ECE 6342 Fall 2016 HW 2 Due 9/08

Problems from Discrete-time Signal Processing - Oppenheim , 2nd edition

Problem 1: Oppenheim problem 2.6

2.6. **(a)** Find the frequency response $H(e^{j\omega})$ of the linear time-invariant system whose input and output satisfy the difference equation

$$y[n] - \tfrac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2].$$

(b) Write a difference equation that characterizes a system whose frequency response is

$$H(e^{j\omega}) = \frac{1 - \frac{1}{2}e^{-j\omega} + e^{-j3\omega}}{1 + \frac{1}{2}e^{-j\omega} + \frac{3}{4}e^{-j2\omega}}.$$

Problem 2. Oppenheim problem 2.8

2.8. An LTI system has impulse response $h[n] = 5(-1/2)^n u[n]$. Use the Fourier transform to find the output of this system when the input is $x[n] = (1/3)^n u[n]$.

Problem 3. Oppenheim problem 2.13

2.13. Indicate which of the following discrete-time signals are eigenfunctions of stable, linear time-invariant discrete-time systems:

(a) $e^{j2\pi n/3}$

(b) 3^n

(c) $2^n u[-n-1]$

(d) $\cos(\omega_0 n)$

(e) $(1/4)^n$

(f) $(1/4)^n u[n] + 4^n u[-n-1]$

Problem 4. Oppenheim problem 2.31

2.31. Consider the difference equation

$$y[n] + \frac{1}{15}y[n-1] - \frac{2}{5}y[n-2] = x[n].$$

(a) Determine the general form of the homogeneous solution to this equation.

(b) Both a causal and an anticausal LTI system are characterized by the given difference equation. Find the impulse responses of the two systems.

(c) Show that the causal LTI system is stable and the anticausal LTI system is unstable.

(d) Find a particular solution to the difference equation when $x[n] = (3/5)^n u[n]$.

Problem 5. Oppenheim problem 2.41. Find the output from the system.

2.41. A linear time-invariant system has frequency response

$$H(e^{j\omega}) = \begin{cases} e^{-j\omega 3}, & |\omega| < \frac{2\pi}{16}\left(\frac{3}{2}\right), \\ 0, & \frac{2\pi}{16}\left(\frac{3}{2}\right) \le |\omega| \le \pi. \end{cases}$$

The input to the system is a periodic unit-impulse train with period $N = 16$; i.e.,

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n + 16k].$$

Problem 6. Oppenheim problem 2.46

2.46. A sequence has the discrete-time Fourier transform

$$X(e^{j\omega}) = \frac{1-a^2}{(1-ae^{-j\omega})(1-ae^{j\omega})}, \qquad |a| < 1.$$

(a) Find the sequence $x[n]$.

(b) Calculate $\int_{-\pi}^{\pi} X(e^{j\omega})\cos(\omega)d\omega/2\pi$.

Problem 7. Oppenheim problem 2.53

2.53. Consider the cascade of LTI discrete-time systems shown in Figure P2.53-1.

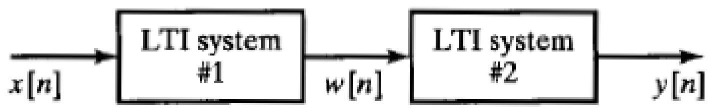

Figure P2.53-1

The first system is described by the equation

$$H_1(e^{j\omega}) = \begin{cases} 1, & |\omega| < 0.5\pi, \\ 0, & 0.5\pi \le |\omega| < \pi, \end{cases}$$

and the second system is described by the equation

$$y[n] = w[n] - w[n-1].$$

The input to this system is

$$x[n] = \cos(0.6\pi n) + 3\delta[n-5] + 2.$$

Determine the output $y[n]$. With careful thought, you will be able to use the properties of LTI systems to write down the answer by inspection.