ECE 6342 Fall 2016 HW 3 Due Thurs 9/15

From Discrete-time Signal Processing - Oppenheim 2nd edition

Problem 1: Oppenheim 3.3

Problem 2. Oppenheim 3.6

Problem 3. Oppenheim 3.9

Problem 4. Oppenheim 3.12

Problem 5. Oppenheim 3.14

Problem 6. Oppenheim 3.22

Problem 7. Oppenheim 3.26

Problem 8. Oppenheim 3.40

3.3. Determine the z-transform of each of the following sequences. Include with your answer the region of convergence in the z-plane and a sketch of the pole–zero plot. Express all sums in closed form; α can be complex.

(a) $x_a[n] = \alpha^{|n|}, \qquad 0 < |\alpha| < 1.$

(b) $x_b[n] = \begin{cases} 1, & 0 \le n \le N-1, \\ 0, & \text{otherwise.} \end{cases}$

(c) $x_c[n] = \begin{cases} n, & 0 \le n \le N, \\ 2N-n, & N+1 \le n \le 2N, \\ 0, & \text{otherwise.} \end{cases}$

Hint: Note that $x_b[n]$ is a rectangular sequence and $x_c[n]$ is a triangular sequence. First express $x_c[n]$ in terms of $x_b[n]$.

3.6. Following are several z-transforms. For each, determine the inverse z-transform using both methods—partial fraction expansion and power series expansion—discussed in Section 3.3. In addition, indicate in each case whether the Fourier transform exists.

(a) $X(z) = \dfrac{1}{1+\frac{1}{2}z^{-1}}, \qquad |z| > \frac{1}{2}$

(b) $X(z) = \dfrac{1}{1+\frac{1}{2}z^{-1}}, \qquad |z| < \frac{1}{2}$

(c) $X(z) = \dfrac{1-\frac{1}{2}z^{-1}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}, \qquad |z| > \frac{1}{2}$

(d) $X(z) = \dfrac{1-\frac{1}{2}z^{-1}}{1-\frac{1}{4}z^{-2}}, \qquad |z| > \frac{1}{2}$

(e) $X(z) = \dfrac{1-az^{-1}}{z^{-1}-a}, \qquad |z| > |1/a|$

3.9. A causal LTI system has impulse response $h[n]$, for which the z-transform is

$$H(z) = \frac{1+z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}.$$

(a) What is the region of convergence of $H(z)$?

(b) Is the system stable? Explain.

(c) Find the z-transform $X(z)$ of an input $x[n]$ that will produce the output

$$y[n] = -\tfrac{1}{3}\left(-\tfrac{1}{4}\right)^n u[n] - \tfrac{4}{3}(2)^n u[-n-1].$$

(d) Find the impulse response $h[n]$ of the system.

3.12. Sketch the pole–zero plot for each of the following z-transforms and shade the region of convergence:

(a) $X_1(z) = \dfrac{1 - \frac{1}{2}z^{-1}}{1 + 2z^{-1}}$, ROC: $|z| < 2$

(b) $X_2(z) = \dfrac{1 - \frac{1}{3}z^{-1}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 - \frac{2}{3}z^{-1}\right)}$, $x_2[n]$ causal

(c) $X_3(z) = \dfrac{1 + z^{-1} - 2z^{-2}}{1 - \frac{13}{6}z^{-1} + z^{-2}}$, $x_3[n]$ absolutely summable.

3.14. If $H(z) = \dfrac{1}{1 - \frac{1}{4}z^{-2}}$ and $h[n] = A_1\alpha_1^n u[n] + A_2\alpha_2^n u[n]$, determine the values of A_1, A_2, α_1, and α_2.

3.22. Consider an LTI system that is stable and for which $H(z)$, the z-transform of the impulse response, is given by

$$H(z) = \frac{3}{1 + \frac{1}{3}z^{-1}}.$$

Suppose $x[n]$, the input to the system, is a unit step sequence.

(a) Find the output $y[n]$ by evaluating the discrete convolution of $x[n]$ and $h[n]$.

(b) Find the output $y[n]$ by computing the inverse z-transform of $Y(z)$.

3.26. Determine the inverse z-transform of each of the following. In Parts (a)–(c), use the methods specified. In Part (d), use any method you prefer.

(a) Long division:

$$X(z) = \frac{1 - \frac{1}{3}z^{-1}}{1 + \frac{1}{3}z^{-1}}, \qquad x[n] \text{ a right-sided sequence}$$

(b) Partial fraction:

$$X(z) = \frac{3}{z - \frac{1}{4} - \frac{1}{8}z^{-1}}, \qquad x[n] \text{ stable}$$

(c) Power series:

$$X(z) = \ln(1 - 4z), \qquad |z| < \tfrac{1}{4}$$

(d) $X(z) = \dfrac{1}{1 - \frac{1}{3}z^{-3}}$, $|z| > (3)^{-1/3}$

3.32. The pole–zero diagram in Figure P3.32-1 corresponds to the z-transform $X(z)$ of a causal sequence $x[n]$. Sketch the pole–zero diagram of $Y(z)$, where $y[n] = x[-n+3]$. Also, specify the region of convergence for $Y(z)$.

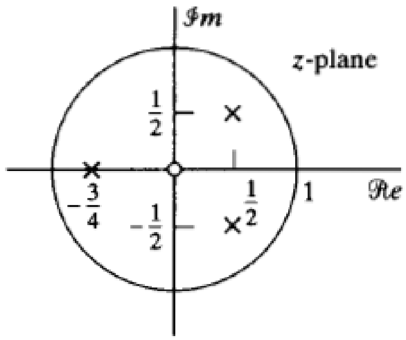

Figure P3.32-1

3.40. In Figure P3.40-1, $H(z)$ is the system function of a causal LTI system.

(a) Using z-transforms of the signals shown in the figure, obtain an expression for $W(z)$ in the form

$$W(z) = H_1(z)X(z) + H_2(z)E(z),$$

where both $H_1(z)$ and $H_2(z)$ are expressed in terms of $H(z)$.

(b) For the special case $H(z) = z^{-1}/(1 - z^{-1})$, determine $H_1(z)$ and $H_2(z)$.

(c) Is the system $H(z)$ stable? Are the systems $H_1(z)$ and $H_2(z)$ stable?

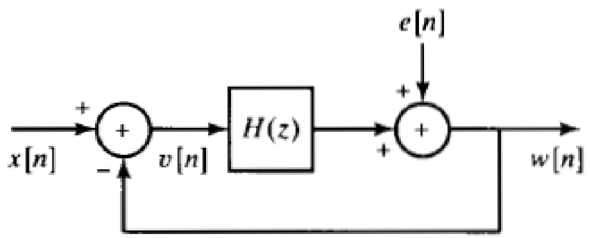

Figure P3.40-1