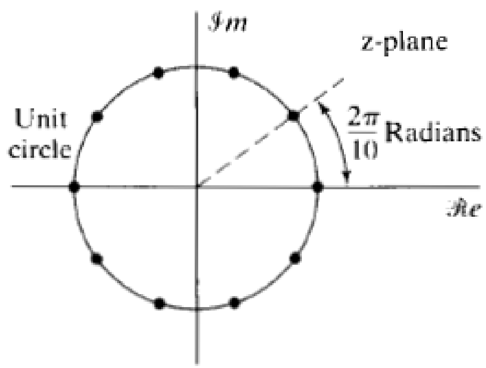

Figure P8.31-1

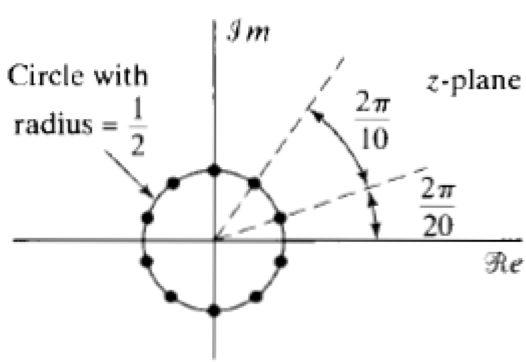

Figure P8.31-2