ECE 6342 Fall 2016 HW 6 Due Thurs 10/06

Problems from Discrete-time Signal Processing - Oppenheim 2nd edition Chapter 5

Problem 1. Problem 5.7 - Oppenheim

5.7. When the input to a linear time-invariant system is

$$x[n] = 5u[n],$$

the output is

$$y[n] = \left[2\left(\tfrac{1}{2}\right)^n + 3\left(-\tfrac{3}{4}\right)^n\right]u[n].$$

(a) Find the system function $H(z)$ of the system. Plot the poles and zeros of $H(z)$, and indicate the region of convergence.

(b) Find the impulse response of the system for all values of n.

(c) Write the difference equation that characterizes the system.

Problem 2 Problem 5.10 - Oppenheim

5.10. If the system function $H(z)$ of a linear time-invariant system has a pole–zero diagram as shown in Figure P5.10-1 and the system is causal, can the inverse system $H_i(z)$, where $H(z)H_i(z) = 1$, be both causal and stable? Clearly justify your answer.

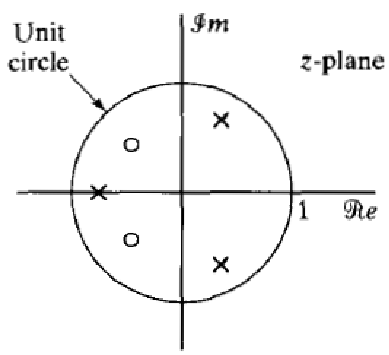

Figure P5.10-1

Problem 3. Problem 5.18 - Oppenheim

5.18. For each of the following system functions $H_k(z)$, specify a minimum-phase system function $H_{\min}(z)$ such that the frequency-response magnitudes of the two systems are equal, i.e., $|H_k(e^{j\omega})| = |H_{\min}(e^{j\omega})|$.

(a)

$$H_1(z) = \frac{1 - 2z^{-1}}{1 + \frac{1}{3}z^{-1}}$$

(b)

$$H_2(z) = \frac{(1 + 3z^{-1})\left(1 - \frac{1}{2}z^{-1}\right)}{z^{-1}\left(1 + \frac{1}{3}z^{-1}\right)}$$

(c)

$$H_3(z) = \frac{(1 - 3z^{-1})\left(1 - \frac{1}{4}z^{-1}\right)}{\left(1 - \frac{3}{4}z^{-1}\right)\left(1 - \frac{4}{3}z^{-1}\right)}$$

Problem 4. Problem 5.24 - Oppenheim

5.24. Let $x[n]$ be a causal, N-point sequence that is zero outside the range $0 \le n \le N-1$. When $x[n]$ is the input to the causal LTI system represented by the difference equation

$$y[n] - \tfrac{1}{4}y[n-2] = x[n-2] - \tfrac{1}{4}x[n],$$

the output is $y[n]$, also a causal, N-point sequence.

(a) Show that the causal LTI system described by this difference equation represents an all-pass filter.

(b) Given that

$$\sum_{n=0}^{N-1} |x[n]|^2 = 5,$$

determine the value of

$$\sum_{n=0}^{N-1} |y[n]|^2.$$

Problem 5. Problem 5.29 - Oppenheim

5.29. The system function of a linear time-invariant system is given by

$$H(z) = \frac{21}{\left(1 - \frac{1}{2}z^{-1}\right)(1 - 2z^{-1})(1 - 4z^{-1})}.$$

It is known that the system is not stable and that the impulse response is two sided.

(a) Determine the impulse response $h[n]$ of the system.

(b) The impulse response found in Part (a) can be expressed as the sum of a causal impulse response $h_1[n]$ and an anticausal impulse response $h_2[n]$. Determine the corresponding system functions $H_1(z)$ and $H_2(z)$.