ECE 6342 Fall 2016 HW 9 due 11/8

Problems from Discrete-time Signal Processing - Oppenheim 2nd edition

Problem 1.
Discrete-time Signal Processing Oppenheim 2nd edition Problem 6.1

6.17. Consider the causal LTI system with system function

$$H(z) = 1 - \frac{1}{3}z^{-1} + \frac{1}{6}z^{-2} + z^{-3}.$$

(a) Draw the signal flow graph for the direct form implementation of this system.
(b) Draw the signal flow graph for the transposed direct form implementation of the system.

Problem 2.
Discrete-time Signal Processing Oppenheim 2nd edition Problem 6.35

6.35. Consider an all-pass system whose system function is

$$H(z) = \frac{z^{-1} - \frac{1}{3}}{1 - \frac{1}{3}z^{-1}}.$$

(a) Draw the direct form I signal flow graph for the system. How many delays and multipliers do you need? (Do not count multiplying by ± 1.)
(b) Draw a signal flow graph for the system that uses one multiplier. Minimize the number of delays.
(c) Now consider another all-pass system whose system function is

$$H(z) = \frac{(z^{-1} - \frac{1}{3})(z^{-1} - 2)}{(1 - \frac{1}{3}z^{-1})(1 - 2z^{-1})}.$$

Find and draw a signal flow graph for the system with two multipliers and three delays.

Problem 3.
Discrete-time Signal Processing Oppenheim 2nd edition Problem 7.1a

7.1. Consider a causal continuous-time system with impulse response $h_c(t)$ and system function

$$H_c(s) = \frac{s + a}{(s + a)^2 + b^2}.$$

(a) Use impulse invariance to determine $H_1(z)$ for a discrete-time system such that $h_1[n] = h_c(nT)$.
(b) Use step invariance to determine $H_2(z)$ for a discrete-time system such that $s_2[n] = s_c(nT)$, where

$$s_2[n] = \sum_{k=-\infty}^{n} h_2[k] \qquad \text{and} \qquad s_c(t) = \int_{-\infty}^{t} h_c(\tau)d\tau.$$

Problem 4. 7.2

7.2. A discrete-time lowpass filter is to be designed by applying the impulse invariance method to a continuous-time Butterworth filter having magnitude-squared function

$$|H_c(j\Omega)|^2 = \frac{1}{1 + (\Omega/\Omega_c)^{2N}}.$$

The specifications for the discrete-time system are those of Example 7.2, i.e.,

$$0.89125 \le |H(e^{j\omega})| \le 1, \quad 0 \le |\omega| \le 0.2\pi,$$
$$|H(e^{j\omega})| \le 0.17783, \quad 0.3\pi \le |\omega| \le \pi.$$

Assume, as in that example, that aliasing will not be a problem; i.e., design the continuous-time Butterworth filter to meet passband and stopband specifications as determined by the desired discrete-time filter.

(a) Sketch the tolerance bounds on the magnitude of the frequency response, $|H_c(j\Omega)|$, of the continuous-time Butterworth filter such that after application of the impulse invariance method (i.e., $h[n] = T_d h_c(nT_d)$), the resulting discrete-time filter will satisfy the given design specifications. Do not assume that $T_d = 1$ as in Example 7.2.
(b) Determine the integer order N and the quantity $T_d\Omega_c$ such that the continuous-time Butterworth filter exactly meets the specifications determined in Part (a) at the passband edge.
(c) Note that if $T_d = 1$, your answer in Part (b) should give the values of N and Ω_c obtained in Example 7.2. Use this observation to determine the system function $H_c(s)$ for $T_d \neq 1$ and to argue that the system function $H(z)$ which results from impulse invariance design with $T_d \neq 1$ is the same as the result for $T_d = 1$ given by Eq. (7.19).

Problem 5. 7.5

7.5. We wish to use the Kaiser window method to design a discrete-time filter with generalized linear phase that meets specifications of the following form:

$$|H(e^{j\omega})| \le 0.01, \qquad 0 \le |\omega| \le 0.25\pi,$$
$$0.95 \le |H(e^{j\omega})| \le 1.05, \qquad 0.35\pi \le |\omega| \le 0.6\pi,$$
$$|H(e^{j\omega})| \le 0.01, \qquad 0.65\pi \le |\omega| \le \pi.$$

(a) Determine the minimum length $(M+1)$ of the impulse response and the value of the Kaiser window parameter β for a filter that meets the preceding specifications.

(b) What is the delay of the filter?

(c) Determine the ideal impulse response $h_d[n]$ to which the Kaiser window should be applied.

Problem 6. 7.9

7.9. Suppose we design a discrete-time filter using the impulse invariance technique with an ideal continuous-time lowpass filter as a prototype. The prototype filter has a cutoff frequency of $\Omega_c = 2\pi(1000)$ rad/s, and the impulse invariance transformation uses $T = 0.2$ ms. What is the cutoff frequency ω_c for the resulting discrete-time filter?

i.e. T_d=0.2 [msec]