ECE 6342 Fall 2016 HW 5 Due Thurs 09/29

Problems from Discrete-time Signal Processing - Oppenheim , 2nd edition Chapter 4

Problem 1. Oppenheim 4.17

Problem 2. Oppenheim 4.19

Problem 3 Oppenheim 4.24

Problem 4. Oppenheim 4.26

Problem 5. Oppenheim 4.29

Problem 6. Oppenheim 4.41

Problem 7. Oppenheim 5.2

Problem 8. Oppenheim 5.4

4.17. Each of the following parts lists an input signal $x[n]$ and the upsampling and downsampling rates L and M for the system in Figure 4.28. Determine the corresponding output $\tilde{x}_d[n]$.

(a) $x[n] = \sin(2\pi n/3)/\pi n,\ L = 4,\ M = 3$

(b) $x[n] = \sin(3\pi n/4),\ L = 3,\ M = 5$

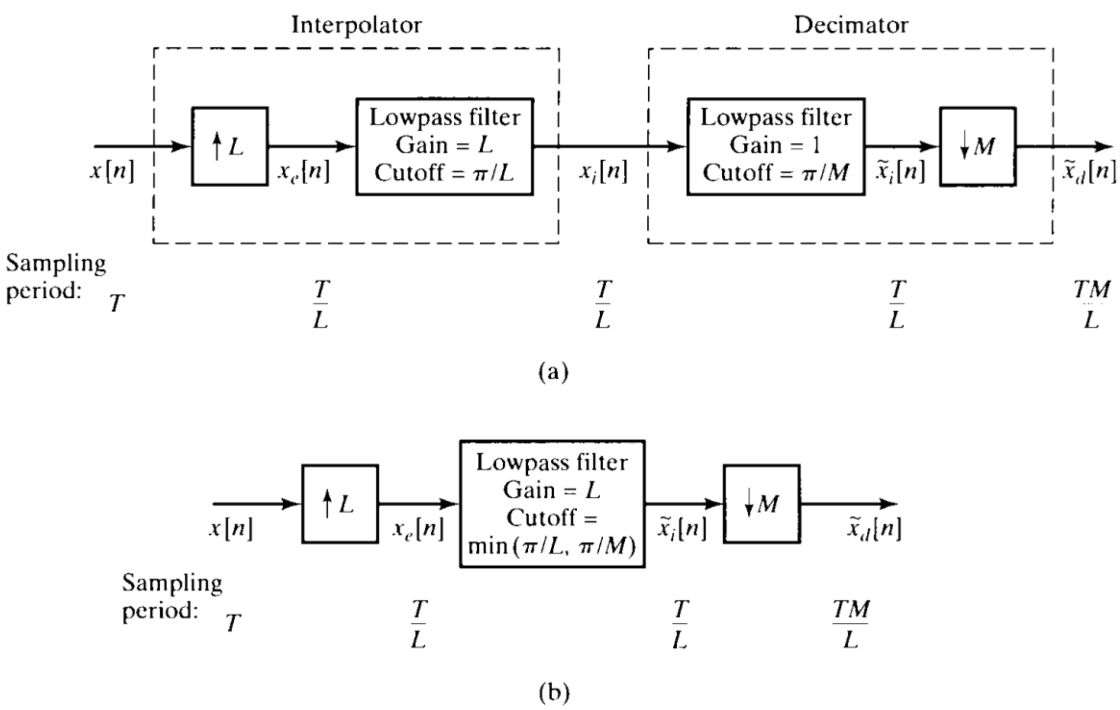

Figure 4.28 (a) System for changing the sampling rate by a noninteger factor. (b) Simplified system in which the decimation and interpolation filters are combined.

4.19. The continuous-time signal $x_c(t)$ with the Fourier transform $X_c(j\Omega)$ shown in Figure P4.19-1 is passed through the system shown in Figure P4.19-2. Determine the range of values for T for which $x_r(t) = x_c(t)$.

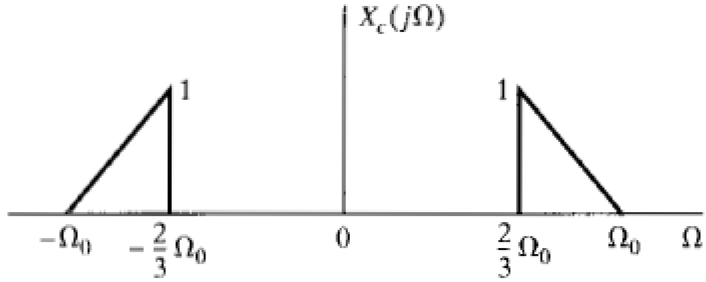

Figure P4.19-1

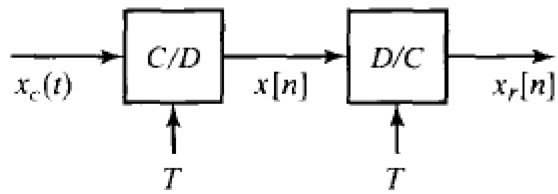

Figure P4.19-2

4.24. In the system of Figure P4.24-1, $X_c(j\Omega)$ and $H(e^{j\omega})$ are as shown. Sketch and label the Fourier transform of $y_c(t)$ for each of the following cases:

(a) $1/T_1 = 1/T_2 = 10^4$

(b) $1/T_1 = 1/T_2 = 2 \times 10^4$

(c) $1/T_1 = 2 \times 10^4, \quad 1/T_2 = 10^4$

(d) $1/T_1 = 10^4, \quad 1/T_2 = 2 \times 10^4$

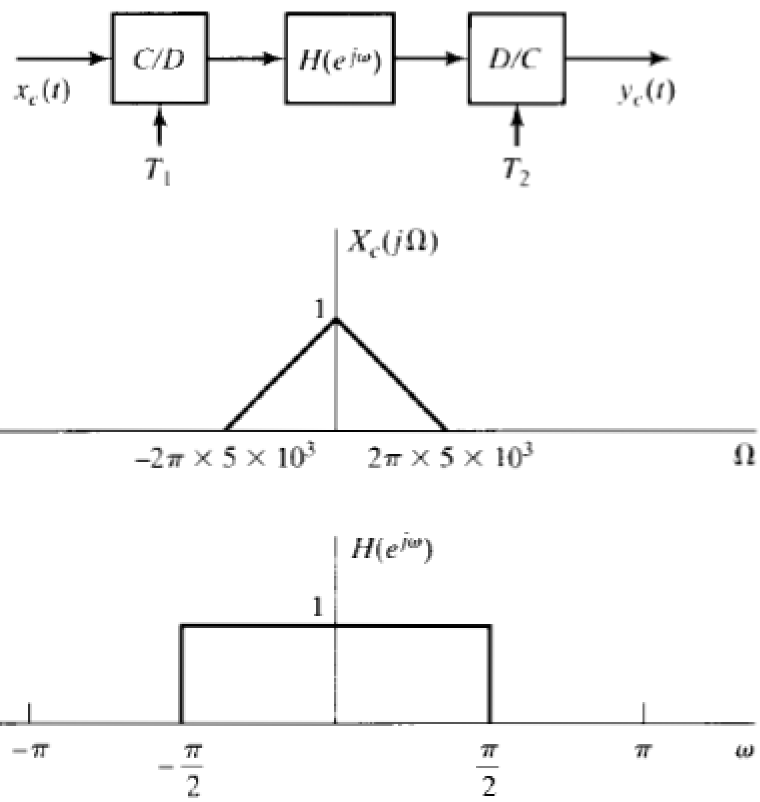

Figure P4.24-1

4.29. Consider the systems shown in Figure P4.29-1. Suppose that $H_1(e^{j\omega})$ is fixed and known. Find $H_2(e^{j\omega})$, the frequency response of an LTI system, such that $y_2[n] = y_1[n]$ if the inputs to the systems are the same.

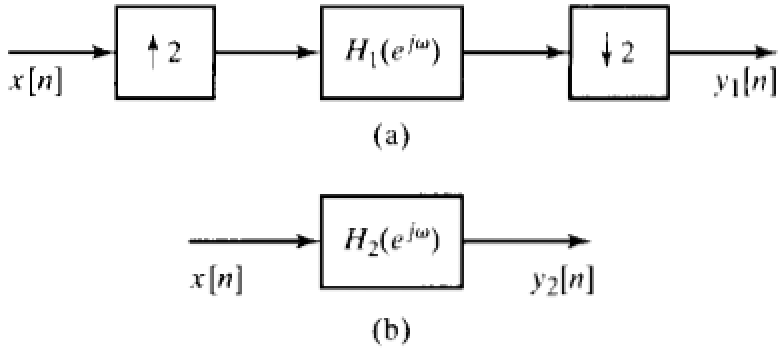

Figure P4.29-1

4.41. Consider the system shown in Figure P4.41-1. The input to this system is the bandlimited signal whose Fourier transform is shown in Figure P4.20-1 with $\Omega_0 = \pi/T$. The discrete-time LTI system in Figure P4.41-1 has the frequency response shown in Figure P4.41-2.

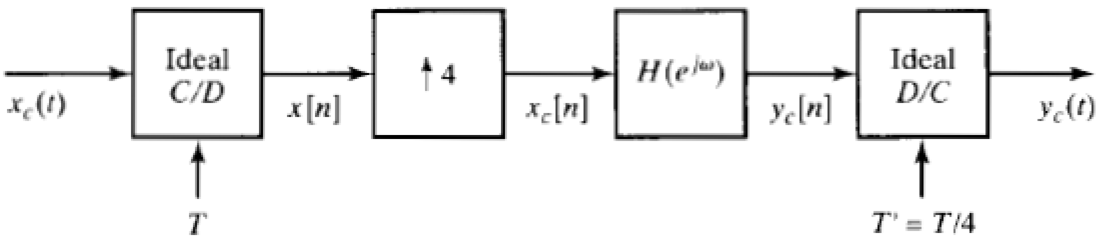

Figure P4.41-1

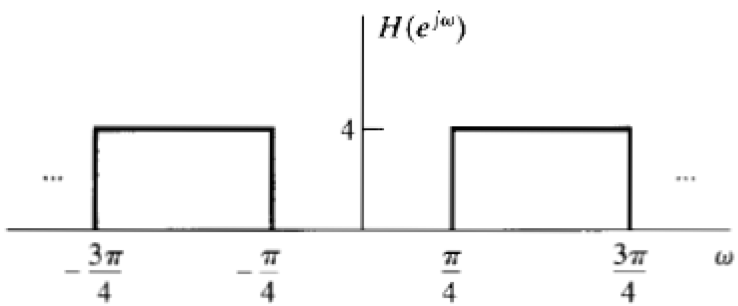

Figure P4.41-2

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c, \\ 0, & \text{otherwise.} \end{cases}$$

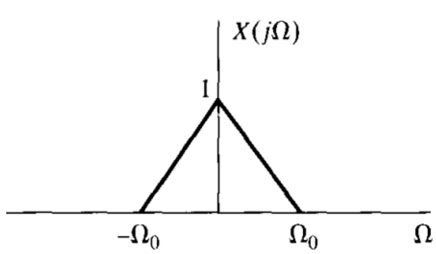

Figure P4.20-1

(a) Sketch the Fourier transforms $X(e^{j\omega})$, $X_e(e^{j\omega})$, $Y_e(e^{j\omega})$, and $Y_c(j\Omega)$.

(b) For the general case when $X_c(j\Omega) = 0$ for $|\Omega| \geq \pi/T$, express $Y_c(j\Omega)$ in terms of $X_c(j\Omega)$. Also, give a general expression for $y_c(t)$ in terms of $x_c(t)$ when $x_c(t)$ is band-limited in this manner.

5.2. Consider a stable linear time-invariant system with input $x[n]$ and output $y[n]$. The input and output satisfy the difference equation

$$y[n-1] - \tfrac{10}{3}y[n] + y[n+1] = x[n].$$

(a) Plot the poles and zeros in the z-plane.

(b) Find the impulse response $h[n]$.

5.4. When the input to a linear time-invariant system is

$$x[n] = \left(\tfrac{1}{2}\right)^n u[n] + (2)^n u[-n-1],$$

the output is

$$y[n] = 6\left(\tfrac{1}{2}\right)^n u[n] - 6\left(\tfrac{3}{4}\right)^n u[n].$$

(a) Find the system function $H(z)$ of the system. Plot the poles and zeros of $H(z)$, and indicate the region of convergence.

(b) Find the impulse response $h[n]$ of the system for all values of n.

(c) Write the difference equation that characterizes the system.

(d) Is the system stable? Is it causal?