ECE 6342 Fall 2016 HW 4 Thurs Due 9/22

Problems from Discrete-time Signal Processing - Oppenheim , 2nd edition Chapter 3

Problem 1. Oppenheim 3.55 (a) only

Problem 2. Oppenheim 4.3

Problem 3. Oppenheim 4.5

Problem 4. Oppenheim 4.6

Problem 5. Oppenheim 4.12

Problem 6. Oppenheim 4.15

3.55. The aperiodic autocorrelation function for a real-valued stable sequence $x[n]$ is defined as

$$c_{xx}[n] = \sum_{k=-\infty}^{\infty} x[k]x[n+k].$$

(a) Show that the z-transform of $c_{xx}[n]$ is

$$C_{xx}(z) = X(z)X(z^{-1}).$$

Determine the region of convergence for $C_{xx}(z)$.

4.3. The continuous-time signal

$$x_c(t) = \cos(4000\pi t)$$

is sampled with a sampling period T to obtain a discrete-time signal

$$x[n] = \cos\left(\frac{\pi n}{3}\right).$$

(a) Determine a choice for T consistent with this information.

(b) Is your choice for T in Part (a) unique? If so, explain why. If not, specify another choice of T consistent with the information given.

4.5. Consider the system of Figure 4.11, with the discrete-time system an ideal lowpass filter with cutoff frequency $\pi/8$ radians/s.

(a) If $x_c(t)$ is bandlimited to 5 kHz, what is the maximum value of T that will avoid aliasing in the C/D converter?

(b) If $1/T = 10$ kHz, what will the cutoff frequency of the effective continuous-time filter be?

(c) Repeat Part (b) for $1/T = 20$ kHz.

4.6. Let $h_c(t)$ denote the impulse response of a linear time-invariant continuous-time filter and $h_d[n]$ the impulse response of a linear time-invariant discrete-time filter.

(a) If

$$h_c(t) = \begin{cases} e^{-at}, & t \geq 0, \\ 0, & t < 0, \end{cases}$$

where a is a positive real constant, determine the continuous-time filter frequency response and sketch its magnitude.

(b) If $h_d[n] = Th_c(nT)$ with $h_c(t)$ as in part (a), determine the discrete-time filter frequency response and sketch its magnitude.

(c) For a given value of a, determine, as a function of T, the minimum magnitude of the discrete-time filter frequency response.

4.12. In the system of Figure 4.11, assume that

$$H(e^{j\omega}) = j\omega/T, \quad -\pi \le \omega < \pi,$$

and $T = 1/10$ sec.

(a) For each of the following inputs $x_c(t)$, find the corresponding output $y_c(t)$.

(i) $x_c(t) = \cos(6\pi t)$

(ii) $x_c(t) = \cos(14\pi t)$

(b) Are the outputs $y_c(t)$ those you would expect from a differentiator?

4.15. Consider the system shown in Figure P4.15-1. For each of the following input signals $x[n]$, indicate whether the output $x_r[n] = x[n]$.

(a) $x[n] = \cos(\pi n/4)$

(b) $x[n] = \cos(\pi n/2)$

(c)

$$x[n] = \left[\frac{\sin(\pi n/8)}{\pi n}\right]^2$$

Hint: Use the modulation property of the Fourier transform to find $X(e^{j\omega})$.

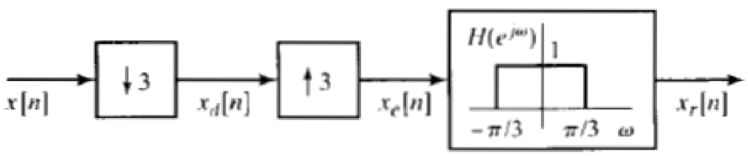

Figure P4.15-1