ECE 6342 Fall 2016 HW 10 due 11/15 Problems from Discrete-time Signal Processing - Oppenheim , 2nd edition ,

Problem 1. **7.10.** We wish to design a discrete-time lowpass filter using the bilinear transformation on a continuous-time ideal lowpass filter. Assume that the continuous-time prototype filter has cutoff frequency $\Omega_c = 2\pi(2000)$ rad/s and we choose the bilinear transformation parameter $T = 0.4$ ms. What is the cutoff frequency ω_c for the resulting discrete-time filter?

Problem 2. **7.12.** An ideal discrete-time highpass filter with cutoff frequency $\omega_c = \pi/2$ was designed using the bilinear transformation with $T = 1$ ms. What was the cutoff frequency Ω_c for the prototype continuous-time ideal highpass filter?

Problem 3. **7.16.** We wish to design an FIR lowpass filter satisfying the specifications

$$0.98 < H(e^{j\omega}) < 1.02, \qquad 0 \le |\omega| \le 0.63\pi,$$
$$-0.15 < H(e^{j\omega}) < 0.15, \qquad 0.65\pi \le |\omega| \le \pi,$$

by applying a Kaiser window to the impulse response $h_d[n]$ for the ideal discrete-time lowpass filter with cutoff $\omega_c = 0.64\pi$. Find the values of β and M required to satisfy this specification.

Problem 4. **7.23.** A continuous-time filter with impulse response $h_c(t)$ and frequency-response magnitude

$$|H_c(j\Omega)| = \begin{cases} |\Omega|, & |\Omega| < 10\pi, \\ 0, & |\Omega| > 10\pi, \end{cases}$$

is to be used as the prototype for the design of a discrete-time filter. The resulting discrete-time system is to be used in the configuration of Figure P7.23-1 to filter the continuous-time signal $x_c(t)$.

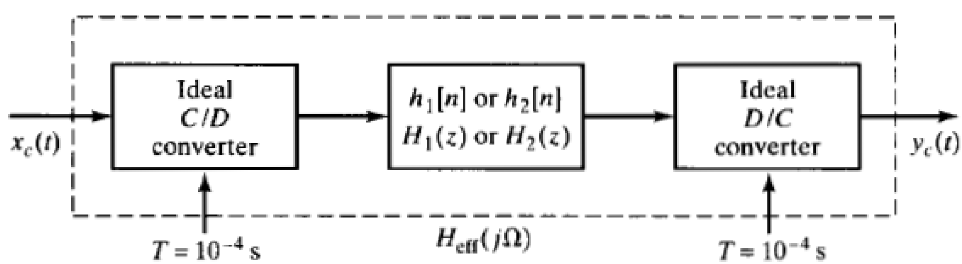

Figure P7.23-1

(a) A discrete-time system with impulse response $h_1[n]$ and system function $H_1(z)$ is obtained from the prototype continuous-time system by impulse invariance with $T_d = 0.01$; i.e., $h_1[n] = 0.01h_c(0.01n)$. Plot the magnitude of the overall effective frequency response $H_{\text{eff}}(j\Omega) = Y_c(j\Omega)/X_c(j\Omega)$ when this discrete-time system is used in Figure P7.23-1.

(b) Alternatively, suppose that a discrete-time system with impulse response $h_2[n]$ and system function $H_2(z)$ is obtained from the prototype continuous-time system by the bilinear transformation with $T_d = 2$; i.e.,

$$H_2(z) = H_c(s)\big|_{s=(1-z^{-1})/(1+z^{-1})}.$$

Plot the magnitude of the overall effective frequency response $H_{\text{eff}}(j\Omega)$ when this discrete-time system is used in Figure P7.23-1.

Problem 5.
7.27

7.27. Suppose that we are given an ideal lowpass discrete-time filter with frequency response

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \pi/4, \\ 0, & \pi/4 < |\omega| \le \pi. \end{cases}$$

We wish to derive new filters from this prototype by manipulations of the impulse response $h[n]$.

(a) Plot the frequency response $H_1(e^{j\omega})$ for the system whose impulse response is $h_1[n] = h[2n]$.

(b) Plot the frequency response $H_2(e^{j\omega})$ for the system whose impulse response is

$$h_2[n] = \begin{cases} h[n/2], & n = 0, \pm 2, \pm 4, \dots, \\ 0, & \text{otherwise}. \end{cases}$$

(c) Plot the frequency response $H_3(e^{j\omega})$ for the system whose impulse response is $h_3[n] = e^{j\pi n}h[n] = (-1)^n h[n]$.

Problem 6.
7.28

7.28. Consider a continuous-time lowpass filter $H_c(s)$ with passband and stopband specifications

$$1 - \delta_1 \le |H_c(j\Omega)| \le 1 + \delta_1, \qquad |\Omega| \le \Omega_p,$$
$$|H_c(j\Omega)| < \delta_2, \qquad \Omega_s < |\Omega|.$$

This filter is transformed to a lowpass discrete-time filter $H_1(z)$ by the transformation

$$H_1(z) = H_c(s)\big|_{s=(1-z^{-1})/(1+z^{-1})},$$

and the same continuous-time filter is transformed to a highpass discrete-time filter by the transformation

$$H_2(z) = H_c(s)\big|_{s=(1+z^{-1})/(1-z^{-1})}.$$

(a) Determine a relationship between the passband cutoff frequency Ω_p of the continuous-time lowpass filter and the passband cutoff frequency ω_{p1} of the discrete-time lowpass filter.

(b) Determine a relationship between the passband cutoff frequency Ω_p of the continuous-time lowpass filter and the passband cutoff frequency ω_{p2} of the discrete-time highpass filter.

(c) Determine a relationship between the passband cutoff frequency ω_{p1} of the discrete-time lowpass filter and the passband cutoff frequency ω_{p2} of the discrete-time highpass filter.

(d) The network in Figure P7.28-1 depicts an implementation of the discrete-time lowpass filter with system function $H_1(z)$. The coefficients A, B, C, and D are real. How should these coefficients be modified to obtain a network that implements the discrete-time highpass filter with system function $H_2(z)$?

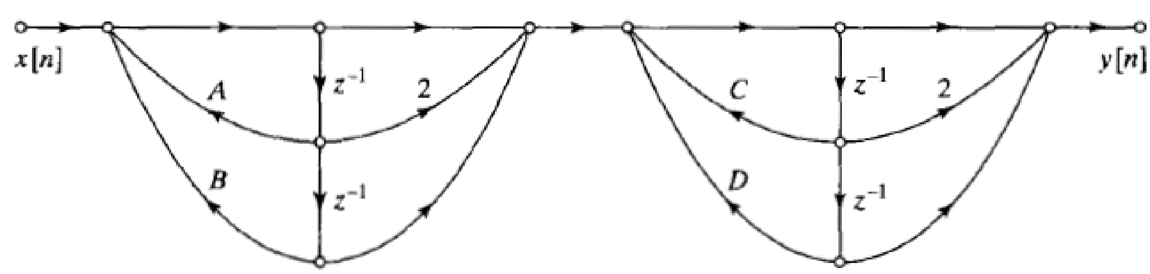

Figure P7.28-1

Problem 7. 8.2

8.2. Suppose $\tilde{x}[n]$ is a periodic sequence with period N. Then $\tilde{x}[n]$ is also periodic with period $3N$. Let $\tilde{X}[k]$ denote the DFS coefficients of $\tilde{x}[n]$ considered as a periodic sequence with period N, and let $\tilde{X}_3[k]$ denote the DFS coefficients of $\tilde{x}[n]$ considered as a periodic sequence with period $3N$.

(a) Express $\tilde{X}_3[k]$ in terms of $\tilde{X}[k]$.

(b) By explicitly calculating $\tilde{X}[k]$ and $\tilde{X}_3[k]$, verify your result in Part (a) when $\tilde{x}[n]$ is as given in Figure P8.2-1.

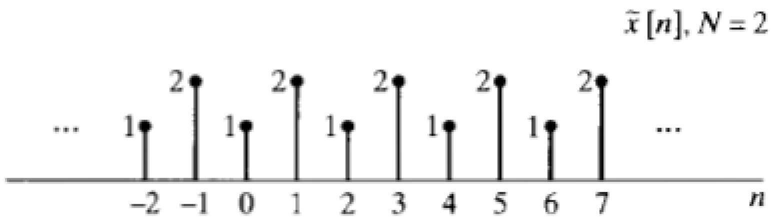

Figure P8.2-1

Problem 8. 8.4

8.4. Consider the sequence $x[n]$ given by $x[n] = \alpha^n u[n]$. A periodic sequence $\tilde{x}[n]$ is constructed from $x[n]$ in the following way:

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n + rN].$$

(a) Determine the Fourier transform $X(e^{j\omega})$ of $x[n]$.

(b) Determine the discrete Fourier series $\tilde{X}[k]$ of $\tilde{x}[n]$.

(c) How is $\tilde{X}[k]$ related to $X(e^{j\omega})$?