All The President's Money: Market Concentration, Oligarchs and Sanctions in Hybrid Regimes*

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Abstract

Countries with political regimes described as "hybrid" also have similarities in their economic systems. They combine a market economy with distortive policies driven by the personal interests of an autocratic leader who extracts income from a clientele of connected firms. We study the economic implications of this system in general equilibrium, derive the leader's incentives for creating economic distortions, and explain the emergence of oligarchs as a function of institutional constraints. We use the model to study the economic impacts of various sanctions, such as the freezing of the leader's or the oligarchs' assets, or the withholding of international transfers. Our results shed light on several recent and historical examples from hybrid regimes around the world.

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1 Introduction

The politics of regimes like contemporary Russia, Turkey, or Hungary is often described as "hybrid," featuring both democratic and autocratic elements, and a large literature analyzes features of the hybrid political model. The economies of these regimes also share a number of similarities. Fundamentally, they each combine a market economy with distortionary economic policies driven by the personal interests of an autocratic leader that lead to the enrichment of a select group of oligarchs. Yet, we lack a corresponding hybrid economic model that would allow a systematic analysis of the economic implications of these regimes.

What is the impact of hybrid leaders on economic (as opposed to political) competition? What is the economic role of oligarchs and entrepreneurs with ties to the leader? More broadly, how do the leader's economic interests shape markets? Studying these questions is all the more important given that hybrid regimes are often the target of economic sanctions from the international community. When considering the potential impact of sanctions, policy evaluation is typically based on informal theoretical discussions. Modeling the economy of a hybrid regime makes it possible to evaluate these arguments formally.

For the purpose of this paper, we define a hybrid economic regime as a general-equilibrium market economy in which the political leader engages in constrained income extraction by capturing industries. This definition incorporates three ideas about hybrid leaders' interaction with the economy. First, the leader extracts income from the private economy through clientelistic arrangements that we refer to as *industry capture*. Dawisha (2015) summarizes the Russian case as follows:

> "Russian leaders needed "private" money [...] and they intended to get it, through more effective taxation but also through new arrangements with oligarchs that would provide more revenue for the state. [...] [This involved] oligarchs sharing their profits with the state and with Kremlin officeholders, including Putin, in return for a license to do business. Putin wanted the oligarchs to understand that they would have rents from these companies only as a reward for loyal state service." (p277).

Industry capture, the extraction of profits in return for a license to do business, may be contrasted with regulatory capture (Stigler, 1971), which is common in democracies, where firms have the power to influence policy makers (e.g., in the form of lobbying). In hybrid regimes, it is the leader who has the bargaining power, and (as in the above quote) uses it to dictate the terms of clientelistic arrangements. The emergence of industry capture reflects a combination of factors, such as leaders' need to finance their political survival (e.g., by paying for propaganda), the weakness of institutional checks and balances that would prevent income extraction, and ultimately the selection of leaders who place high value on this income.

Second, profit extraction by the leader faces constraints, which distinguishes hybrid regimes from full-scale dictatorships. Entrepreneurs cannot simply be forced to become a leader's clients and operate firms. There are economic opportunities that are not tied to the leader, and entrepreneurs can choose an occupation that is not in the leader's orbit. This gives rise to a *participation constraint*. Even if entrepreneurs agree to enter into a clientelistic contract with the leader, the leader's power to control and monitor them is not unlimited. The contract has to ensure that clients do not have an incentive to abscond with or hide their profits instead of sharing it with the leader. This creates an *enforcement constraint*.

Third, industry capture by the leader is a *regime* that is widespread enough to affect a large part of the economy. This is in contrast with episodes of clientelism in established democracies, the impact of which is typically more limited. We represent this by modeling the effects of industry capture in a multisector general equilibrium economy.

The above elements appear in a variety of countries and political systems. Without attempting to pinpoint any particular country's location on an autocracy-hybrid-democracy scale, we will argue that a model focusing on these elements is a useful vehicle to study systems that others have described as hybrid, like Russia under Yeltsin and Putin, Turkey under Erdogan, or Hungary under Orban.

The setup, which we present in Section 2, features a continuum of agents who establish firms, work, and consume in a multisector economy with monopolistic competition akin to Dixit and Stiglitz (1977). We do not model the politics of how the leader gets elected or acquires specific powers. Instead, we simply assume that he has the ability to capture industries: (i) he determines who can become an entrepreneur and establish firms, and (ii) he extracts income from these entrepreneurs in return for the right to operate. These actions are subject to the participation and enforcement constraints described above. When deciding on firm entry and income extraction, the leader considers both his income and social welfare.

Section 3 derives the core implications of this model. In a benchmark with a purely welfare-maximizing leader, the clientelistic system replicates a free-entry economy: the leader chooses the same number of firms as the market would, and extracts no income. When his value of extracted income is positive, however, the leader chooses to restrict entry in order to increase profits. The reason for this is that, in equilibrium, increasing market concentration is necessary to create the extra income that can be extracted. This in turn leads to welfare losses. An immediate implication is that checks and balances that limit the opportunities for industry capture, or improving the political selection of leaders with low value for extracting income, lead to more competitive markets and higher social welfare.

As the value of extracted income grows (reflecting, e.g., weaker institutions or worse political selection), the distortions from industry capture become more severe. However, the leader's limited power over his clients endogenously mitigates this effect. As clients' profits increase, their incentive to abscond also rises, and this can only be offset by providing them with rents. A leader with low power over his clients or a moderate value of extracted income finds this too costly, and prefers to extract less.

As the leader's power or the value of extracted income grows beyond this range, a qualitatively different regime emerges. Entry restrictions and profits rise, but the latter is now shared with clients who receive rents - they become "oligarchs."[1] Thus, oligarchs are associated with particularly severe economic distortions and welfare losses. However, oligarchs with large rents are a symptom, not a cause of these negative effects: they reflect a leader who values his income enough that he extracts more even if this requires providing rents to clients. In fact, for such a leader the need to deal with oligarchs acts as a moderating force on his actions. As the leader's power rises, this moderating force weakens. Hybrid regimes with generally strong leaders (i.e., leaders with more power over their clients across many industries) can eschew rents *and* are particularly harmful for competition and welfare.

As we explain in detail, these results can shed light on the rise of Russian oligarchs under Yeltsin and their subsequent weakening under Putin, and predict an increase in market concentration in both cases. In other countries, hybrid leaders' incentive to restrict economic competition can explain protectionist policies or why privatization is not accompanied by market liberalization.

We also study settings where the leader's power over clients is industry-specific. For example, the leader is likely to have more power over a real estate developer whose business is heavily dependent on government licenses and regulations, than over a technology company producing for the international market. We show that leaders optimally concentrate clientelism, and hence distortions, in industries where they have more power. We also show that competition in an industry benefits (in a second-best sense) from a leader with extensive powers in *other* industries. Intuitively, if the leader's power in an industry goes up, he will choose to extract more profit from that industry, which increases the marginal utility of raising welfare by allowing more competition elsewhere in the economy. At the same time, we show that the leader's power in other industries gives him an incentive to impose entry restrictions even in an industry from which he cannot extract income.

These general equilibrium effects show the potential of clientelism anywhere in the econ-

[1] It may seem unusual that our use of the term "oligarch" does not presuppose any political influence. In our model, oligarchs are entrepreneurs who receive rents based on their clientelistic arrangement with the leader. In turn these rents can shape their political preferences, and we study this explicitly in an extension.

omy to impact the entire economy. In theory, the free-entry outcome in *any* industry can only be achieved by eliminating the leader's power over *all* industries.

In Section 4, we use our model of the economy under a hybrid leader to study various sanctions - a tool of international policy making that has seen a sharp increase since the end of the Cold War. The overarching message from our model is clear: the impact of sanctions depends on how the various actors (firms, oligarchs, and the leader) will respond to them as they pursue their objectives. Broad economic sanctions can hurt consumers and social welfare without any offsetting benefits. Sanctions that directly target the leader or the oligarchs (sometimes called "smart" sanctions in the literature) can lower the leader's income but give rise to undesirable policy responses. For example, a sanction that freezes the leader's assets will increase his marginal utility of income. The leader responds by extracting more income from his clients, which requires higher profits and more entry restrictions. Freezing oligarchs' assets has essentially the same consequences, because an oligarch with reduced income will need to be compensated with higher rents by the leader in order to keep him from absconding. To be successful at lowering the leader's income without imposing large welfare losses, sanctions need to be even "smarter" and must take into account the *mechanism* through which the leader extracts his income.

In Section 5, we study several extensions. First, we use our model to study the economic interests of oligarchs in supporting a hybrid regime, and how this support can be affected by sanctions. In some cases, oligarchs are willing to support even the most distortive leader. In these economies, sanctions targeting the leader or the oligarchs tend to increase oligarchs' utility by incentivizing the leader to increase distortions. These sanctions only strengthen oligarchs' support for hybrid leaders. However, in other economies oligarchs are sensitive to the tradeoff between undesirable economic distortions and obtaining higher rents. In this case, imposing increasingly severe sanctions can lead to a sudden shift in oligarchs' support toward a welfare-maximizing regime and away from the hybrid leader.

Second, we study the implications of hybrid regimes for productivity. In line with recent empirical findings, we show that industry capture increases incentives to invest in productivity due to the higher profits that economic distortions create. However, we also show that this increase in productivity is welfare-reducing because it comes at the cost of inefficient labor allocation by entrepreneurs.

Finally, we study another area of government activity where hybrid leaders play a key economic role: public goods provision through procurement. To study this, we consider a world where the government hires private companies to provide public goods, and pays for them using tax revenues earmarked for this purpose. The leader cannot simply divert tax revenues into his private income. However, he can use his powers over the public procurement process to achieve essentially the same goal. In particular, the leader will purchase public goods from his client-entrepreneurs at an inflated price, and then extract the resulting extra profit. We show that the importance of public procurement in hybrid regimes is due to the constraints faced by the leader. The model also provides an explanation for why, empirically, client-entrepreneurs are often clustered in industries involved in public procurement.

In this context too, successful sanctions are those that do more than just limit resources. For example, limiting international transfers will both reduce public good spending and incentivize the leader to restrict competition. By contrast, policies that target the income extraction mechanism by limiting overpricing in public procurement can lower the leader's income while *increasing* welfare.

In Section 6 we use our results to interpret a number of examples of hybrid regimes around the world, including Russian oligarchs, income extraction in Turkey, the use of European Union funds in Hungary, privatization in Latin America, the "License Raj" in India, and crony firms in North Africa.

Related literature. A growing set of studies investigates the political processes that are emblematic of hybrid regimes - for example, how political leaders survive by exerting pressure on the media and controlling the flow of information to voters (Besley and Prat, 2006; Guriev and Treisman, 2020; Egorov and Sonin, 2024), by pitting rival groups against each other (Acemoglu et al., 2004; Padró i Miquel, 2007), or through policy concessions and power sharing arrangements (Bueno De Mesquita et al., 2003; François et al., 2015; Bidner et al., 2015). While these studies treat the economy in a reduced-form way in order to focus on politics, we do the opposite.[2]

Our model differs from settings in which voters choose their optimal leaders and economic distortions arise from agency considerations (e.g., Acemoglu et al. (2008)), or from settings where oligarchs simply choose their preferred leader (e.g., Guriev and Sonin (2009)) or directly choose policies (e.g., Acemoglu (2008)). In our view, these approaches are not ideal to describe how strongmen like Putin, Erdogan or Orban interact with crony entrepreneurs and other economic agents in the "steady state," once their power is secure. In this sense, our model studies the incentives for limiting economic competition once political competition has been sufficiently curtailed.[3]

[2]A larger empirical literature studies the economic role of political connections in hybrid regimes and other non-democracies - see, e.g., Fisman (2001), González and Prem (2020), and Szeidl and Szűcs (2021).

[3]Also related are models of (autocratic) regimes in which economic policies are chosen to avert a coup or revolution (e.g., Acemoglu and Robinson (2001, 2006); Gallego and Pitchik (2004); Shadmehr (2019)). Again, we assume that the hybrid leader has already solidified his power enough that these considerations are not first-order in his interactions with economic agents. Section 5.1 presents an extension analyzing the sources of oligarchs' support for the leader.

Our paper is related to a literature investigating the general equilibrium effects of lobbying (e.g., Grossman and Helpman (1994); Bombardini and Trebbi (2012); Huneeus and Kim (2021)).[4] While lobbying is a key channel linking firms and politicians in established democracies, its relative importance is lower in hybrid regimes where the bargaining power rests squarely with the leader. This is particularly apparent in the examples of industry capture that are the focus of this paper. As we show, the implications of this channel can differ from those of lobbying models, particularly in general equilibrium.

Our paper is also related to a growing stream of papers on the causes and consequences of increasing market concentration. Focusing mostly on the US and other developed countries, this literature distinguishes between "good" market concentration driven by changes in preferences and technology, and "bad" market concentration driven by increasing entry barriers (see e.g. Autor et al., 2017; Haskel and Westlake, 2018; Covarrubias et al., 2020). Our model describes a novel source of the latter type of market concentration in hybrid regimes.

Finally, because income extraction by the leader is a form of corruption, our paper is related to the massive corruption literature. Shleifer and Vishny (1993)'s seminal observation that corrupt officials have an incentive to create scarcity operates in our model through entry restrictions, and indeed, empirically, corruption is often positively correlated with entry restrictions and market concentration (Ades and Di Tella, 1999; Djankov et al., 2002). However, few papers have combined this aspect of corruption with an explicit economic model. One exception is Aidt and Dutta (2008), who model the economic implications of corruption in a democracy, focusing on how economic growth affects a corrupt politician's incentive to maintain entry barriers.[5] Our contribution to this literature is to provide an explicit model of a type of corruption that is particularly relevant in hybrid regimes, and embed it in a general equilibrium model in order to analyze its economic implications. This allows us to study how specific features of corruption affect entry restrictions, how these translate into economic outcomes (such as competition, product diversity, and social welfare), and how international policies or sanctions may affect these regimes.

2 Setup

There is a continuum of agents and a leader. Agents work, consume and produce in a general equilibrium economy with monopolistic competition akin to Dixit and Stiglitz (1977). We do not model the politics of how the leader gets elected or how he acquires specific powers.

[4] There is also a large literature on political influence in a partial equilibrium economy. Recent work includes Cowgill et al. (2023); Akcigit et al. (2023), and Callander et al. (2022). See also Shleifer and Vishny (1994) and Boycko et al. (1996) on state-owned firms.

[5] See also Bliss and Di Tella (1997) and Emerson (2006).

Instead, we simply assume that the leader has powers that, empirically, seem fundamental to hybrid regimes. Specifically, the leader has the power to capture industries: (i) he determines who can become an entrepreneur and establish firms, and (ii) he extracts income from these entrepreneurs in return for the right to operate. This section introduces the details of this environment.

2.1 Consumption, production, and the free-entry equilibrium

There is a unit mass of ex-ante identical agents, each endowed with a unit of labor. Agents have Cobb-Douglas utility across the products of $J+1$ industries indexed $j = 0, ..., J$, with a CES aggregator across varieties produced within an industry. The utility of each agent is

$$\prod_j Q_j^{\hat{\beta}_j} \tag{1}$$

where $\sum_j \hat{\beta}_j = 1$, and

$$Q_j = \left[\int_0^{\Omega_j} q_j (\omega)^{\frac{1}{\mu}} \, d\omega \right]^{\mu}$$

is the quantity index of industry $j > 0$. The term $q_j (\omega)$ is the quantity of variety ω while Ω_j is the mass of varieties produced in industry $j > 0$. The elasticity of substitution is the same in each industry and is given by $\frac{\mu}{\mu-1}$ so that the parameter $\mu > 1$ captures the lack of substitutability between products within an industry. Industry $j = 0$ produces a homogeneous good, and Q_0 is the quantity consumed of that good. It will be convenient to use the following notation: $\beta_j \equiv \hat{\beta}_j \frac{\mu-1}{\mu}$ and $\bar{\beta} \equiv \sum_{j>0} \beta_j = \frac{\mu-1}{\mu} - \beta_0$.

All $J+1$ industries use labor as the only input. Agents use some of their labor as workers and some as entrepreneurs. We do not restrict this choice to be binary: agents can divide their unit of labor endowment between the two occupations (for example, they can spend some of their time on entrepreneurial tasks involved in setting up and managing a firm, and the rest on production tasks). Worker labor is fully mobile across industries, and we normalize its wage to 1. Entrepreneurial labor is industry-specific, i.e., each agent can only work as an entrepreneur in a specific industry.[6]

In each industry $j > 0$, production has both a fixed cost f_j and a variable cost c_j in terms of labor. Specifically, producing quantity $q_j (\omega)$ of variety ω requires f_j units of labor provided by entrepreneurs to set up a firm, and $c_j q_j (\omega)$ units of labor provided by workers.

The homogeneous good in industry $j = 0$ is produced with unit variable input requirement, $c_0 = 1$, and no fixed cost, $f_0 = 0$. We assume free entry in this industry, so that this

[6]This assumption is necessary to allow heterogeneity across industries in equilibrium.

good is provided in perfectly elastic quantity for a price $p_0 = 1$. This is the numeraire good. From now on, when it does not cause any confusion, we use the index j for industries $j > 0$, excluding the numeraire.

Each firm produces one variety, choosing its price $p_j(\omega)$ to maximize profit $\pi_j(\omega) \equiv q_j(\omega)(p_j(\omega) - c_j)$. Suppose agent $i \in [0,1]$ has income $Y(i)$, and let Y denote aggregate income. By standard arguments in Appendix A, we obtain that equilibrium prices and quantities will be the same across varieties within an industry (so we drop the index ω from now on). Specifically, for all $j > 0$,

$$p_j = \mu c_j \tag{2}$$

$$q_j(i) = \frac{\beta_j}{\Omega_j(\mu - 1)c_j} Y(i) \tag{3}$$

$$Q_j(i) = \frac{\beta_j \Omega_j^{\mu-1}}{(\mu - 1)c_j} Y(i) \tag{4}$$

$$\pi_j = \frac{\beta_j}{\Omega_j} Y \tag{5}$$

where $q_j(i)$ and $Q_j(i)$ denote individual i's consumed quantity and quantity index, respectively. Note that π_j does not include the "fixed cost" of entrepreneurs' labor f_j. For the numeraire industry, $Q_0(i) = \hat{\beta}_0 Y(i)$ and $\pi_0 = 0$.

Income $Y(i)$ comes from two sources: labor and profits. Workers earn wages (normalized to 1 per unit of labor), while entrepreneurs receive a share of the firm's profit equal to π_j/f_j per unit of labor. Given Ω_j, total income of workers is $1 - \sum_j \Omega_j f_j$ and total income of entrepreneurs is $\sum_j \Omega_j \pi_j$, so that

$$Y = 1 + \sum_{j=1}^{J} \Omega_j(\pi_j - f_j). \tag{6}$$

To close the model, we need to determine the number of entrepreneurs and hence the number of firms. In our model, this will be set by the leader subject to various constraints. As a benchmark, assume for a moment that there is free entry into entrepreneurship. In this case, equilibrium requires that agents be indifferent between using their labor as workers or as entrepreneurs. Thus, it must be that

$$\frac{\pi_j}{f_j} = 1. \tag{7}$$

Using (5), (6), and (7), we get that in this *free-entry equilibrium*, the number of firms in industry $j > 0$ is given by

$$\Omega_j = \Omega_j^{FE} \equiv \frac{\beta_j}{f_j}.$$

2.2 The leader-client contract

To model the economy of a hybrid regime, we assume that there is a leader who has the power to determine who can become an entrepreneur. We consider a unitary leader, but this can represent a small inner circle, such as a strongman chief executive and his family members, close allies, or the upper echelons of his party.

In return for the right to operate firms, entrepreneurs must hand over part of their profit to the leader. In other words, the only way to become an entrepreneur is to enter into a clientelistic contract. These contracts specify the amount of profit that entrepreneurs in a firm can keep, w_j, with the remaining $\pi_j - w_j$ handed over to the leader.[7]

In our model, the clientelistic contract is subject to two fundamental constraints. The first is that agents cannot be forced to become clients against their will: they can always use their labor as workers, and will only become entrepreneurs if this is worth it for them. Since workers earn a wage of 1 per unit of labor, this *participation constraint (PC)* is

$$w_j \geq f_j \tag{8}$$

The second constraint is that enforcement of the contract is limited, in the spirit of Kehoe and Levine (1993). Specifically, we assume that clients can abscond with a share $(1 - \phi_j)$ of the profits. For example, entrepreneurs can move their profit abroad or to the shadow economy (Johnson et al., 1997), or shield it from the leader through defensive ownership structures (Earle et al., 2022).[8] Absconding clients forfeit their payment w_j, but they cannot be subjected to any other punishment by the leader. In this sense, the hybrid leader's clients benefit from limited enforcement. This gives rise to an *enforcement constraint* (EC):

$$w_j \geq (1 - \phi_j)\pi_j. \tag{9}$$

Limited enforcement creates an "efficiency wage" role for w_j in incentivizing clients not to

[7] In practice, this profit sharing sometimes takes place in creative ways. In Turkey, firms that win procurement contracts make "donations" to an NGO run by the president's son; in Hungary, they subcontract with a company owned by the prime minister's father. Instead of handing over funds directly, firms can also spend on behalf of the leader, e.g., by buying propaganda. See Section 6.

[8] We implicitly assume that the (out-of-equilibrium) decision to abscond with the firm's profit would be made jointly by all the entrepreneurs working in a firm. This ignores potential collective action problems between clients (which a sophisticated leader might be able to exploit). Studying such problems may be an interesting avenue for future research.

abscond with their profits. Throughout, we assume that $\phi_j > 0$ for at least some j.

The parameter ϕ_j can be interpreted as the leader's power over his clients. More powerful leaders need to give up less of their income in order to incentivize their clients. Power may arise from the leader's personal ties to clients: for example, a close social contact or party member may find it harder to abscond with his profits than an entrepreneur at arm's length from the leader. Power may also arise from the nature of the industry's activities. For instance, the leader may have more power over an oil company or a property developer whose business heavily depends on government licenses and regulations, compared to a technology firm producing for the international market who may be able to relocate its business to another country. In human-capital intensive industries, entrepreneurs with specialized skills may be able to take more of a firm's profit with them, implying a lower power for the leader. Below, we study the impact of changes, as well as heterogeneity, in ϕ_j on clientelistic contracts and the economy.[9]

While stylized, we believe the participation and enforcement constraints capture important features of hybrid regimes that distinguish them from either totalitarian dictatorships or established democracies. Although clientelism can also be pervasive in totalitarian systems, a dictator's power over his clients tends to be quite extensive, e.g., he may simply expropriate a firm's profit and throw managers in jail if they stand in the way. Thus, we do not expect the enforcement constraint (and perhaps not even the participation constraint) to matter. In established democracies, clientelism is relatively uncommon. "Profit sharing" between firms and the government takes place through legally codified channels, such as the tax system. Because they are backed by the legal system, such profit sharing contracts are easy to enforce. Thus, again, we do not expect the enforcement constraint to play an important role.

2.3 The leader's problem

In a hybrid regime, the leader chooses the number of firms in each industry,[10] $\Omega = (\Omega_1, ..., \Omega_J)$, and the clients' payments $w = (w_1, ..., w_J)$ to maximize a combination of social welfare W (the sum of all agents' utility) and the income he obtains from his clients, $Y_L \equiv \sum_j \Omega_j(\pi_j -$

[9] In a dynamic version of our setup, (9) could be rationalized by assuming that the client can abscond with and sell a share of the firm's final products, while the leader can exclude the absconding client from future profit sharing contracts. Then the client has to be paid above the net expected gain from such self-dealing. (See Kehoe and Levine (1993) and Rampini and Viswanathan (2010) for similar dynamic arguments.) Alternatively, ϕ_j can also be interpreted as the leader's bargaining power relative to his clients. If profit sharing between the leader and his clients takes place through Nash bargaining with weights ϕ_j for the leader and $1 - \phi_j$ for clients, we again obtain expression (9).

[10] As will be clear below, some industries could be excluded from the leader's choice set without affecting the analysis. These industries would have free entry, and their profits would just cover the fixed costs f.

w_j).

In some hybrid regimes, the leader values his income Y_L because it is essential to maintain his power by financing political propaganda or vote-buying. In others, this income represents the funds the leader can keep out of the public eye and use for his family's personal consumption.[11] Regardless of the deeper determinants of his preferences, we simply take it as given that Y_L is valued by the leader. A key parameter in our analysis is a weight, denoted with $\lambda \in [0, 1)$, that the leader places on this income relative to social welfare. Strong democratic institutions with robust checks and balances limit the leader's value from extracted income, resulting in a small λ. As institutions weaken, λ is likely to grow. We do not model where λ comes from - we use it to summarize the exogenous, institutional determinants of leaders' value from extracting income from the economy. This can be contrasted with the endogenous determinants of income extraction that we study below.

Total income of all entrepreneurs is $\sum_j \Omega_j w_j$, so total income of all agents in the economy (without the leader) is

$$\int_0^1 Y(i)di = 1 + \sum_j \Omega_j (w_j - f_j). \tag{10}$$

Social welfare is

$$W \equiv \int_0^1 \prod_j Q_j(i)^{\hat{\beta}_j} di,$$

and using (4) and (10) this can be written as

$$W(\Omega, w) = \hat{\beta}_0^{\hat{\beta}_0} \int_0^1 Y(i)di \prod_j \left(\frac{\hat{\beta}_j \Omega_j^{\mu-1}}{\mu c_j} \right)^{\hat{\beta}_j} = \hat{\beta}_0^{\hat{\beta}_0} \left(1 + \sum_j \Omega_j (w_j - f_j) \right) \prod_j \left(\frac{\hat{\beta}_j \Omega_j^{\mu-1}}{\mu c_j} \right)^{\hat{\beta}_j}.$$

For tractability, we specify the leader's objective as Cobb-Douglas: $Y_L(\Omega, w)^{\lambda} W(\Omega, w)^{1-\lambda}$. Thus, taking logs and dropping the constants, the leader solves

$$\max_{\Omega, w} \lambda \ln \left(\sum_j \Omega_j (\pi_j - w_j) \right) + (1 - \lambda) \left[\ln \left(1 + \sum_j \Omega_j (w_j - f_j) \right) + \sum_j \beta_j \mu \left(\ln \Omega_j - \frac{\ln c_j}{\mu - 1} \right) \right] \tag{11}$$

subject to (5),(6),(8), and (9).[12]

[11] In Turkey, something akin to Y_L is legislated in the form of an extrabudgetary discretionary fund that the president can allocate without any oversight (see Section 6).

[12] Because we model the economy in general equilibrium, how the leader spends his income affects prices, quantities, and profits. To keep things simple, this formulation assumes that the leader allocates his income across varieties and industries in the same proportion as every other agent. Hence, for any Ω and w, the equilibrium in this economy is still described by the system (2)-(6), with total spending given by (6).

The choice of Ω and w determines the level of competition, prices and quantities in each industry, as well as all agents' income in the economy (as reflected in the constraints (5) and (6)). When the value of extracted income (λ) is zero, the objective function in (11) nests a welfare-maximizing leader.

3 Regulation and income extraction in a hybrid regime

3.1 A benchmark: the welfare-maximizing leader

The following proposition describes the leader's choice and the economy in the special case of $\lambda = 0$, i.e., a welfare-maximizing leader. (All proofs are in the Appendix.)

Proposition 1 *When* $\lambda = 0$, *we have* $\pi_j = f_j$ *and* $\Omega_j = \Omega_j^{FE}$.

For a welfare-maximizing leader, we obtain the same solution as the free-entry equilibrium described in Section 2.1. Here, entrepreneurs keep all their profits, and the leader allows the same number of firms to operate as would emerge in equilibrium if entry into entrepreneurship was free. Clientelism plays no role: in effect, all firms remain independent from the leader.

3.2 The logic of industry capture

When the value of extracted income is positive ($\lambda > 0$), the leader will choose to restrict the number of firms below its free-entry level. Intuitively, in the free-entry solution profits are only large enough to cover the fixed costs f_j of operation. Since entrepreneurs cannot be forced to participate in the clientelistic contract, in order to extract income the leader must first raise profits. This is accomplished by restricting entry.

From the clients' perspective, once some of their profit is extracted by the leader, their firms only stay in business *because* the leader restricts entry and creates enough profit to cover both the fixed costs and the extracted share. In this sense, incumbent firms become dependent on the regulation of entry, and hence the leader, for their survival.

We characterize the different economic regimes that arise in equilibrium as a function of the parameters. Here we focus on the case when the leader's power is the same across industries $\phi_j = \phi$ and study the case of industry-specific power $\phi_j \neq \phi_k$ in Section 3.4. The

(Studying a leader who overspends in certain sectors would be a simple extension of our analysis.)

following thresholds play a key role in the analysis

$$\lambda'(\phi) \equiv \frac{\frac{\phi}{1-\phi}\mu(1-\bar{\beta})\bar{\beta}}{1+\frac{\phi}{1-\phi}\mu(1-\bar{\beta})\bar{\beta}}$$

$$\lambda''(\phi) \equiv 1 - \frac{1-\phi}{\mu(1-\bar{\beta})}.$$

(Note that $\lambda'(\phi) < \lambda''(\phi)$, and both thresholds are increasing in ϕ.)

Proposition 2 *Suppose* $0 < \lambda \leq \lambda''(\phi)$. *Then entrepreneurs get no rents:* $w_j = f_j$. *In addition:*
(i) If λ *is small and/or* ϕ *is large, i.e.,* $\lambda \leq \lambda'(\phi)$, *then* $w_j = f_j$, $\Omega_j = \frac{\beta_j}{f_j}\frac{\bar{\beta}\mu}{\mu\bar{\beta}+\frac{\lambda}{1-\lambda}}$, *and* $\frac{\pi_j}{f_j} = 1 + \frac{\lambda}{1-\lambda}\frac{1}{\mu(1-\bar{\beta})\bar{\beta}}$.
If λ *and* ϕ *are intermediate, i.e.,* $\lambda'(\phi) < \lambda \leq \lambda''(\phi)$, *then* $w_j = f_j$, $\Omega_j = \frac{\beta_j}{f_j}\frac{1-\phi}{1-\phi\bar{\beta}}$, *and* $\frac{\pi_j}{f_j} = \frac{1}{1-\phi}$.
(ii) As λ *or* ϕ *increases, profits increase, while the number of firms in each industry, workers' and entrepreneurs' utility (and therefore welfare) all decrease.*

Figure 1 illustrates Proposition 2. Note that the equilibrium number of firms Ω_j is symmetric across industries up to the scaling factor $\frac{\beta_j}{f_j}$, hence the discussion below applies to each industry. As λ rises above 0, the leader attaches more importance to extracting income relative to raising welfare. Given the constraints, extracting more income is only possible if profits rise, which is accomplished by limiting the number of firms on the market.

As long as λ is low (case (i) of Proposition 2), profits remain relatively low, which limits clients' incentive to abscond. In this range the enforcement constraint is irrelevant and profit extraction can take place without giving rents to clients (i.e., $w_j = f_j$).[13] Here, entrepreneurs' income, like workers', is fixed at 1, and both groups are hurt equally by economic distortions as λ rises.

Once λ exceeds the threshold λ', the leader wishes to make profits so high that the enforcement constraint becomes binding. Increasing profit extraction further becomes more costly as this would require providing rents to clients (i.e., $w_j > f_j$). As long as $\lambda \leq \lambda''(\phi)$, this extra cost is not worth it for the leader, and this halts the increase in profits (and the corresponding decline in competition and welfare). On Figure 1 this is indicated by the flat segment on each graph for medium values of λ. Here the enforcement constraint limits the

[13] As the proof of Proposition 2 shows, in the optimal contract either the participation constraint or the enforcement constraint will bind. To see why the PC $w_j \geq f_j$ binds for low values of λ, recall that this constraint binds in the free entry equilibrium and hence the $\lambda = 0$ case (Proposition 1). With $\lambda > 0$, the leader has more incentive to lower w_j, so this constraint will still bind as λ rises above 0.

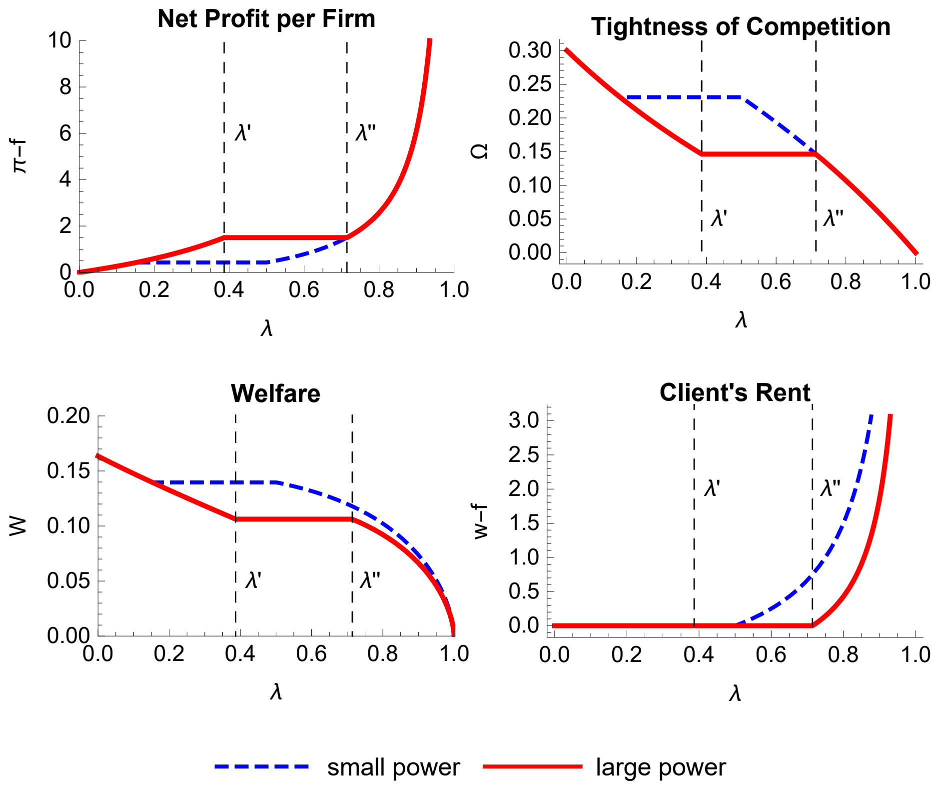

Figure 1: The effect of the value of extracted income, λ, and the leader's power, ϕ, on each firm's profit, industry competition, welfare, and each client's rent. The thresholds λ' and λ'' correspond to large power. Entrepreneurs become oligarchs once $\lambda > \lambda''$. Parameters are $\mu = 2$, $f = c = 1$, $\hat{\beta} = 0.6$, and $\phi = 0.3$ and $\phi = 0.6$ for small and large power, respectively.

leader's incentive to extract income. A leader with smaller power ϕ will stop raising profits sooner: the threshold λ' shifts to the left. Compared to a more powerful leader, this results in lower profits and hence more firms and higher welfare.[14] Thus, the model indicates a negative economic impact of stronger leaders in hybrid regimes. Because they are able to extract more surplus from their clients, stronger leaders have added incentives to increase profits through economic distortions.

While our model focuses on entry restrictions, in practice the incentives of hybrid leaders to restrict competition have broader impacts on the actions of government. These incentives can give rise to protectionist measures like restrictions on inbound FDI, and can explain why privatization programs often fail to break up state monopolies and thus do not result in increased competition (see Section 6).

[14] Comparing Proposition 1 and 2 shows that $\lambda > 0$ results in a socially suboptimal number of firms Ω_j. Because a less powerful leader increases the number of firms, it follows that welfare goes up.

3.3 Oligarchs

For high λ, the leader values his private income so much that he extracts more in spite of the extra cost created by the enforcement constraint. As λ increases above $\lambda''(\phi)$, income extraction starts to rise again, with the associated increase in profits and decline in competition and welfare. However, the leader's clients now receive some of the profits as rents - they become "oligarchs."

While the literature sometimes associates oligarchs with political influence, in our model "oligarchy" is simply a regime of industry capture with rents, and the distinguishing feature of oligarchs are these rents they receive. This is consistent with the view that any desire for political influence *derives* from oligarchs' economic interests. We explore the implications of this idea further in Section 5.1.

Proposition 3 *If λ is large and/or ϕ is small, i.e., $\lambda''(\phi) < \lambda$, then entrepreneurs receive rents: $w_j > f_j$. In addition,*
(i) $\Omega_j = \frac{\beta_j}{f_j}\frac{1-\lambda}{\frac{1}{\mu}+(1-\lambda)\bar{\beta}}$, and $\frac{\pi_j}{f_j} = \frac{1}{(1-\lambda)\mu(1-\bar{\beta})}$
(ii) As λ or ϕ increases, worker utility and social welfare decrease. As ϕ increases, entrepreneur utility decreases. As λ increases, entrepreneur utility decreases monotonically if $\frac{1}{\mu\bar{\beta}} < \lambda''(\phi)$, increases monotonically if $\frac{1}{\mu\bar{\beta}} > 1$ and is non-monotonic otherwise.

Economic outcomes under oligarchy are illustrated on Figures 1 and 2. Note in particular the rapid increase in profits and rents as λ increases above λ'' - indeed, the rapid enrichment of select entrepreneurs accompanied by the erosion of democratic institutions is a common phenomenon in hybrid regimes.

Comparing across economic regimes in Propositions 2 and 3, it is interesting to note that oligarchs are not always detrimental to welfare. According to Proposition 3, under a leader with a given power ϕ, oligarchs emerge when the value of extracted income λ is high enough. In this case, oligarchs reflect the leader's willingness to distort the economy *despite* having to provide rents to clients, and therefore welfare goes down. However, under a leader with a given λ, oligarchs will also emerge when the leader's power ϕ is low enough. In this case, oligarchs reflect the leader's limited ability to extract income, which lowers his incentive to distort. Here, the presence of oligarchs acts as a moderating force on the leader's actions and *increases* welfare.

In Section 6 we discuss how these observations can be used to understand the rise of Russian oligarchs under Yeltsin (driven by an increase in λ) and their weakening under Putin (due to an increase in ϕ). Proposition 3 can also be used to derive predictions on how economic fundamentals affect the emergence of oligarchs. For example, if products in

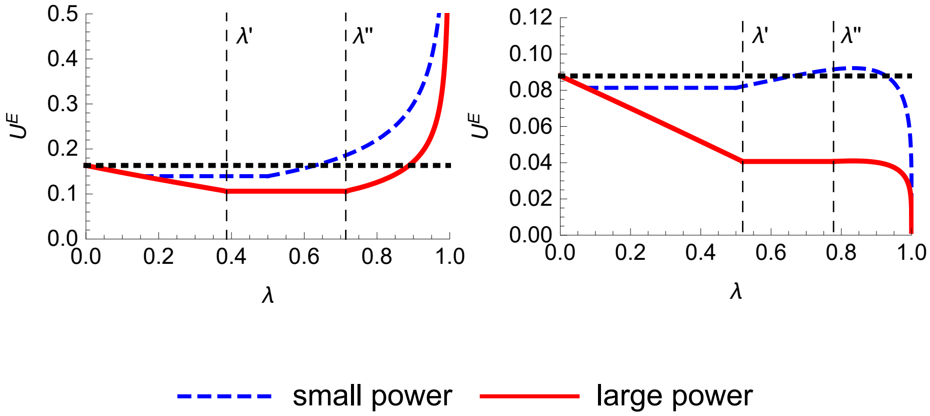

Figure 2: Entrepreneur utility as a function of λ. The two panels correspond to $\bar{\beta}\mu < 1$ and $\bar{\beta}\mu > 1$, respectively. The thresholds λ' and λ'' correspond to large power. Entrepreneurs become oligarchs once $\lambda > \lambda''$. Parameters are $\mu = 2, f = c = 1, \hat{\beta} = 0.6$, and $\phi = 0.3$ and $\phi = 0.6$ for small and large power, respectively on the left, while $\mu = 3$ and $\phi = 0.1$ and $\phi = 0.6$ on the right. The horizontal line is entrepreneur utility under a benevolent leader $\lambda = 0$ for comparison.

an industry are closer substitutes (the parameter μ is lower), this lowers the welfare cost of reducing competition. This gives the leader added incentives to raise profits, which in turn means that entrepreneurs are more likely to get rents and become oligarchs (λ'' shifts to the left).

While oligarchs' utility is decreasing in the leader's power, the effect of the value of extracted income λ may be non-monotonic (Figure 2). Once they become oligarchs, λ creates a tradeoff for entrepreneurs: on the one hand, they continue to be hurt by the increased distortions; on the other hand, they benefit from the increase in rents. How this tradeoff is resolved depends on $\mu\bar{\beta}$, which reflects the disutility of distortions, and the leader's power ϕ. When both $\mu\bar{\beta}$ and ϕ are high ($\lambda''(\phi) > \frac{1}{\mu\bar{\beta}}$), the rents are too low to compensate for the increased distortions, so oligarchs' utility continues to fall. In this case, entrepreneurs would be better off under a benevolent leader $\lambda = 0$. When $\mu\bar{\beta}$ is low ($1 < \frac{1}{\mu\bar{\beta}}$), once they become oligarchs, entrepreneurs' utility increases in λ because of the increasing rents they receive (left panel of Figure 2). In this case, entrepreneurs' utility is highest under the most extractive leader ($\lambda \to 1$). Finally, when $\mu\bar{\beta}$ is high but the leader's power ϕ is low ($\lambda''(\phi) < \frac{1}{\mu\bar{\beta}} < 1$), then the rents are sufficient that they initially compensate oligarchs for the increased distortions, until eventually distortions become so large that their utility goes down (right panel of Figure 2). In this case, conditional on oligarchy, entrepreneurs' utility is highest under a leader with an interior λ that balances the tradeoff between rents and distortions.

3.4 Spillovers across industries

Hybrid leaders may have clients in a few specific industries (e.g., natural resources), or in many industries throughout the economy. What is the impact of clientelism in one industry on other industries? This question is relevant for understanding the normative implications of hybrid regimes, because the full welfare effect of the leader's power over his clients includes any spillover effects. The question is also relevant for thinking about the impact of reforms that limit a leader's ability to extract income in some industries but not others.

To study these issues, we solve a version of the model where the leader's power is industry-specific. To maximize transparency, we assume $J = 2$ and drop the numeraire industry ($\beta_0 = 0$). The solution, which we present in Appendix C, implies the following results.

Proposition 4 *Suppose the leader has no power over industry 1: $\phi_1 = 0$. The leader will still restrict entry in this industry as long as he has power over industry 2: $\Omega_1 < \Omega_1^{FE}$ as long as $\phi_2 > 0$.*

Proposition 4 shows that when the leader's power is industry-specific, he will limit entry even in industries over which he has no power. Clientelism in industry 2 (where the leader has power) spills over onto industry 1 (where he does not), and results in entry restrictions in *both* industries. The reason for this is that a leader with no power over industry 1 can still raise income in the economy by raising profits in industry 1 through entry restrictions. Higher income means higher profits in *all* industries - including industries where $\phi_j > 0$, and whose income the leader is therefore able to extract. The idea is reminiscent of what d'Aspremont et al. (1996) have called the "Ford effect:" Henry Ford apparently observed that a (large) firm should take into account how increasing its price will, by raising profits, increase consumers' income, and therefore affect demand. In our case, instead of a firm setting prices, it is the leader setting entry regulations who optimally considers how general equilibrium effects can raise his profit.

Interestingly, the impact of these general equilibrium considerations differs from those typically seen in lobbying models. Lobbying tends to create asymmetries because factor owners in a given industry have fundamentally asymmetric interests regarding policies for other sectors.[15] By contrast, industry capture tends to create symmetries across industries,

[15] For example, in Grossman and Helpman (1994), factor owners in lobbying sectors obtain trade protection for themselves, but promote competition in other sectors in order to lower prices on their personal consumption.

because the leader's interests are fundamentally symmetric: he benefits from raising profits everywhere in the economy, including in industries that are not captured.

An immediate implication of Proposition 4 for institutional design is that eliminating the leader's power over a specific industry may be ineffective in limiting economic distortions. This is true even if the goal is to limit distortions in one industry only: due to the spillover effects, the free-entry outcome in *any* industry can only be achieved by eliminating the leader's power over *all* industries.

The next proposition asks whether more power implies more economic distortions. We pose this question in two ways: first, by asking what happens if the leader's power over a given industry increases, and second, by comparing industries where the leader has different power.

Proposition 5 *1. Suppose that the leader's power in industry 1 (ϕ_1) rises. Then Ω_1 decreases and Ω_2 increases.*
2. Suppose that $\beta_1 = \beta_2$, $f_1 = f_2$, and $\phi_2 > \phi_1$. Then $\Omega_1 \geq \Omega_2$.

According to part 1 of the proposition, an increase in the leader's power over industry 1 reduces competition in industry 1 but increases it in industry 2. The own-industry effect is a generalization of the corresponding result from Propositions 2 and 3 and reflects the fact that more power improves the technology of income extraction, raising the marginal utility of each additional dollar of profit for the leader. The cross-industry effect, however, goes in the opposite direction, which is due to an income effect. Because an increase in ϕ_1 leads to more profit extraction in industry 1, it increases the leader's income and reduces clients' income (and hence social welfare). This raises the marginal utility of increasing clients' income relative to the leader's private income, and this in turn incentivizes the leader to allow a higher Ω_2.

Although in the symmetric case an increase in power was always detrimental to competition, with industry-specific power there are offsetting cross-industry effects. An increase in the leader's power over industry 1 can create enough income for the leader that he becomes more willing to increase competition in industry 2. In this sense, competition in some industries may benefit (in a second-best sense) from a leader with extensive powers over other industries in the economy.

An important implication of Part 1 of Proposition 5, stated in Part 2, is that all else equal, distortions will be concentrated in industries where the leader has more power. Intuitively, it is more profitable for the leader to extract income when the clients' enforcement constraint is not binding. As distortions are increased, and clients' profits rise, this constraint will first become binding in the industry where the leader has less power. This limits the leader's incentive to distort in that industry.

In this way, our analysis can be used to shed light on the heterogeneity in clientelism across industries under a given leader, driven by the value of extracted income λ and the distribution of the leader's power ϕ_j across industries. For example, a high value of extracted income combined with similar power across industries will lead to a regime with oligarchs in several industries. Similar power across industries can arise, e.g., if the leader has a large network that allows for close monitoring of clients across the economy, or if there is broad dependence on local markets or natural resources that makes it difficult for clients in any industry to abscond. By contrast, when the leader's power is high in some industries but low in others, we expect to find oligarchs in the latter but not the former.[16]

4 Sanctions against hybrid regimes

The use of economic sanctions in foreign policy has increased dramatically since the end of the Cold War (Drezner, 2011), with the international response to Russia's invasion of Ukraine providing a salient recent example.[17] As noted by Morgan et al. (2023), "[o]ur theoretical and empirical understanding of sanctions has not kept up with these changes." (p5).

Sanctions are typically imposed on non-democracies, with the aim of incentivizing policy changes or weakening the country's current leadership (Marinov, 2005). In order to understand sanctions' full impact, however, it is important to consider how the targeted leader might respond to them (Oechslin, 2014; De Bassa et al., 2021). One concern is whether, in equilibrium, sanctions could lead to excessive welfare losses for the population at large.

In this section, we use our framework to study how various sanctions impact (i) a leader's profit extraction, (ii) his incentive to create entry barriers, and (iii) the resulting change in social welfare relative to the leader's income. The leader's income is a relevant consideration to a sanctioner who wants to create policy changes, if the leader's incentive to change policy is tied to the income he loses as a result of the sanctions. It is also relevant to a sanctioner who wants to weaken the leader, if the leader's political fortunes depend on his income (e.g., if he needs this income to pay for propaganda). In these cases, from the sanctioner's perspective, reductions in the leader's income represent a benefit, while a decrease in social welfare is a cost.[18]

[16]Specifically, Proposition C.1 implies that both industries will have oligarchs if and only if $\lambda > \max(\phi_1, \phi_2)$. By contrast, if $\lambda < \phi_2$, then industry 2 will not have oligarchs even if industry 1 does.

[17]Here we use the term "sanctions" very broadly to include trade restrictions, withholding international aid, freezing assets, etc.

[18]In reality, sanctions have other benefits and costs, including direct economic costs to the sanctioner, which

To demonstrate how our framework can shed light on the differential effects of various sanctions, we consider the following version of the leader's problem (11) in the symmetric case ($\phi_j = \phi$):

$$\max_{\Omega,w} \lambda \ln \left(\sum_j \Omega_j (\pi_j - w_j) - B \right) + (1-\lambda) \ln \left(1 + \sum_j \Omega_j (w_j - f_j - Cf_j) \right) + (1-\lambda) \sum_j \beta_j \mu \left(\ln \Omega_j - \frac{\ln(c_j + A)}{\mu - 1} \right) \quad (12)$$

with the participation and enforcement constraints

$$w_j - Cf_j \geq f_j \quad (13)$$

$$w_j - Cf_j \geq \pi_j D (1 - \phi) \quad (14)$$

The parameters $A, B, C, D \geq 0$ represent various sanctions described below.

4.1 Broad sanctions increasing input costs

Common economic sanctions, like restrictions on a country's ability to purchase inputs or technology on the international market, lead to an increase in the costs of production ($A > 0$). On the one hand, this is costly for social welfare. On the other hand, in principle it is possible that the increase in production costs will hurt the leader, particularly when he obtains private income from firm profits.

Our model highlights a simple fact: the extent to which economic sanctions translate into profits, and hence the leader's income, depends on the nature of competition in the economy. In particular, if firms are able to fully pass on cost increases to their consumers by raising prices, then the leader's income will not be affected. This is exactly what happens under monopolistic competition.

Proposition 6 *Sanctions increasing the cost of production (an increase in A) have no impact on profit extraction ($\pi - w$) or entry restrictions Ω. They lower social welfare W while leaving the leader's income Y_L unchanged.*

According to the proposition, in this model economic sanctions that raise production costs must have some other justification than a desire to disrupt the leader's profit extraction. More generally, the ineffectiveness of broad economic sanctions is consistent with the

we do not model. See Clayton et al. (2024) for a general model of economic statecraft from a hegemon's perspective. In extensions, we study the impact of sanctions on the leader's support among oligarchs (Section 5.1), and also analyze sanctions in the context of public procurement (Section 5.3).

historical trend where broad sanctions are increasingly replaced by so-called "smart" sanctions that target specific actors and activities (Drezner, 2011).

4.2 Smart sanctions targeting the leader

Another form of sanctions aims to directly reduce the leader's income, for example, by freezing his foreign assets. We model this by increasing B in problem (12). To focus on the reduction of the leader's income, rather than the reduction of total income in the economy, we assume that the sanctioner spends B in the economy in the same way that the leader would have.[19]

Proposition 7 *Consider an oligarchy with no sanctions initially. A marginal increase in B lowers both the leader's income Y_L and social welfare W and leads to restricted competition. If $\lambda > \frac{1}{2}\left(1+\frac{1}{\mu\beta}\right)$, then $\frac{\partial \ln W}{\partial B} / \frac{\partial \ln Y_L}{\partial B} > 1$.*

Naturally, B directly lowers the leader's income. However, this also raises its marginal utility, which can give the leader an incentive to offset B by restricting competition and raising profits. This in turn lowers welfare. As the proposition shows, if λ is sufficiently large, so that the leader is motivated more by extracted income than social welfare, his incentive to shield his interest will be sufficiently strong that social welfare drops more than extracted income.

In this model, smart sanctions that target the leader are more effective at reducing the leader's income than broad economic sanctions in the sense of Proposition 6. However, directly targeting the leader's income is no "silver bullet" and may not avoid welfare losses once the leader's response is taken into account.

Previous studies have noted that leaders may have the ability to directly offset sanctions - for example, by transferring resources to strategic firms hurt by the sanctions (Ahn and Ludema, 2020). Our analysis highlights that even a leader who cannot directly offset sanctions may be able to do so indirectly, by adjusting the economic policies that ultimately govern income extraction.

4.3 Smart sanctions targeting oligarchs

We contrast sanctions on the leader with two ways to target his clients' income. First, the sanctioner could introduce a wedge C between the income the leader pays to the client and

[19] In the main text, we highlight the results corresponding to an oligarchy. Lemmas D.7-D.9 in the Appendix give a complete characterization for all values of λ.

what the client can spend. This might be done by seizing a portion of the client's wealth held in foreign banks (a common form of sanctions, e.g., on Russian oligarchs).[20]

Alternatively, sanctions may treat oligarchs who defect by absconding with their profits differently from those who stay with the leader. These differential sanctions are captured by the parameter D in (14). Namely, $D < 1$ if an oligarch absconding with his profits expects that some of it will be seized by foreign powers, while $D > 1$ captures a situation when a defecting oligarch enjoys an additional reward.

Proposition 8 *Consider an oligarchy with no sanctions initially.*

1. *A marginal increase in C lowers both the leader's income Y_L and social welfare W and leads to more restricted competition. If $\lambda > \frac{1}{2}$, then $\frac{\partial \ln W}{\partial C} / \frac{\partial \ln Y_L}{\partial C} > 1$.*

2. *A reduction in D increases the leader's income Y_L and lowers social welfare: $\frac{\partial \ln W}{\partial D} > 0$, $\frac{\partial \ln Y_L}{\partial D} < 0$.*

If C increases, so that oligarchs expect part of their income flows to be seized, they require larger transfers w from the leader to dissuade them from absconding. This reduces the leader's income, and his reaction is therefore similar to sanctions that target him directly, as above. To compensate for the lost income, the leader increases profits by restricting competition further. This in turn lowers welfare, and for sufficiently high λ this effect can be larger than the decline in the leader's income.

It is interesting to contrast these results with the effect of D, capturing the differential effect of sanctions on the (expected) income of defecting oligarchs. A reduction in D, i.e., a "tax" on defecting oligarchs' income, unambiguously decreases social welfare and increases the leader's income. Intuitively, if clients can only abscond with less, their terms of contracting with the leader worsens. Thus, the effect of a smaller D is akin to an increase in the leader's power ϕ. As we saw in section 3.2, a more powerful leader enjoys an improved trade-off between extracting funds and the corresponding welfare reduction. As a result, he extracts more even as this reduces welfare.

One lesson from this discussion is that replacing broad economic sanctions with smart sanctions that target the leader or his clients may not be sufficient to avoid negative welfare consequences. In our model, these sanctions are still too broad in that they are not focused on the *mechanism* through which the leader's income is generated. As illustrated by the example of sanctions on defecting oligarchs, sanctions that ignore the mechanism may inadvertently reinforce it, and lead to negative welfare consequences. By the same token, sanctions that

[20]Note that in our formulation (12-14) this sanction is expected: C enters the equilibrium decision of both the leader and the clients and has general equilibrium consequences.

disrupt the income extraction mechanism may be more successful. For example, the mirror image of the above argument is that ensuring higher incomes for defecting oligarchs (through an increase in D) would lead to less income extraction and higher welfare.[21]

5 Extensions

5.1 Hybrid leaders' support among oligarchs and the impact of sanctions

5.1.1 Hybrid leaders' support among oligarchs

Our model has focused on how oligarchs shape a hybrid system through their participation in profit-sharing arrangements with the leader. But oligarchs may also have the ability to affect who comes to power (Guriev and Sonin, 2009). A full analysis of this question is beyond the scope of this paper, which has deliberately kept the politics of the model reduced-form in order to focus on the economics. Here we nevertheless provide a simple analysis of the tradeoffs that oligarchs face in deciding which leader to support.[22] When is it in entrepreneurs' economic interest to support a particular hybrid regime characterized by (λ, ϕ)? When would they prefer a welfare-maximizing leader ($\lambda = 0$) instead? When would they prefer the most distortive leader ($\lambda \to 1$)?

Let U^E denote an entrepreneur's utility, and define the set of entrepreneur-supported hybrid leaders

$$\Lambda(\phi) = \{\lambda^* | U^E(\lambda^*, \phi) > U^E(\lambda = 0)\}$$

as the set of λ-s under which the entrepreneur is better off compared to a welfare-maximizing leader.[23] Graphically, on Figure 2 $\Lambda(\phi)$ is the set of λ-s for which the curve lies above the $\lambda = 0$ intercept.

Note that $\Lambda(\phi)$ never contains a leader who does not provide rents. Without rents, entrepreneurs are always hurt by economic distortions, and therefore they prefer the benevolent leader. Only oligarchs can support a hybrid leader.

We have the following result:

[21] Rewarding defecting oligarchs is not without precedent: in July 2023, the UK removed Russian tycoon Oleg Tinkov from its list of sanctioned individuals after he spoke out against the invasion of Ukraine (`https://www.ft.com/content/fe6ab027-fb19-4593-9ef1-bb751aeeb14b`). See also Section 5.3, where we study other sanctions designed to disrupt income extraction in the context of public procurement.

[22] Guriev and Sonin (2009) discuss one such tradeoff, showing that oligarchs will sometimes opt for a weak leader who does not protect their property rights from looting by others, because such a leader also extracts less rents from them than a strong leader would.

[23] From the characterization in Propositions 2 and 3 we know that in equilibrium the entrepreneur's industry does not affect U_E and therefore $\Lambda(\phi)$.

Proposition 9 *When $1 < \frac{1}{\mu\bar{\beta}}$, then there is a sufficiently high λ such that oligarchs will support the hybrid leader over the benevolent leader: $\Lambda(\phi) = [\underline{\lambda}(\phi), 1)$, where $\frac{\partial \underline{\lambda}}{\partial \phi} > 0$.*

When $1 > \frac{1}{\mu\bar{\beta}}$, then if $1-(\bar{\beta}\mu-1)\frac{1-\bar{\beta}}{\bar{\beta}}\left(\frac{\bar{\beta}^2\mu}{\bar{\beta}\mu-1}\right)^{\bar{\beta}\mu} < \phi$, oligarchs never support a hybrid leader: $\Lambda(\phi) = \emptyset$.

Conversely, if $1-(\bar{\beta}\mu-1)\frac{1-\bar{\beta}}{\bar{\beta}}\left(\frac{\bar{\beta}^2\mu}{\bar{\beta}\mu-1}\right)^{\bar{\beta}\mu} > \phi$ then oligarchs will support hybrid leaders with an interior λ: $\Lambda(\phi) = [\underline{\lambda}(\phi), \bar{\lambda}(\phi)] \subset (\lambda''(\phi), 1)$, where $\frac{\partial \underline{\lambda}}{\partial \phi} > 0$ and $\frac{\partial \bar{\lambda}}{\partial \phi} < 0$.

When oligarchs' disutility from distortions, captured by $\mu\bar{\beta}$, is low, their utility is monotonically increasing in λ because of the increasing rents they receive (the left panel of Figure 2). In this case, the set $\Lambda(\phi)$ is non-empty for any ϕ, containing all regimes where λ is sufficiently close to 1. This result tells us that even the most distortive leaders can be supported by oligarchs. As long as distortions do not have a large direct impact on entrepreneurs' well-being, a sufficiently distortive leader will increase their profit enough to be appealing.

Because the leader's power ϕ reduces rents and therefore oligarchs' payoff (Section 3.3), it shrinks the set of entrepreneur-supported hybrid leaders. For λ just above $\underline{\lambda}$, an increase in the hybrid leader's power will make entrepreneurs want to support a benevolent leader instead.

When oligarchs care sufficiently about economic distortions, they will support a hybrid leader as long as his power ϕ is not too large. Even though a powerful leader would raise their profit, he would extract so much of it that the set Λ is empty. In this case, supporting a welfare-maximizing leader is in the economic interest of oligarchs. On the other hand, when the hybrid leader's power is more limited, there is a range of oligarch-supported hybrid leaders and $\Lambda(\phi)$ is an interior set (this set is larger the smaller is ϕ). Leaders in this set create more moderate distortions, reflecting oligarchs' trade-off between economic costs and rents.

One implication of Proposition 9 is that, empirically, powerful leaders (high ϕ) should also have a high value for extracted income (high λ). This is not due to any intrinsic relationship between power and the leader's preferences. Instead, it reflects selection: powerful leaders are costly to oligarchs, and therefore such a leader will only receive their support if he also creates large profits through economic distortions - which requires a high value for extracted income.

More generally, the above results provide an explanation for why entrepreneurs can support a distortive regime instead of a benevolent leader. This is relevant for the observation that mass privatization programs (such as those after the fall of the Soviet Union) often fail to create extensive support for democratic processes and the rule of law among the new business elite. Some previous explanations focus on these entrepreneurs' incentive to strip the companies' assets, which would be hindered by the rule of law (Hoff and Stiglitz, 2004). However, such an explanation is difficult to reconcile with evidence on the increased productivity of the privatized firms acquired by oligarchs (see Section 5.2 below). Our model provides an explanation for the distortive leaders emerging with oligarchs' support even in the absence of asset-stripping.

5.1.2 Can sanctions reduce oligarchs' support?

We now use our setup to ask about the impact of sanctions on oligarchs' support of a hybrid leader. Specifically, how do the sanctions analyzed in Section 4 - either broad economic sanctions or smart sanctions targeting the leader or the oligarchs - affect the set $\Lambda(\phi)$ of leaders supported by oligarchs?

Consider a parameter combination where the set of oligarch-supported hybrid leaders $\Lambda(\phi)$ is non-empty. Recall from Proposition 9 that $\Lambda(\phi)$ is either a segment of the form $[\underline{\lambda}, 1]$ (if $\frac{1}{\mu\beta} > 1$), or a segment of the form $[\underline{\lambda}, \bar{\lambda}] < 1$ (if $\frac{1}{\mu\beta} < 1$). The following Proposition describes how this set is affected by the different sanctions.

Proposition 10 *Suppose that an outside authority imposes sanctions.*

1. *Sanctions increasing the cost of production (an increase in A) reduce the set of oligarch-supported leaders Λ (they raise $\underline{\lambda}$ and lower $\bar{\lambda}$).*

2. *Starting from no sanctions initially, a marginal reduction in the leader's income (an increase in B) or a marginal reduction in oligarchs' income (an increase in C), increases the set of oligarch-supported leaders by lowering $\underline{\lambda}$ if $\frac{1}{\mu\beta} > 1$; it shifts the set of oligarch-supported leaders down by lowering both $\underline{\lambda}$ and $\bar{\lambda}$ if $\frac{1}{\mu\beta} < 1$.*

3. *A sanction that reduces the leader's power over oligarchs (such as an increase in benefits D to defecting oligarchs) increases the set of oligarch-supported leaders by lowering $\underline{\lambda}$ and raising $\bar{\lambda}$.*

These results show that the case for some sanctions becomes stronger if the goal is to reduce a hybrid leader's support among oligarchs. In particular, broad economic sanctions that raise production costs always lower oligarchs' utility from supporting such a leader. Interestingly, as the severity of sanctions increases, the supported set $\Lambda(\phi)$ can become empty - in other words, oligarchs may suddenly favor a transition to a welfare-maximizing regime from *any* hybrid leader. The reason for this is the non-monotonicity of oligarchs' utility, as on the right panel of Figure 2. When oligarchs prefer an interior λ, sanctions can cause both leaders who do not distort enough *and* leaders who distort too much to lose oligarchs' support, until $\underline{\lambda} = \bar{\lambda}$ and $\Lambda(\phi)$ becomes the empty set.

"Smart" sanctions targeting either the leader's or the oligarchs' income can also remove relatively distortive hybrid leaders from the supported set. This happens when oligarch-supported leaders have interior λ-s: then for the more distortive leaders, oligarchs' utility is decreasing in distortions. As we saw in Section 4, the leader's response to these sanctions is to create even more distortions, which removes leaders with λ close to $\bar{\lambda}$ from the supported set.

At the same time, smart sanctions can now also have additional unintended consequences beyond those discussed in Section 4, by increasing hybrid leaders' support. This happens when oligarchs' utility is increasing in distortions. In this case, because sanctions increase distortions, they increase oligarchs' utility from the hybrid leader.

Similarly, sanctions that lower the leader's power over oligarchs can also backfire. Because in this case the leader can only extract less of their profits, oligarchs find him more appealing. This can give rise to a "rally around the flag" effect where oligarchs become more willing to support a hybrid leader who is facing sanctions.

5.2 Productivity in hybrid regimes

What are the implications of hybrid regimes for innovation and productivity? A long-standing view is that non-democracies discourage investment in productivity growth due to their lack of commitment to secure property rights (North and Weingast, 1989), and block innovation to maintain the status quo balance of economic and political power (Acemoglu and Robinson, 2006, 2012). Several recent studies have qualified this view. Empirical evidence in Guriev and Rachinsky (2005) and Gorodnichenko and Grygorenko (2008) indicates that in the early 2000s oligarchs in Russia and Ukraine improved the productivity of their firms relative to other types of owners. Aghion et al. (2008) show evidence that non-democracy can benefit output growth in industries far from the technological frontier, while Beraja et al. (2023) show that the Chinese government's investment in face recognition technology resulted in productivity growth for AI firms. These studies indicate that, in some cases, non-democracies can result in higher productivity.

To study the implications of our model for productivity, we endogenize the technology that firms use. Suppose that different technologies imply different productivity, captured by (the inverse of) the marginal cost of production c_j. It will be convenient to measure productivity by $\kappa_j \equiv (1/c_j)^{\frac{1}{\mu-1}}$. Increasing productivity requires entrepreneurial time: a higher κ_j implies a higher fixed cost, as described by the increasing function $f_j(\kappa_j)$. We assume that the benefit-cost ratio of investing in productivity, $\frac{\kappa_j}{f_j(\kappa_j)}$ is single-peaked and concave (this will guarantee that both the welfare-maximizing and the equilibrium level of productivity are unique).

In Appendix F we formally introduce an R&D sector where forward-looking inventors develop the technology which will maximize each firm's profits before any other decisions are made. For simplicity, here we use an equivalent shortcut: we let each firm choose their profit-maximizing technology and assume that all other agents, including the leader, take technology choices as given.

Holding constant what other firms are doing, firms that invest in higher productivity can charge lower prices and sell larger quantities, earning higher profits (see Appendix F). Specifically, given choices of $\kappa_j(\omega)$, the profit of a firm producing variety ω, previously given in (5), modifies to

$$\pi_j(\kappa_j(\omega)) = \beta_j Y \frac{\kappa_j(\omega)}{K_j} \tag{15}$$

where $K_j \equiv \int_0^{\Omega_j} \kappa_j(\omega)\, d\omega$ is average productivity.

Each firm takes average productivity K_j as given, so that the profit-maximizing choice κ_j^* is determined by the first order condition $\beta_j Y/K_j = f_j'\left(\kappa_j^*\right)$. As this choice is identical across firms in a given industry, $K_j = \Omega_j \kappa_j^*$, leading to the equilibrium condition[24]

$$\beta_j Y \frac{1}{\Omega_j} = \kappa_j^* f_j'\left(\kappa_j^*\right). \tag{16}$$

Is firms' choice of technology welfare-maximizing? The following proposition answers this question.

Proposition 11 *In our setup with endogenous productivity:*

1. *The welfare-maximizing productivity is equal to* $\kappa_j^{**} \equiv \arg\max_{\kappa_j} \frac{\kappa_j}{f_j(\kappa_j)}$, *independent of* λ.

2. *Consider an equilibrium* $(\Omega^*, w^*, \kappa^*)$ *where* κ_j^* *satisfies* (16) *for* $\Omega_j = \Omega_j^*$, *and the leader chooses* (Ω^*, w^*) *as in* (11) *with* $f_j = f_j(\kappa_j^*)$ *and* $c_j = \frac{1}{\left(\kappa_j^*\right)^{\mu-1}}$.

   (a) *For* $\lambda = 0$, *equilibrium productivity is welfare-maximizing:* $\kappa_j^* = \kappa_j^{**}$.

   (b) *Equilibrium productivity* κ_j^* *is monotonically increasing in* λ, *implying increasing overinvestment in productivity for* $\lambda > 0$.

[24] Appendix F.3 shows that equilibrium productivity is unique.

In this model, firms with higher productivity receive a larger share of consumer spending, and because spending is proportional to income, firms' incentive to raise κ is increasing in income (see (15)). As λ rises and the leader increases profits, income goes up, and this gives firms added incentives to choose more productive technologies. However, when entrepreneurs use more of their labor on these technologies, there is less labor available to create new firms, which results in lower product variety. Thus, from a social welfare perspective, this increase in productivity is undesirable.

These results provide a qualification to the empirical findings showing that non-democracies can sometimes increase productivity. In our model, such increase in productivity is socially excessive.

The discussion so far has focused on the relationship between productivity enhancements and the leader's choice of entry restrictions. Another way in which productivity can affect industry capture is through entrepreneurs' ability to abscond with the firm's profit (i.e., ϕ_j). For example, more innovative (higher productivity) technologies may imply more complex firm structure, multiple plants, or offshore operations, offering more opportunities to abscond (in a similar vein, Gorodnichenko and Grygorenko (2008) argue that the "looting" of firms by their oligarch owners is less likely to be noticed when productivity is high). In this case, we expect ϕ_j to increase in κ_j.

Propositions 2 and 3 immediately imply that if a higher productivity technology results in higher ϕ_j, this will tamper the leader's profit extraction and reduce distortions (increase Ω_j). This creates an offsetting, welfare-improving effect from productivity enhancements in hybrid regimes.

5.3 Public procurement and sanctions affecting external transfers

Our analysis so far has focused on the economic role of the leader through the regulation of entry in markets for private goods. In practice, another important tool for leaders is the public procurement process, and hybrid leaders are notorious for obtaining private gains through this channel (see, e.g., Szűcs (2023)). How does public procurement interact with clientelism in hybrid regimes?

The procurement process also presents a major dilemma to potential sanctioners, because external funding can account for a significant portion of government expenditures. A prime example of this is Hungary, which relies on transfers from the European Union worth billions of Euros each year. Could withholding some of these funds be effective at weakening hybrid regimes, or are there other sanctions better suited for this?

To investigate these important questions, we modify our setup to include public procurement financed in part from domestic taxes and in part from external transfers. We explain the role of procurement in the hybrid leader's toolkit, then study the impact of sanctions in this context.

5.3.1 Income extraction and public procurement

Suppose that firms in the last industry, J, do not produce for the private market - instead, they produce (varieties of) public goods purchased by the government and consumed by the consumers. For example, industry J could be the road construction industry, with firms specializing in different types of roads, or roads in different geographic areas. To model the leader's extensive powers in dealing with firms in the public goods industry, we assume that the leader makes a take-it-or-leave it offer for the markup m to be paid over the cost c_J.[25]

The leader finances public procurement using a lump sum tax T and external funds Δ, and we assume that both of these are earmarked for the provision of public goods. Leaders in hybrid regimes face constraints: they cannot simply pocket the taxes that the government collects or the transfers that international organizations provide.

By choosing m, Ω_J and T, the leader effectively decides on the share of income that society will allocate to the public good. Assuming that the leader does not tax his own income, denote this share with $\tau \equiv \frac{T}{Y-Y_L}$. Then the government's budget constraint can be written as

$$\tau \left(Y - \sum_j \Omega_j (\pi_j - w_j) \right) + \Delta = m c_J q_J \Omega_J, \tag{17}$$

where the left-hand-side is total revenue, and the right-hand-side is total spending on the products of industry J.

Just as in the baseline model, the chosen markup and quantity have to be such that firms are willing to participate in procurement, that is, the participation constraint (8) and the enforcement constraint (9) are satisfied for industry J.

We provide a detailed formulation of the general problem and a full characterization for a case when industries with private and public goods coexist in Appendix G.2. For our purposes, here it is sufficient to focus on the $J = 1$ case, that is, when the public good is produced by the single increasing-return-to-scale industry. The following Proposition describes the main properties of the equilibrium in this variant of our economy

Proposition 12 *Let* $J = 1$.

[25] As we show below, a welfare-maximizing leader would set the same markup $m = \mu$ as the equilibrium of the baseline economy with no public goods (see (2)). Thus if $m > \mu$, then the leader chooses to overpay for these goods.

1. *The equilibrium features oligarchs if and only if $\lambda > \lambda^{PP}(\phi)$, where $\lambda^{PP}(\phi) > \lambda'(\phi)$.*

2. *For any $\lambda > 0$ and $\Delta \geq 0$, as the value of extracted income λ increases, the leader increasingly overprices public procurement ($m > \mu$ and $\frac{\partial m}{\partial \lambda} > 0$), and overspends on the public good ($\frac{\partial \tau}{\partial \lambda} > 0$) while providing less of it ($\frac{\partial Q_J}{\partial \lambda} < 0$). In addition, market concentration increases ($\frac{\partial \Omega_J}{\partial \lambda} < 0$).*

The equilibrium with procurement is illustrated on Figure 3, which also shows the corresponding no-procurement baseline for comparison.[26] As before, the leader increases profits by restricting competition and extracts the resulting income, providing rents to oligarchs for λ high enough. In addition, the leader has an incentive to increase the cost of public goods by overpricing government procurement. By purchasing public goods at inflated prices and then extracting the elevated profits from his clients, the leader can effectively transform tax revenues (and external funds) into his private income. Even though taxes are earmarked for public goods, the combination of public procurement and clientelism allows the leader to extract some of the surplus that is created. In this sense, the importance of public procurement to the hybrid leader derives precisely from the constraints he faces, namely his inability to divert tax revenue directly.

5.3.2 Sanctions and procurement: Withholding external funds or improving the oversight of their allocation

Consider two potential interventions by an actor, such as an international organization, that provides external funds to a hybrid regime. First, external funds may be withheld. Second, the actor may try to improve oversight of the public procurement process where these funds are used in order to limit overpricing. The latter may be achieved through what is known as "conditionality" (Stokke, 2013): sanctions that are explicitly conditioned on specific policy changes. For example, as of 2022 the EU had suspended more than €13bn of funding to Hungary over concerns regarding democratic institutions and corruption. Improving the oversight of the procurement process and stamping out corruption in the allocation of these funds was one of the EU's main requirements to resume their flow.[27]

We contrast the effects of these two types of sanctions by comparing the equilibrium effect of reducing external funds, Δ, and of imposing an additional constraint $m \leq \bar{m}$ in the

[26] As shown in the Figure, for low λ welfare with public procurement can be larger than in the baseline. This is because public procurement allows even a welfare-maximizing leader to alleviate the deadweight loss inherent in monopolistic competition. Thus, the welfare-maximizing solution is different with and without procurement (see Appendix G.2 for details).

[27] https://www.theguardian.com/world/2022/nov/30/Brussels-seeking-to-freeze-13bn-of-eu-funds-to-hungary-over-corruption-fears

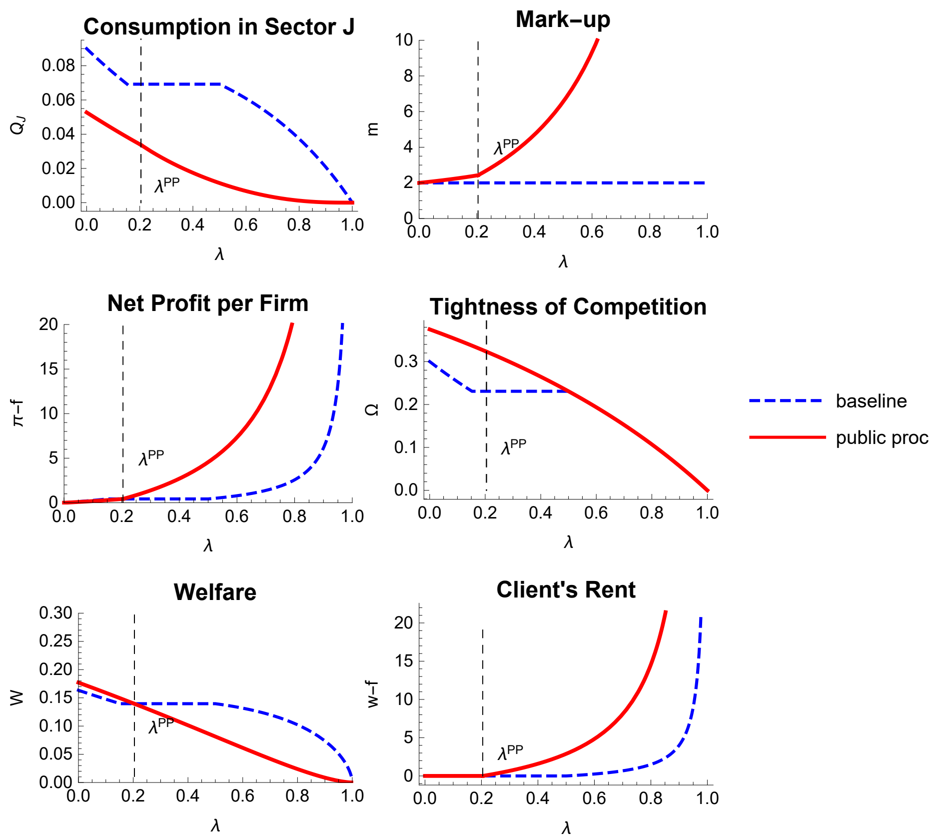

Figure 3: Industry capture with and without public procurement. λ^{PP} is the threshold given in Proposition 12. Parameters are $\mu = 2, f = c = 1, \hat{\beta} = 0.6$, and $\phi = 0.3$.

leader's problem.

Proposition 13 *1. A reduction in external funds, Δ implies smaller market concentration, $\frac{\partial \Omega_J}{\partial \Delta} > 0$, and less public good provision, $\frac{\partial Q_J}{\partial \Delta} > 0$, leading to a reduction in both welfare and the leader's income. For any λ and Δ, the relative effect is given by*

$$\frac{\partial \ln W}{\partial \Delta} / \frac{\partial \ln Y_L}{\partial \Delta} = \mu(1 - \hat{\beta}_0) + \hat{\beta}_0 > 1.$$

2. A stricter limit $\bar{m}$ on overpricing implies (weakly) smaller market concentration, $\frac{\partial \Omega_J}{\partial \bar{m}} \geq 0$, but more public good provision, $\frac{\partial Q_J}{\partial \bar{m}} < 0$, leading to a reduction in the leader's income but an increase in welfare:

$$\frac{\partial \ln Y_L}{\partial \bar{m}} > 0 > \frac{\partial \ln W}{\partial \bar{m}}.$$

Part 1 of the Proposition shows that withholding external funds has an unambiguously larger negative effect on welfare than on the leader's income. By contrast, Part 2 shows that tighter limits on overpricing increase welfare while decreasing the leader's income. Thus, withholding external funds earmarked for public procurement and limiting overpricing in public procurement can have starkly different consequences.

The first sanction leads to a loss in available funds in the economy, which directly lowers both welfare and the leader's income. But because the leader controls tax revenues, markups, and entry, he has considerable flexibility in mitigating his losses by distributing them across the economy. In equilibrium, the leader responds to the decrease in his income by further restricting competition and reducing the quantity of the public good, which reduces welfare beyond the sanction's direct impact. The second intervention, instead of reducing funds in the economy, directly disrupts the mechanism the leader uses for income extraction, which raises welfare. Although the leader can undo this somewhat by limiting competition and increasing profits, he will extract less from the economy, and this results in *higher* welfare.

These results echo the findings in Section 4 showing that, unless interventions directly target the mechanism of income extraction, actions taken by the leader in order to protect his income can easily have undesirable welfare consequences.

6 Model and Facts

For the purpose of this paper, our definition of hybrid regimes has been that their leader extracts income from the private economy through industry capture, and this is extensive enough to have general equilibrium effects, but at the same time is subject to constraints arising from entrepreneurs' ability to pursue other economic opportunities.

In this section we illustrate elements of our model from different countries and time periods, and use our results to interpret specific features of these systems. Clearly, political regimes are multidimensional and change constantly, and we will not attempt to pinpoint any particular regime's location on an autocracy-hybrid-democracy scale. Our goal is simply to show that our model has explanatory power for understanding several examples that are commonly viewed as being neither fully democratic nor fully autocratic.

Income extraction in Turkey Contemporary Turkey provides several examples illustrating elements of our model. The president has a clientele of favored businessmen who regularly receive lucrative public contracts, and in some cases purchase entire public companies at steep discounts.[28] In turn, these businessmen are expected to make "donations" to specific organizations – for example, firms winning government contracts are expected to

[28] Examples include Eti Aluminium, Oymapinar Hydroelectric Company, and the public motor company BMC - the latter was sold to an ally of Erdogan at an estimated 25% below its value (Yildirim, 2015).

donate a sum equal to 10-20 percent of the winning bid to the non-governmental organization TURGEV, which is run by President Erdogan's son (Yildirim, 2015). By contrast, disfavored businessmen are being pushed out of the market through investigations and fines by tax authorities and other government agencies. For example, in 2009 the Tax Ministry fined a media conglomerate owned by Aydin Dogan, an outspoken critic of the president; the amount of the fine was $2.5 billion, nearly as much as the value of the parent organization of the conglomerate.[29]

One unusual element of the Turkish system is that the leader's use of extracted income is formalized, as an extrabudgetary discretionary fund that the president can allocate without any oversight. Such a fund was originally used by Erdogan as prime minister, and after he became president, a similar fund was created for that office in 2015. Spending from the fund grew from $70 million per year in 2003 to $60 million per month in 2024. While it is impossible to know exactly what the fund is being spent on, observers noted a 238 percent increase in spending from the fund ahead of the 2024 local elections.[30]

Oligarchs in Russia Russia offers two particularly famous, and historically consequential, examples. The first is a defining moment in the privatization of Soviet enterprises: President Yeltsin's controversial "loans-for-shares" program. This program sold off state companies at low prices to a group of oligarchs, who then provided resources to finance Yeltsin's 1996 reelection campaign (Shleifer and Treisman, 2005). The second example is from the early 2000s, when bargaining powers between the current president, Putin, and the oligarchs were markedly different. As described in the Dawisha quote in the Introduction, Putin extracted resources from oligarchs in return for letting them stay in business.

Based on our model, the Yeltsin episode reflects the start of an oligarchy in the sense of Proposition 3, as a leader with low power ϕ faces an increase in λ driven by the need for money to get reelected. This results in clients receiving rents. By contrast, the Putin episode reflects a leader whose increasing power ϕ allows him to reduce rents.

Our model predicts that both of these regimes will be associated with entry barriers and increased market concentration in order to deliver higher profits. As far as we know, this prediction has yet to be formally tested. It certainly seems plausible that a privatization program different from loans-for-shares may have resulted in more competition. Similarly, the Putin regime encouraged market concentration through mergers and the creation of conglomerates in several industries including aircraft design, shipbuilding, and defense (Aslund, 2019, p28).

[29] https://www.nytimes.com/2009/09/10/world/europe/10istanbul.html

[30] https://www.turkishminute.com/2024/03/21/erdogans-monthly-discretionary-fund-use-soared-to-59-mln-ahead-of-elections/

Public procurement in Hungary A prime opportunity for industry capture is the public procurement process, as modeled explicitly in Section 5.3. Profit-sharing through procurement is well illustrated by the case of the highest net-worth Hungarian, Lőrinc Mészáros.

Mészáros and Viktor Orbán (the prime minister since 2010), became close in 1999 in Felcsút, a small village and Orbán's birthplace. At the time Mészáros owned a small firm building gas pipelines in neighboring villages, and he sponsored the local football team where Orbán played as a striker. Since 2010, Mészáros's net worth has been increasing exponentially. It was estimated at $30M in 2013 when he first made it to the top 100 nationally. By 2024, it reached $3.2B according to Forbes, making him the wealthiest Hungarian. By then his activities had spread to other sectors, including public construction, energy, banking, agriculture, manufacturing, media, and hospitality.[31]

It is well documented that Mészáros's wealth accumulation was supported by the increasing share of public procurement contracts his firms won throughout the years. While this share was negligible in 2010 when Orbán came to power, it grew to 5% of the value of *all* public procurement contracts by 2017, and to 17% by 2021. The bulk of these contracts were financed by EU funds.[32]

Investigative journalists have documented various ways in which, similarly to our model, Mészáros's profits are channeled back to support Orbán. One common method is to use the procurement revenues to pay for subcontractors linked to the prime minister. One such subcontractor is Dolomit Ltd, a mining company owned by Győző Orbán, the prime minister's father.[33] Another channel is through the media sector. In 2015 and 2016 Mészáros acquired several local and national newspapers as well as a national TV channel, then in 2017 gifted it all to a newly established foundation which has been running the government's propaganda machine since.[34]

Privatization in Latin America A common criticism of privatization programs around the world is that while they achieve the transfer of control and ownership of former state monopolies to the private sector, they do not necessarily increase competition, and monopolies remain. This was the case in the privatization programs of several Latin American countries in the 1990s (Manzetti, 1999). Our model provides an immediate explanation: market power (a low Ω) makes these companies more valuable to buyers, which is an important consideration to leaders if they hope to extract private income from these transactions.

[31] For background information see the Wikipedia article on Lőrinc Mészáros and citations therein.

[32] Estimates are from the non-profit Corruption Research Center Budapest, see `http://www.crcb.eu/?p=3400` and `http://www.crcb.eu/?p=3183`

[33] `https://www.direkt36.hu/en/igy-folyik-tovabb-a-kozpenz-meszaros-lorinctol-az-orban-csaladhoz/`

[34] `https://media1.hu/2019/06/05/meszaros-lorinc-mediaworks-talentis-vetelar-elajandekozas/`

For example, during President Menem's privatization program in Argentina "the symbiosis between political and economic power reached an unprecedented level as the largest conglomerates generously funded Menem's campaign re-election and their chief executive officers figured prominently on the list of special guests travelling abroad with the President to procure new business." (Manzetti, 1999, p140) These same conglomerates were the ones acquiring the privatized state companies. In some cases conglomerates expanded to new industries by acquiring state monopolies, while in sectors such as petroleum or steel, they bought their previously state-owned competitors, thereby increasing their market power. (Manzetti, 1999, p135).

The License Raj in India In most cases, clientelistic contracts between the leader and private firms remain hidden, but in others they are part of a quasi-official system of favor exchange, justified by an ideology of promoting economic growth or equity. A salient example of this is India in the 1970s and 80s. One of Nehru's legacies was a development model based on central planning and government regulation. The stated goal was to facilitate the allocation of resources to high-priority industries to promote growth and accomplish various social objectives. But when the ruling Congress party faced shortages in party finances, the system transformed into what became known as the "License Raj," a system where Congress obtained funding from companies, while the latter "depended on the system to secure and maintain monopoly, protection, and guaranteed profitability." (Kochanek, 1987, p1284).

Over time, this system incentivized firms to evade taxes, and engage in black-market operations. On the one hand, this gave managers access to discretionary funds that could more easily be used for political payments; on the other hand, it all allowed reducing firms' assets that were visible to the state and that politicians could therefore make claims on (Root, 2006, Chapter 7). This illustrates the fundamental difficulty that hybrid leaders face in their efforts to extract resources: the possibility that cronies may shield some of these resources from them. Our model captures this through the enforcement constraint that clientelistic contracts are subject to.

Crony firms in North Africa Hybrid regimes in North Africa prior to the Arab Spring offer another set of examples consistent with our model. Systematic evidence exists on at least two countries, Tunisia and Morocco. Rijkers et al. (2017) document the economic success of firms connected to President Ben Ali, who ruled Tunisia between 1987-2011. Most industries in which these firms operated had two important characteristics: (i) they required government authorization for running a business, and (ii) they had restrictions on inbound Foreign Direct Investment. In turn, these entry barriers allowed connected firms to generate abnormal profits.

Ruckteschler et al. (2022) study a trade liberalization episode between the EU and Morocco in 2000. They show that to offset the increased competition from foreign firms, Morocco introduced non-tariff measures (NTMs) such as input regulations, labeling requirements and shipping inspections. These protectionist measures were especially likely in industries with many "crony firms" – firms connected to politicians and to the royal family. Particularly interesting for our results is the fact that, although protection was more likely in industries with many cronies, *all* sectors where trade was liberalized experienced a subsequent rise in NTMs. This would be difficult to explain with either a welfare-maximizing government or a partial equilibrium model. However, it is consistent with our Proposition 3 which shows why, in general equilibrium, a leader with positive λ can benefit by restricting entry even in sectors in which he has no cronies.

7 Conclusion

This paper seeks to shed light on the economics of hybrid regimes. To do this, we model industry capture by a leader who enters into clientelistic contracts with firms. These contracts create profits through entry restrictions, which are then divided between the leader and his clients as a function of the model's parameters and the leader's constraints. Two kinds of hybrid regimes emerge, one where clients obtain no rents, and one where they do and become oligarchs. We study the welfare implications of these regimes in a number of scenarios, including heterogeneity in the leader's power, and under different sanctions imposed by an outside actor such as an international organization.

When institutions are such that the leader has high value for extracted income, market concentration and associated welfare losses will be particularly large. A leader with more power over clients also creates larger distortions, even as this is accompanied by a reduction in oligarchs' rents. In general equilibrium, clientelism in some industries spills over in the form of distortive policies in the rest of the economy, though more power over some industries can alleviate restrictions in others.

Descriptively, our model sheds light on a number of examples from hybrid regimes. Normatively, it can be used to study the impact of sanctions. In particular, we show why even "smart sanctions" that directly target the leader's income may be costly for welfare, unless they can also target the mechanism responsible for creating this income. We also show that the case for some economic sanctions can be strengthened in settings where the leader's support hinges on the preferences of oligarchs.

While most of the literature on non-democracies considers detailed models of the politics of these regimes, it treats the economy as a black box. Our paper takes the complementary approach of combining a detailed economic model with a reduced form treatment of politics. A natural next step in this research would be to combine these approaches, for example, by endogenizing the weights placed by the leader on private income and social welfare through explicit models of political competition or propaganda. Another useful extension would be to study different models of the economy, such as different market structures or allowing for international trade, in order to derive further economic implications of hybrid regimes. Finally, along the lines of our extension on productivity, a dynamic model with investment could shed light on the impact of hybrid leaders on economic development and growth.

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Online Appendix

A Appendix: The benchmark economy

In this section, we derive (2)-(5), which take Ω_j as exogenously given. Utility maximization implies that consumer i's demand for product ω of industry $j > 0$ is

$$q_j(\omega, i) = \hat{\beta}_j Y(i) P_j^{\frac{1}{\mu-1}} p_j(\omega)^{\frac{\mu}{1-\mu}}, \tag{A.1}$$

where

$$P_j = \left[\int_0^{\Omega_j} p_j(\omega)^{\frac{1}{1-\mu}} d\omega\right]^{1-\mu}$$

is a price index. Demand for good 0 is $Q_0(i) = \hat{\beta}_0 Y(i)$. Aggregating across consumers, firms in industry $j > 0$ solve

$$\max_{p_j(\omega)} \hat{\beta}_j Y P_j^{\frac{1}{\mu-1}} (p_j(\omega) - c_j) p_j(\omega)^{\frac{\mu}{1-\mu}}$$

taking P_j as given. This yields the same price for each variety ω,

$$p_j(\omega) = \mu c_j, \tag{A.2}$$

and

$$P_j = \Omega_j^{1-\mu} \mu c_j. \tag{A.3}$$

Therefore, produced quantities are also the same within industry j,

$$q_j(\omega) = q_j = \hat{\beta}_j Y P_j^{\frac{1}{\mu-1}} (\mu_j c_j)^{\frac{\mu}{1-\mu}} = \frac{\beta_j Y}{\Omega_j(\mu-1)c_j}$$

where we used $\beta_j = \hat{\beta}_j \frac{\mu-1}{\mu}$ and (A.44).

The definition of Q_j gives

$$Q_j = \left[\int_0^{\Omega_j} q_j(\omega)^{\frac{1}{\mu}} d\omega\right]^{\mu} = \Omega_j^{\mu} q_j = \frac{\beta_j \Omega_j^{\mu-1}}{(\mu-1)c_j} Y$$

Profit, disregarding the fixed cost is

$$\pi_j = \hat{\beta}_j Y P_j^{\frac{1}{\mu-1}} p_j(\omega)^{\frac{\mu}{1-\mu}} (p_j(\omega) - c_j) = \frac{\beta_j}{\Omega_j} Y.$$

Using (A.1), (A.43) and (A.44), individual i's consumption is

$$Q_j(i) = \frac{\hat{\beta}_j \Omega_j^{\mu-1}}{\mu c_j} Y(i).$$

B Appendix: Proofs for Sections 3.1-3.3

We first prove Proposition 1, then present a series of lemmas and use them to prove Propositions 2 and 3.

Proof of Proposition 1. Clearly, if $\lambda = 0$ then the leader does not extract any income, so $w_j = \pi_j$ and the enforcement constraint becomes irrelevant. The problem in (11) is

$$\max_{\Omega} \ln\left(1 + \sum_j \Omega_j \left(\pi_j - f_j\right)\right) + \sum_j \beta_j \left(\mu \ln \Omega_j - \frac{\mu}{\mu - 1} \ln c_j\right)$$

$s.t.$

$$\pi_k = \frac{\beta_k}{\Omega_k} Y \quad \forall k \tag{A.4}$$

$$Y = 1 + \sum_j \Omega_j (\pi_j - f_j) \tag{A.5}$$

$$\pi_k \geq f_k \quad \forall k \tag{A.6}$$

Using (A.4) and the fact that $\sum_{j>0} \beta_j = \frac{\mu-1}{\mu} - \beta_0$, (A.5) can be expressed as

$$Y = \mu \frac{1 - \sum_j \Omega_j f_j}{\mu \beta_0 + 1}. \tag{A.7}$$

Then the problem becomes

$$\max_{\Omega} \ln\left(1 - \sum_j \Omega_j f_j\right) + \sum_j \beta_j \left(\mu \ln \Omega_j - \frac{\mu}{\mu - 1} \ln c_j\right)$$

$$s.t. \frac{1 - \sum_j \Omega_j f_j}{\mu \beta_0 + 1} \frac{\mu \beta_k}{\Omega_k} \geq f_k \quad \forall k$$

The derivative of the objective w.r.t. Ω_k is

$$\frac{-f_k}{1 - \sum \Omega_j f_j} + \frac{\mu \beta_k}{\Omega_k}. \tag{A.8}$$

If the constraint is slack for any k, then $\frac{-f_k}{1-\sum \Omega_j f_j} + \frac{\mu\beta_k}{\Omega_k} > \frac{-1}{\mu\beta_0+1} \frac{\mu\beta_k}{\Omega_k} + \frac{\mu\beta_k}{\Omega_k} > 0$, so this cannot be an optimum. If all constraints bind, then $Y = 1$ and hence $\pi_k = \frac{\beta_k}{\Omega_k}$ and $\Omega_k = \Omega_k^{FE}$. ■

Lemma B.1 *At least one of the constraints (8) or (9) must bind for every industry j.*

Proof. Substituting A.7 into A.4, define

$$\pi_j\left(\Omega_J\right) \equiv \frac{\beta_j}{\Omega_j} \frac{1}{1 - \bar{\beta}} \left(1 - \sum_{j=1}^{J} \Omega_j f_j\right) \tag{A.9}$$

Then, we can write the Lagrangian corresponding to problem (11) as

$$\max_{\Omega,w} \lambda \ln \sum_j \Omega_j(\pi_j(\Omega) - w_j) + (1-\lambda)\ln[1 + \sum_j \Omega_j(w_j - f_j)] + (1-\lambda)\sum_j \beta_j \mu \ln \Omega_j$$
$$- \sum_j \gamma^j_{PC}(w_j - f_j) - \sum_j \gamma^j_{EC}[w_j - (1-\phi_j)\pi_j(\Omega)], \quad \text{(A.10)}$$

where and $\gamma^j_{PC} \geq 0$ and $\gamma^j_{EC} \geq 0$ are the Lagrange multipliers corresponding to the PC and EC constraints of industry j, respectively.

The first-order conditions for w_k and Ω_k can be written as

$$\frac{-\lambda}{\sum_j \Omega_j(\pi_j(\Omega) - w_j)} + \frac{1-\lambda}{1 + \sum_j \Omega_j(w_j - f_j)} - \frac{\gamma^k_{EC} + \gamma^k_{PC}}{\Omega_k} = 0 \quad \text{(A.11)}$$

$$\lambda \frac{-\frac{f_k \bar{\beta}}{1-\bar{\beta}} - w_k}{\sum_j \Omega_j(\pi_j(\Omega) - w_j)} + (1-\lambda)\frac{w_k - f_k}{1 + \sum_j \Omega_j(w_j - f_j)} + (1-\lambda)\frac{\beta_k \mu}{\Omega_k} + \sum_j (1-\phi_j)\gamma^j_{EC}\frac{\partial \pi_j(\Omega)}{\partial \Omega_k} = 0 \quad \text{(A.12)}$$

Suppose both constraints were slack for some industry j'. Then $\gamma^{j'}_{PC} = \gamma^{j'}_{EC} = 0$, therefore

$$\frac{\lambda}{\sum_j \Omega_j(\pi_j(\Omega) - w_j)} = \frac{1-\lambda}{1 + \sum_j \Omega_j(w_j - f_j)} \quad \text{(A.13)}$$

from (A.11). But because (A.13) is independent of j', (A.11) implies that we must also have $\gamma^k_{PC} + \gamma^k_{EC} = 0$ for all $k \neq j'$. This is only possible if $\gamma^k_{PC} = \gamma^k_{EC} = 0$ for all k.

Using this observation together with (A.9) and (A.11), (A.12) can be rewritten as

$$\beta_k \mu \left(1 - \bar{\beta}\right)\left(1 + \sum_j \Omega_j(w_j - f_j)\right) = f_k \Omega_k \ \forall k. \quad \text{(A.14)}$$

Using (A.9), (A.13) can be written as

$$\frac{\lambda}{\frac{\bar{\beta}}{1-\bar{\beta}}(1 - \sum_j \Omega_j f_j) - \sum_j \Omega_j w_j} = \frac{1-\lambda}{(1 - \sum_j \Omega_j f_j) + \sum_j \Omega_j w_j}$$

implying

$$\sum_j \Omega_j w_j = \left((1-\lambda)\frac{\bar{\beta}}{1-\bar{\beta}} - \lambda\right)\left(1 - \sum_{j=1}^{J} \Omega_j f_j\right). \quad \text{(A.15)}$$

Substituting (A.15) into (A.14), we obtain

$$f_k \Omega_k = \beta_k \mu (1-\lambda)(1 - \sum_j \Omega_j f_j) \ \forall k. \quad \text{(A.16)}$$

implying

$$\bar{\beta}\mu(1-\lambda)\left(1-\sum_{j=1}^{J}\Omega_j f_j\right)=\sum_{j=1}^{J}\Omega_j f_j.$$

Our starting assumption was that the participation constraint (8) is slack for at least one industry. This would imply $\sum_j \Omega_j w_j > \sum_j \Omega_j f_j$. Using (A.15), this would mean

$$\frac{\bar{\beta}(1-\lambda)}{1-\bar{\beta}}-\lambda > \bar{\beta}\mu(1-\lambda)$$

which in turn would require $\frac{1}{1-\bar{\beta}} > \mu$, or $\mu\beta_0 < 0$. This is not possible, hence at least one of the constraints must bind for every industry. ■

Lemma B.2 *Let $\phi_j = \phi$ for all j. If $\lambda < \lambda'(\phi)$, then the optimal solution is $w_j = f_j$ and $\Omega_j = \frac{(1-\lambda)\bar{\beta}\mu}{\left(\lambda+(1-\lambda)\bar{\beta}\mu\right)}\frac{\beta_j}{f_j}$, implying $\frac{\pi_j}{f_j} = 1+\frac{\lambda}{1-\lambda}\frac{1}{\mu(1-\bar{\beta})\bar{\beta}}$.*

Proof. We consider the problem where the EC's (9) are ignored for every j. We show that in this relaxed problem the solution in the statement is optimal. Then, we show that under the restriction on λ, all constraints (9) are slack. Hence, the proposed solution remains optimal in the original problem.

Note first that if all the EC's are slack, then by Lemma B.1, each PC has to bind, that is $w_j = f_j$. But in this case, (A.12) implies

$$(1-\lambda)\frac{\beta_k\mu}{\Omega_k f_k}=\frac{\lambda}{1-\bar{\beta}}\frac{1}{\sum_j \Omega_j(\pi_j(\Omega)-f_j)}$$

substituting in (A.9) and summing up implies

$$\frac{\frac{1-\lambda}{\lambda}\bar{\beta}^2\mu}{1+\frac{1-\lambda}{\lambda}\bar{\beta}\mu}=\sum_{j=1}^{J}\Omega_j f_j$$

hence

$$\frac{(1-\lambda)\bar{\beta}\mu}{\left(\lambda+(1-\lambda)\bar{\beta}\mu\right)}\frac{\beta_k}{f_k}=\Omega_k.$$

Therefore, by (A.9) profit per firm is

$$\frac{\pi_j}{f_j}=\frac{\lambda}{\left(1-\bar{\beta}\right)(1-\lambda)\bar{\beta}\mu}+1$$

Hence, as long as

$$\frac{\lambda}{\left(1-\bar{\beta}\right)(1-\lambda)\bar{\beta}\mu}+1<\frac{1}{1-\phi}$$

this solution satisfies all EC constraints in (9). Rearranging, we obtain the condition in the Lemma. ■

Lemma B.3 *Let $\phi_j = \phi$ for all j. If $\lambda'(\phi) \leq \lambda \leq \lambda''(\phi)$, then $w_j = f_j$, $\frac{\pi_j}{f_j} = \frac{1}{1-\phi}$, and $\Omega_j = \frac{\beta_j}{f_j}\frac{1-\phi}{1-\phi\bar{\beta}}$.*

Proof. Note that for the proposed solution all ECs (9) and PCs (8) bind. Note also that substituting in the proposed solution

$$\sum_j \Omega_j \pi_j(\Omega) - \sum_j \Omega_j f_j = \frac{\bar{\beta}\phi}{1-\phi\bar{\beta}}$$

and (A.11) gives

$$1 - \lambda \frac{1}{\bar{\beta}\phi} = \frac{\gamma_{EC}^k + \gamma_{PC}^k}{\Omega_k}. \tag{A.17}$$

Also as

$$\frac{\partial \pi_k(\Omega)}{\partial \Omega_k} = \frac{1}{1-\bar{\beta}}\frac{\beta_k}{\Omega_k}\left(-\frac{1}{\Omega_k} + \frac{1}{\Omega_k}\sum_{j=1}^{J}\Omega_j f_j - f_k\right) = -\frac{1}{1-\bar{\beta}}\frac{f_k^2}{\frac{1-\phi}{1-\phi\bar{\beta}}}\left(\frac{1-\phi\bar{\beta}}{\beta_k(1-\phi)} + 1 - \frac{\bar{\beta}}{\beta_k}\right)$$

$$\frac{\partial \pi_j(\Omega)}{\partial \Omega_k} = -\frac{1}{1-\bar{\beta}}\frac{\beta_j}{\Omega_j} f_k = -\frac{1}{1-\bar{\beta}}\frac{1-\phi\bar{\beta}}{1-\phi} f_j f_k$$

(A.12) implies

$$\beta_k \frac{-\lambda \frac{1}{\bar{\beta}\phi\left(1-\bar{\beta}\right)} + (1-\lambda)\frac{\mu}{1-\phi} - \frac{1}{1-\bar{\beta}}\sum_j \gamma_{EC}^j f_j}{\frac{1}{1-\bar{\beta}}\left(\frac{1-\phi\bar{\beta}}{1-\phi} - \bar{\beta}\right)} = \gamma_{EC}^k f_k. \tag{A.18}$$

Summing up by industry and solving for $\sum_{j>0} \gamma_{EC}^j f_j$ gives

$$\frac{\bar{\beta}\frac{-\lambda\frac{1}{\bar{\beta}\phi} + (1-\lambda)\frac{(1-\bar{\beta})\mu}{1-\phi}}{\frac{1-\phi\bar{\beta}}{1-\phi} - \bar{\beta}}}{\frac{\bar{\beta}\phi - 1}{\bar{\beta}-1}} = \sum_k \gamma_{EC}^k f_k$$

and substituting back to (A.18) implies

$$\frac{\bar{\beta}\mu\phi(1-\lambda)\left(1-\bar{\beta}\right) - \lambda(1-\phi)}{\bar{\beta}\phi\left(1-\bar{\beta}\phi\right)}\frac{\beta_k}{f_k} = \gamma_{EC}^k.$$

Using (A.17) also gives

$$\frac{-\mu - \phi + \beta\mu + \lambda\mu - \beta\lambda\mu + 1}{1-\phi} = \frac{\gamma_{PC}^k}{\Omega_k}$$

These two expressions imply that $\gamma^k_{EC} \geq 0$ if $\lambda \geq \lambda'(\phi)$ while $\gamma^k_{PC} \geq 0$ if $\lambda \leq \lambda''(\phi)$. Hence under the condition of the Lemma, the proposed solution solves the Lagrangian (A.10). ■

Lemma B.4 *Let $\phi_j = \phi$ for all j. If $\lambda > \lambda''(\phi)$, then the optimal solution is $\Omega_j = \frac{\beta_j}{f_j}\frac{(1-\lambda)}{\frac{1}{\mu}+(1-\lambda)\bar{\beta}}$, implying $\frac{\pi_j}{f_j} = \frac{1}{(1-\lambda)\mu(1-\bar{\beta})}$.*

Proof. Consider the problem where we ignore (8) for all j. By Lemma B.1, we know that in this case all EC's must bind, i.e., $w_j = (1-\phi)\pi_j(\Omega)$. The problem of the leader simplifies to

$$\max_{\Omega} \lambda \ln \phi \sum_j \Omega_j \pi_j(\Omega) + (1-\lambda)\ln[1 + \sum_j \Omega_j((1-\phi)\pi_j(\Omega) - f_j)] + (1-\lambda)\sum_j \beta_j \mu \ln \Omega_j.$$

Substituting in (A.9) and omitting constants, this simplifies to

$$\max_{\Omega} \ln\left(1 - \sum_{j=1}^{J} \Omega_j f_j\right) + (1-\lambda)\sum_j \beta_j \mu \ln \Omega_j$$

The FOC w.r.t. Ω_j is

$$(1-\lambda)\beta_j\mu\left(1 - \sum_{j=1}^{J} \Omega_j f_j\right) = f_j \Omega_j$$

which gives

$$\frac{(1-\lambda)\bar{\beta}\mu}{1+(1-\lambda)\bar{\beta}\mu} = \sum_{j=1}^{J} f_j \Omega_j$$

Hence

$$\frac{\beta_j\mu}{f_j}\frac{(1-\lambda)}{1+(1-\lambda)\bar{\beta}\mu} = \Omega_j$$

and substituting back to (A.9)

$$\pi_j = \frac{f_j}{\mu\left(1-\bar{\beta}\right)(1-\lambda)}$$

The solution satisfies each PC (8) if

$$\frac{(1-\phi)}{\mu\left(1-\bar{\beta}\right)(1-\lambda)} > 1$$

which gives the condition in the statement. ■

Proof of Proposition 2 and 3. Lemmas B.2-B.4 verify the characterizations in part (i) of Proposition 2 and 3.

For the effect of λ and ϕ on the number of firms, it is easy to verify that Ω_j is continuous in $(1-\phi)$. Raising $(1-\phi)$ moves the solution from Lemma B.2 to Lemma B.3 to Lemma B.4. Holding all else fixed, this raises Ω_j. In addition, under Lemma B.3, it increases the number of firms since $\frac{\partial \Omega_j}{\partial \phi} < 0$. Thus, a higher ϕ weakly reduces the number of firms.

As λ rises, we move from from Lemma B.2 to Lemma B.3 to Lemma B.4. In addition, note that $\frac{\partial \Omega_j}{\partial \lambda} < 0$ under Lemma B.2 and Lemma B.4, while Ω_j is independent of λ under Lemma B.3. Again, it is easy to verify that Ω_j is continuous in λ. Thus, raising λ reduces the number of firms.

Turning to utility and welfare, from (1) and (4) individual utility is given by

$$U_i = Y(i)\hat{\beta}_0^{\hat{\beta}_0} \prod_{j>0} \left(\frac{\hat{\beta}_j \Omega_j^{\mu-1}}{\mu c_j} \right)^{\hat{\beta}_j} \tag{A.19}$$

For workers, $Y(i) = 1$, and as just shown, both λ and ϕ reduce Ω_j and hence U_i.

For entrepreneurs in industry k, $Y(i) = w_k/f_k$. For $\lambda < \lambda''$, $w_k/f_k = 1$, so that entrepreneurs' and workers' utility is the same, and both (as well as welfare) are decreasing in λ and ϕ.

For $\lambda > \lambda''$, the equilibrium value of an entrepreneur's utility U^E can be obtained from the characterization in Lemma B.4. Denoting

$$\Gamma \equiv \hat{\beta}_0^{\hat{\beta}_0} \prod_{j>0} \left(\frac{\hat{\beta}_j}{\mu c_j} \right)^{\hat{\beta}_j} \left(\frac{\beta_j}{f_j} \right)^{\hat{\beta}_j(\mu-1)}, \tag{A.20}$$

we have

$$U^E = \frac{1-\phi}{(1-\lambda)\mu(1-\bar{\beta})} \left(\frac{1-\lambda}{\frac{1}{\mu} + (1-\lambda)\,\bar{\beta}} \right)^{\bar{\beta}\mu} \Gamma \tag{A.21}$$

Clearly, U^E is decreasing in ϕ. The only part that is not obvious is how λ affects U^E. Denote $X = (1-\lambda)\mu$ so $\frac{\partial U^E}{\partial X} = \frac{\partial}{\partial X} \left(\frac{1-\phi}{X(1-\bar{\beta})} \left(\frac{X}{1+X\bar{\beta}} \right)^{\bar{\beta}\mu} \right)$, which is proportional to

$$\frac{\bar{\beta}\mu - 1}{X} - \frac{\bar{\beta}^2 \mu}{1 + X\bar{\beta}}$$

If $1 < \frac{1}{\bar{\beta}\mu}$, this is negative, so U^E is monotonically increasing in λ. If $1 > \frac{1}{\bar{\beta}\mu}$, then this is negative iff $\lambda < \frac{1}{\bar{\beta}\mu}$. Thus, U^E is increasing in λ for $\lambda''(\phi) < \lambda < \frac{1}{\mu\bar{\beta}}$ and decreasing in λ for $\max(\lambda''(\phi), \frac{1}{\mu\bar{\beta}}) < \lambda$.

Finally, for $\lambda > \lambda''$ the negative welfare impact of λ can be verified as follows. Using Ω_j from Lemma B.4, compute total income Y using (A.7). Replacing $Y(i)$ in (A.19) with Y

yields an expression for social welfare:

$$W=\frac{1}{1-\bar{\beta}}\left(1-\frac{(1-\lambda)\bar{\beta}\mu}{1+(1-\lambda)\bar{\beta}\mu}\right)\left(\frac{1-\lambda}{\frac{1}{\mu}+(1-\lambda)\,\bar{\beta}}\right)^{\bar{\beta}\mu}\Gamma \tag{A.22}$$

Taking the derivative yields $\frac{\partial W}{\partial \lambda}<0$. ■

C Appendix: Industry-specific power (proofs for Section 3.4)

To characterize the equilibrium with industry specific power, we first prove two lemmas that will be used extensively. We then prove two propositions that characterize the equilibrium when power in one industry is either "low" or "high" in a well-defined sense. Finally, we use these propositions to prove Propositions 4 and 5 in the text.

Lemma C.5 *Let $J=2$ and assume that only EC1 and PC2 bind. Then $w_1>f_1$, $w_2=f_2$, $\Omega_1=\frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$, and*

$$\Omega_2=\frac{1}{f_2}\frac{(2A(\mu(1-A)-1)+1)\,(1-\lambda)\mu\beta_2+\lambda+A(\mu(1-A)-1)-S}{2A\mu\,(1-A)\,(\lambda+\mu-\lambda\mu)} \tag{A.23}$$

where $A\equiv(1-\phi_1)\beta_1$ and

$$S\equiv\sqrt{\mu^2\left(\lambda-1\right)^2\beta_2^2-2\mu\left(1-\lambda\right)\left(2A\lambda-A-\lambda+A\mu\left(2\lambda-1\right)\left(A-1\right)\right)\beta_2+\left(A-\lambda-A\mu+A^2\mu\right)^2}$$

.

Proof of Lemma C.5. Assume that only EC1 and PC2 bind: $w_2=f_2$ and $w_1=(1-\phi)\pi_1>f_1$. The latter implies

$$\Omega_1 f_1<(1-\phi_1)\mu\beta_1(1-\sum_j\Omega_j f_j). \tag{A.24}$$

Define $\xi\equiv\frac{\Omega_2 f_2}{1-\Omega_1 f_1}$. We know that $A<\frac{\mu-1}{\mu}=\sum_j\beta_j$, and $\xi\in[0,1]$ because $1\geq\sum_j\Omega_j f_j$.

The first-order conditions w.r.t. Ω_1 and Ω_2 can be written respectively as

$$\frac{-(\mu-1-\mu A)\lambda}{(1-\xi)(\mu-1-\mu A)-\xi}\frac{\Omega_1 f_1}{1-\Omega_1 f_1}+\frac{1-\lambda}{1+\mu A(1-\xi)}(-\mu A-1)\frac{\Omega_1 f_1}{1-\Omega_1 f_1}+(1-\lambda)\mu\beta_1=0 \tag{A.25}$$

$$\frac{-\xi\mu\lambda(1-A)}{(1-\xi)(\mu-1-\mu A)-\xi}-\frac{1-\lambda}{1+\mu A(1-\xi)}\mu A\xi+(1-\lambda)\mu\beta_2=0. \tag{A.26}$$

Expression (A.26) yields

$$F(\xi) \equiv (1-\xi)^2\mu^2(1-A)A(1-\lambda)\beta_2+(1-\xi)\mu[(1-2A)(1-\lambda)\beta_2-\xi A(1-A)]-\xi(\lambda-A)-(1-\lambda)\beta_2 = 0. \tag{A.27}$$

This is a quadratic equation in ξ. Solving, it can be verified that only the lower root satisfies $\pi_2 \geq f_2$. This is

$$\xi(\phi_1) = \frac{(2A(\mu(1-A)-1)+1)\,(1-\lambda)\mu\beta_2 + \lambda + A(\mu(1-A)-1) - S}{2A\mu\,(\mu\beta_2(1-\lambda)+1)\,(1-A)}, \tag{A.28}$$

where S is defined in the statement of the proposition. Substituting (A.28) into (A.25) yields

$$\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}. \tag{A.29}$$

Substituting this into (A.28) and solving for Ω_2 yields (A.23). ■

Lemma C.6 *For $\xi(\phi_1)$ given by (A.28), $\frac{\partial\xi}{\partial A} < 0$.*

Proof. Because the denominator of (A.28) is positive, we have that $\frac{\partial\xi}{\partial A}$ is proportional to

$$2\,(\mu(1-2A)-1)\,(1-\lambda)\mu\beta_2 + \mu(1-2A) - 1 - \frac{\partial S}{\partial A} - 2\mu\,(\mu\beta_2(1-\lambda)+1)\,(1-2A)\,\xi.$$

Taking the derivative of S and using (A.28), algebra shows that this expression is always negative. ■

The following Proposition describes situations where the leader has little power in one of the industries (industry 1).

Proposition C.1 *Suppose that ϕ_1 is small enough that $PC1$ is slack. Then $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$, and there exists $\widetilde{\phi}_2 \in (\lambda, 1)$ such that*
(i) if $\phi_2 < \lambda$, then $w_1 > f_1$, $w_2 > f_2$, and $\Omega_2 = \frac{\beta_2}{f_2}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$.
(ii) if $\widetilde{\phi}_2 < \phi_2$, then $w_1 > f_1$, $w_2 = f_2$, and Ω_2 is given by (A.23)
(iii) if $\lambda \leq \phi_2 \leq \widetilde{\phi}_2$, then $w_1 > f_1$, $w_2 = f_2$, and $\Omega_2 = \frac{\beta_2}{f_2}\frac{\mu(\lambda+(1-\lambda)(1+\mu\beta_2))}{(\frac{1}{1-\phi_2}+\mu\beta_2)(\lambda+\mu-\lambda\mu)}$.

Proof of Proposition C.1. By assumption PC1 is slack (which will be the case, e.g., for $\phi_1 = 0$), therefore EC1 binds. There are only 3 cases to consider for $j = 2$'s constraints: only PC2 binds, only EC2 binds, both PC2 and EC2 bind.

Step 1. Assume that only PC2 binds: $w_2 = f_2 > (1-\phi_2)\pi_2$, which implies

$$\Omega_2 f_2 > (1-\phi_2)\mu\beta_2(1-\sum_j \Omega_j f_j). \tag{A.30}$$

From Lemma C.5, we have $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$, and Ω_2 is given by (A.23). Using the notation $\xi(\phi_1)$ from (A.28), (A.30) can be rewritten as

$$\xi(\phi_1) > \frac{\mu(1-\phi_2)\beta_2}{1+(1-\phi_2)\beta_2\mu}. \tag{A.31}$$

Note that (A.31) holds for $\phi_2 = 1$, and the right-hand side is decreasing in ϕ_2. Using (A.28), it can also be shown that (A.31) fails for $\phi_2 = \lambda$. It follows that there must be some $\widetilde{\phi}_2 \in (\lambda, 1)$ such that (A.31) holds if and only if $\phi_2 > \widetilde{\phi}_2$.

Step 2. Assume that only EC2 binds: $w_2 = (1-\phi_2)\pi_2 > f_2$. Then the first order condition w.r.t. Ω_k is

$$\frac{-f_k}{1-\sum_j \Omega_j f_j} + (1-\lambda)\frac{\beta_k \mu}{\Omega_k} = 0,$$

which can be solved to obtain $\Omega_k = \frac{\beta_k}{f_k}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$. The condition $(1-\phi_2)\pi_2 > f_2$ yields $\lambda > \phi_2$.

Step 3. As shown in Step 1, condition (A.31) fails for $\phi_2 \leq \lambda < \widetilde{\phi}_2$. Hence if a solution where only PC2 binds is feasible, then a solution where only EC2 binds is not, and vice versa. Since the only other possibility (when both PC2 and EC2 binds) imposes an extra constraint relative to either of these, that solution cannot be optimal when one of these less constrained solutions is feasible.

Step 4. The only remaining case is when $\lambda < \phi_2 < \widetilde{\phi}_2$. Then it must be that both PC2 and EC2 bind: $\Omega_2 f_2 = \mu A(1-\sum_j \Omega_j f_j)$, from which

$$\Omega_2 f_2 = \frac{\mu(1-\phi_2)\beta_2(1-\Omega_1 f_1)}{1+\mu(1-\phi_2)\beta_2}.$$

Substituting into the objective, and solving, we obtain $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$ and $\Omega_2 = \frac{\beta_2}{f_2}\frac{\mu(\lambda+(1-\lambda)(1+\mu\beta_2))}{(\frac{1}{1-\phi_2}+\mu\beta_2)(\lambda+\mu-\lambda\mu)}$. ■

The following Proposition turns to cases where the leader's power in one of the industries (industry 2) is large.

Proposition C.2 *Suppose that ϕ_2 is large enough that EC2 is slack. There exists $\widetilde{\phi}_1 \in (0, \frac{\mu\lambda}{\lambda+\mu-1})$ such that*
(i) if $\frac{\mu\lambda}{\lambda+\mu-1} < \phi_1$, then $w_1 = f_1$, $w_2 = f_2$, and $\Omega_j = \frac{\beta_j}{f_j}\frac{(\mu-1)(1-\lambda)}{\lambda+(\mu-1)(1-\lambda)}$, $j = 1, 2$.
(ii) if $\phi_1 < \widetilde{\phi}_1$, then $w_1 > f_1$, $w_2 = f_2$, $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$, and Ω_2 is given by (A.23)
(iii) if $\widetilde{\phi}_1 \leq \phi_1 \leq \frac{\mu\lambda}{\lambda+\mu-1}$, then $w_1 = f_1$, $w_2 = f_2$, Ω_2 is the lower root of

$$-\frac{\lambda}{1-\lambda}\mu f_2\Omega_2(1-f_2\Omega_2) + [\mu(1-f_2\Omega_2) - 1 - \mu A][\mu\beta_2 - (\mu-1)f_2\Omega_2] = 0, \tag{A.32}$$

and $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\phi_1)(1-\Omega_2 f_2)}{1+(1-\phi_1)\mu\beta_1}$.

Proof of Proposition C.2. By assumption EC2 is slack (which will be the case, e.g., for

$\phi_2 = 1$), therefore PC2 binds. There are only 3 cases to consider for $j = 1$'s constraints: only PC1 binds, only EC1 binds, both PC1 and EC1 bind.

Step 1. Assume that only PC1 binds, $w_1 = f_1 > (1 - \phi_1)\pi_1$, which implies

$$\Omega_1 f_1 > (1 - \phi_1)\mu\beta_1(1 - \sum_j \Omega_j f_j). \tag{A.33}$$

Then the first order condition w.r.t. Ω_k is

$$\lambda \frac{-f_k \mu}{\sum_j \Omega_j(\pi_j(\Omega) - f_j)} + (1 - \lambda)\frac{\beta_k \mu}{\Omega_k} =$$

$$\lambda \frac{-f_k \mu}{\mu(1 - \sum_j \Omega_j f_j) - 1} + (1 - \lambda)\frac{\beta_k \mu}{\Omega_k} = 0. \tag{A.34}$$

Summing over k yields

$$1 - \sum_j \Omega_j f_j = \frac{\lambda + \frac{\mu-1}{\mu}(1-\lambda)}{\lambda + (\mu - 1)(1 - \lambda)},$$

and (A.34) yields

$$\Omega_k f_k = \beta_k \frac{(\mu - 1)(1 - \lambda)}{\lambda + (\mu - 1)(1 - \lambda)}. \tag{A.35}$$

Using (A.35), the condition (A.33) can be rewritten as

$$\frac{\mu\lambda}{\lambda + \mu - 1} < \phi_1. \tag{A.36}$$

Step 2. Assume that only EC1 binds. From Lemma C.5, we have $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$, and Ω_2 is given by (A.23).

Using (A.29), condition (A.24) can be rewritten as

$$\frac{(1 - \lambda)\beta_1}{\mu\beta_2(1 - \lambda) + 1} < A(1 - \xi(\phi_1)). \tag{A.37}$$

Step 3. Conditions (A.36) and (A.37) are necessary for the corresponding solution to be optimal. To establish that they are also sufficient, we show that (A.37) cannot hold if (A.36) does. That is, if a solution where only PC1 binds is feasible, then a solution where only EC1 binds is not, and vice versa. Since the only other possibility (when both PC 1 and EC 1 binds) imposes an extra constraint relative to either of these, that solution cannot be optimal when one of these less constrained solutions is feasible.

Suppose that (A.36) holds. This is equivalent to $A < A^* \equiv (1 - \frac{\mu\lambda}{\lambda+\mu-1})\beta_1$. Lemma C.6 shows that $\frac{\partial \xi}{\partial A} < 0$, therefore we also have $A(1 - \xi(A)) < A^*(1 - \xi(A^*))$. To establish that (A.37) cannot hold, it is thus sufficient to show that $A^*(1 - \xi(A^*)) < \frac{(1-\lambda)\beta_1}{\mu\beta_2(1-\lambda)+1}$. Rewrite this as

$$1 - \xi(A^*) < \frac{\lambda + \mu - 1}{(\mu - 1)(\mu\beta_2(1 - \lambda) + 1)}.$$

Plugging $A = A^*$ into (A.27) it can be directly verified that this condition holds.

Step 4. Suppose $A = \beta_1$ (i.e., $\phi_1 = 0$). Plugging into (A.27) and solving, it is easily verified that (A.37) holds. From the result in Step 3 and Lemma C.6, it follows that there must be some $\tilde{A} \in [A^*, \beta_1)$ such that (A.37) holds if and only if $A > \tilde{A}$. Equivalently, there exists $\widetilde{\phi}_1 \in (0, \frac{\mu\lambda}{\lambda+\mu-1}]$ such that (A.37) holds if and only if $\phi_1 < \widetilde{\phi}_1$.

Step 5. The only remaining case is when neither (A.36) nor (A.37) holds. Then it must be that both PC1 and EC1 bind: $\Omega_1 f_1 = \mu A(1 - \sum_j \Omega_j f_j)$, from which

$$\Omega_1 f_1 = \frac{\mu A(1 - \Omega_2 f_2)}{1 + \mu A}.$$

Substituting into the objective, the first order condition w.r.t. Ω_2 yields (A.32). This is a quadratic equation, and it can be verified that only the lower root satisfies $\pi_2 \geq f_2$. ■

Using the above characterizations, we turn to the propositions stated in the text.

Proof of Proposition 4. From Proposition C.1, $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1} < \frac{\beta_1}{f_1} = \Omega_1^{FE}$. ■

Proof of Proposition 5.

Part 1. Notice that, by switching 1 and 2 when needed, Propositions C.1 and C.2 characterize all possible solutions except when both EC and PC bind in both industries. In this last case, $\Omega_j = \mu\frac{\beta_j}{f_j}(1-\phi_j)(1 - \sum_j \Omega_j f_j)$, from which

$$\Omega_j = \frac{\beta_j}{f_j}\frac{1-\phi_j}{1 - \beta_1\phi_1 - \beta_2\phi_2}$$

Taking derivatives shows that $\frac{\partial\Omega_1}{\partial\phi_1} < 0$ and $\frac{\partial\Omega_2}{\partial\phi_1} > 0$. For the cases covered in Propositions C.1 and C.2, we can prove the proposition by showing that $\frac{\partial\Omega_k}{\partial\phi_j} \leq 0$ and $\frac{\partial\Omega_j}{\partial\phi_j} \geq 0$ for *both* $j = 1, 2$, $j \neq k$.

In Proposition C.1, Ω_1 is unaffected by ϕ_1 or ϕ_2. In case (i), Ω_2 is also unaffected by ϕ_1 and ϕ_2. In case (ii), Lemma C.6 implies that $\frac{\partial\Omega_2}{\partial\phi_1} > 0$, while ϕ_2 has no effect. In case (iii), Ω_2 is unaffected by ϕ_1, and, taking the derivative, $\frac{\partial\Omega_2}{\partial\phi_2} > 0$.

In Proposition C.2, the number of firms is unaffected by ϕ_2. In case (i), the number of firms is also unaffected by ϕ_1. Case (ii) corresponds to case (ii) of Propositions C.1, and the same argument applies. In case (iii), from (A.32), $\frac{\partial\Omega_2}{\partial A}$ is proportional to $\mu(\mu-1) f_2\Omega_2 - \mu\beta_2$. Using Ω_2, algebra shows that this is negative, implying $\frac{\partial\Omega_2}{\partial\phi_1} > 0$. From $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\phi_1)(1-\Omega_2 f_2)}{1+(1-\phi_1)\mu\beta_1}$, we get $\frac{\partial\Omega_1}{\partial\phi_1} < 0$.

Part 2. From the characterization in Propositions 2 and 3, we know that $\beta_1 = \beta_2$, $f_1 = f_2$ and $\phi_1 = \phi_2$ imply $\Omega_1 = \Omega_2$. This together with Part 1 immediately imply the statement in Part 2. ■

D Appendix: Sanctions (proofs for Section 4)

For Proposition 6], observe that A only enters the leader's problem (11) as a constant lowering W.

For the other sanctions, Lemmas D.7-D.9 below give a complete characterization of the solution of the problem with sanctions, along with comparative statics on W and Y_L. Propositions 7 and 8 in the main text highlight the results corresponding to the oligarchy regime, that is, those in Lemma D.9.

Lemma D.7 *Let*

$$\lambda'(\phi, B, C) = \frac{\left(\bar{\beta}-1\right)\mu\left(\bar{\beta}(\phi(B+C+1)-C)-B\right)}{\bar{\beta}^2\mu(\phi(B+C+1)-C)-\bar{\beta}(\mu B(\phi+1)+C(\mu+1)(\phi-1))+B\mu+(\phi-1)(C+1)}$$

If $\lambda < \lambda'(\phi, B, C)$*, then* $\Omega_j = \frac{\beta_j}{f_j}\frac{(1-\lambda)\mu\left(\bar{\beta}-B(1-\bar{\beta})\right)}{\left(C(1-\bar{\beta})+1\right)\left(\bar{\beta}(1-\lambda)\mu+\lambda\right)}$*, and*

$$\frac{\partial \ln Y_L}{\partial B} = \frac{\bar{\beta}-1}{\bar{\beta}-B(1-\bar{\beta})}$$
$$\frac{\partial \ln Y_L}{\partial C} = 0$$

and

$$\frac{\partial \ln W}{\partial B} = \mu\bar{\beta}\frac{\bar{\beta}-1}{\bar{\beta}-B(1-\bar{\beta})}$$
$$\frac{\partial \ln W}{\partial C} = \mu\bar{\beta}\frac{\bar{\beta}-1}{C(1-\bar{\beta})+1}.$$

Proof. Following the logic of the baseline case, when λ is small, only (13) binds, hence $w_j = (1+C)\,f_j$, and using (5), problem (12) simplifies to

$$\max_{\Omega} \lambda \ln Y_L(\Omega, B, C) + (1-\lambda)\sum_j \left(\beta_j\mu\left(\ln\Omega_j - \frac{\ln(c_j+A)}{\mu-1}\right)\right)$$

where

$$Y_L(\Omega, B, C) = \frac{\bar{\beta}\left(1+B+C\sum_j f_j\Omega_j\right)-\left(B+(1+C)\sum_j f_j\Omega_j\right)}{1-\bar{\beta}}. \tag{A.38}$$

The first order conditions give the solution

$$\Omega_j = \frac{\beta_j}{f_j}\frac{(1-\lambda)\mu\left(\bar{\beta}-B(1-\bar{\beta})\right)}{\left(C(1-\bar{\beta})+1\right)\left(\bar{\beta}(1-\lambda)\mu+\lambda\right)} \tag{A.39}$$

And the implied profit is

$$\frac{\pi_j}{f_j} = \frac{\beta_j \mu \left(1-\bar{\beta}\right)(1-\lambda)(B+C+1) + \lambda \left(C\left(1-\bar{\beta}\right)+1\right)}{\left(1-\bar{\beta}\right)(1-\lambda)\mu\left(\bar{\beta}-B(1-\bar{\beta})\right)}$$

Solving $\frac{\pi_j}{f_j} = \frac{1}{1-\phi}$ for λ gives $\lambda'(\phi, B, C)$. Substituting (A.39) into (A.38) and differentiating yields $\frac{\partial \ln Y_L}{\partial B}$ and $\frac{\partial \ln Y_L}{\partial C}$ as stated.

Substituting (A.39) into the welfare function and differentiating gives

$$\frac{\partial \ln W}{\partial B} = \frac{\partial}{\partial B} \sum_j \beta_j \mu \left(\ln \frac{\beta_j}{f_j} \frac{(1-\lambda)\mu\left(\bar{\beta}-B(1-\bar{\beta})\right)}{\left(C(1-\bar{\beta})+1\right)\left(\bar{\beta}(1-\lambda)\mu+\lambda\right)} \right) = -\mu\bar{\beta}\frac{1-\bar{\beta}}{\bar{\beta}-B\left(1-\bar{\beta}\right)}$$

$$\frac{\partial \ln W}{\partial C} = \frac{\partial}{\partial C} \sum_j \beta_j \mu \left(\ln \frac{\beta_j}{f_j} \frac{(1-\lambda)\mu\left(\bar{\beta}-B(1-\bar{\beta})\right)}{\left(C(1-\bar{\beta})+1\right)\left(\bar{\beta}(1-\lambda)\mu+\lambda\right)} \right) = -\mu\bar{\beta}\frac{1-\bar{\beta}}{C\left(1-\bar{\beta}\right)+1}.$$

■

Lemma D.8 *Let*

$$\lambda''(\phi, B, C, D) = \frac{\left((1-\phi)-\mu\left(1-\bar{\beta}\right)\right)\left(\left(\phi\bar{\beta}-1\right)B+\bar{\beta}(\phi-1)(C+D)+\bar{\beta}\right)}{\left(\phi\bar{\beta}-1\right)\left((1-\phi)-\mu\left(1-\bar{\beta}\right)\right)B+(\phi-1)\left(\bar{\beta}\left(-\mu-\phi+\mu\bar{\beta}\right)+1\right)C+\mu\bar{\beta}\left(\bar{\beta}-1\right)(1-D(1-\phi))}$$

If $\lambda'(\phi, B, C) < \lambda < \lambda''(\phi, B, C, D)$, *then* $\Omega_j = \frac{\beta_j}{f_j}\frac{1-\phi}{1-\phi\bar{\beta}}$, *and*

$$\frac{\partial \ln Y_L}{\partial B}|_{C=0} = -\frac{1-\phi\bar{\beta}}{(1+B)\,\phi\bar{\beta}-B}$$

$$\frac{\partial \ln Y_L}{\partial C}|_{B=0} = -\frac{1-\phi}{\phi - C\,(1-\phi)}$$

$$\frac{\partial \ln W}{\partial B} = \frac{\partial \ln W}{\partial C} = 0.$$

Proof. Similarly as in the baseline model, we can consider the Lagrangian

$$\max_{\Omega_1} \lambda \ln Y_L(\Omega, B, C) + (1-\lambda)\sum_j \left(\beta_j \mu \left(\ln \Omega_j - \frac{\ln(c_j+A)}{\mu-1} \right) \right) + \sum_j \gamma^j_{PC}\left(w_j - (1+C)\,f_j\right)$$
$$+ \sum_j \gamma^j_{EC}\left(w_j - \left(D\,(1-\phi)\,\pi_j + C f_j\right)\right)$$

and conjecture that both (14) and (13) bind. This implies $w_j = (1+C)\,f_j = D\,(1-\phi)\,\pi_j + C f_j$, which, using (5) implies

$$\Omega_j = \frac{\beta_j}{f_j}\frac{1-\phi}{1-\phi\bar{\beta}}.$$

Then, calculating the implied γ^j_{PC} and γ^j_{EC} from the first order conditions, the requirement $\gamma^j_{PC}, \gamma^j_{EC} > 0$ gives the condition $\lambda'(\phi, B, C) < \lambda < \lambda''(\phi, B, C, D)$.

Substituting the solution into $Y_L(\Omega_1, B, C)$ and welfare

$$\ln W(\Omega, B, C) = \sum_j \beta_j \mu \left(\ln \Omega_j - \frac{\ln(c_j + A)}{\mu - 1} \right)$$

and differentiating give the results. ■

Lemma D.9 *If $\lambda''(\phi) < \lambda$, then $\Omega_j = \frac{\beta_j}{f_j}\frac{X}{\bar{\beta}}$, where $X \equiv \sum_k \Omega_k f_k$ is the lower root of the quadratic equation*

$$-X\lambda \frac{(1-(1-\phi)D)\frac{\bar{\beta}}{1-\bar{\beta}} + C}{(1-(1-\phi)D)\frac{\bar{\beta}}{1-\bar{\beta}}(1-X) - CX - B} - \frac{(1-\lambda)X}{1-X} + (1-\lambda)\mu\bar{\beta} = 0 \qquad \text{(A.40)}$$

Moreover,

$$\lim_{B\to 0} \frac{\partial \ln Y_L|_{C=0,D=1}}{\partial B} = -\frac{\left(1-\bar{\beta}\right)\left(1+(1-\lambda)^2\mu\bar{\beta}\right)}{\phi\bar{\beta}}$$
$$\lim_{C\to 0} \frac{\partial \ln Y_L|_{B=0,D=1}}{\partial C} = -\frac{(1-\lambda)^2\left(1-\bar{\beta}\right)\mu}{\phi}$$
$$\lim_{D\to 1} \frac{\partial \ln Y_L|_{B=C=0}}{\partial D} = -\frac{1-\phi}{\phi},$$

and

$$\lim_{B\to 0} \frac{\partial \ln W|_{C=0,D=1}}{\partial B} = -\frac{\lambda^2\left(1-\bar{\beta}\right)\mu}{\phi}$$
$$\lim_{C\to 0} \frac{\partial \ln W|_{B=0,D=1}}{\partial C} = -\frac{\lambda^2\left(1-\bar{\beta}\right)\mu}{\phi}$$
$$\lim_{D\to 1} \frac{\partial \ln W|_{B=C=0}}{\partial D} = \frac{(1-\phi)\bar{\beta}}{1-\bar{\beta}\phi}.$$

Proof. Following the logic of the baseline case, if λ is large, only (14) binds, hence $w_j = D(1-\phi)\pi_j + Cf_j$, and problem (12) simplifies to

$$\max_{\Omega} \lambda \ln Y_L(\Omega, B, C, D) + (1-\lambda)\left(\ln I(\Omega, B, C, D) + \sum_j \beta_j \mu \left(\ln \Omega_j - \frac{\ln(c_j + A)}{\mu - 1} \right) \right)$$

where, using (5),

$$\ln Y_L(\Omega, B, C, D) = \sum_j \Omega_j \left(\pi_j - D(1-\phi)\pi_j - Cf_j\right) - B = \qquad \text{(A.41)}$$
$$= (1-(1-\phi)D)\frac{\bar{\beta}}{1-\bar{\beta}}(1-X) - CX - B$$

and $I(\Omega, B, C, D)$ denotes workers' and clients' aggregate income:

$$\ln I(\Omega, B, C, D) = \sum_j (1 + \Omega_j (D(1-\phi)\pi_j - f_j)) =$$
$$= (1-X)\left(1 + (1-\phi)D\frac{\bar{\beta}}{1-\bar{\beta}}\right)$$

Taking the first order conditions with respect to Ω_j yields

$$-f_j\lambda\frac{(1-(1-\phi)D)\frac{\bar{\beta}}{1-\bar{\beta}} + C}{Y_L} - f_j\frac{1-\lambda}{1-X} + \frac{(1-\lambda)\beta_j\mu}{\Omega_j} = 0$$

Summing over $j = 1, ..., J$ yields the quadratic equation (A.40). It can be verified that the solution is the lower root (the larger root yields $Y_L < 0$). Expressing Ω_j from (D) yields the expression in the statement.

We substitute Ω_j into (A.41), differentiate and take the limit to get the derivatives with respect to the sanction parameters.

Similarly, using

$$\ln W(\Omega, A, B, C, D) = \ln I(\Omega, B, C, D) + \sum_j \beta_j\mu\left(\ln\Omega_j - \frac{\ln(c_j + A)}{\mu - 1}\right)$$

we differentiate with respect to the sanction parameters and take the limit to obtain the expressions in the statement.

Comparing the derivatives of W and Y_L verifies the statements in Propositions 7 and 8.

■

E Appendix: Proofs for Section 5.1 (Hybrid leaders' support among oligarchs)

Proof of Proposition 9. When $1 < \frac{1}{\mu\bar{\beta}}$, $U^E = \infty$ for $\lambda = 1$ (see (A.21)), so oligarchs will always support a leader with λ high enough. The threshold $\underline{\lambda}$ equates U^E from (A.21) with entrepreneur utility at $\lambda = 0$, which is simply Γ from (A.20). Because U^E is decreasing in ϕ and increasing in λ over this range, the threshold is increasing in ϕ.

When $1 > \frac{1}{\mu\bar{\beta}}$, Proposition 3 implies that oligarchs will only support a hybrid leader if $\lambda'' < \frac{1}{\mu\bar{\beta}}$. Moreover, it must be that the highest utility under oligarchy (achieved when $\lambda = \frac{1}{\mu\bar{\beta}}$) is at least as large as the utility for $\lambda = 0$, which is Γ. Using (A.21) this requires

$$\frac{1-\phi}{(\bar{\beta}\mu - 1)\frac{1-\bar{\beta}}{\bar{\beta}}}\left(\frac{\bar{\beta}\mu - 1}{\bar{\beta}^2\mu}\right)^{\bar{\beta}\mu} \geq 1$$

or

$$1 - (\bar{\beta}\mu - 1)\frac{1-\bar{\beta}}{\bar{\beta}}\left(\frac{\bar{\beta}^2\mu}{\bar{\beta}\mu - 1}\right)^{\bar{\beta}\mu} \geq \phi \qquad \text{(A.42)}$$

Note that when (A.42) holds, the requirement that $\lambda''(\phi) < \frac{1}{\mu\bar{\beta}}$ is automatically satisfied. To see this, write this condition as $1 - \frac{\mu\bar{\beta}-1}{\bar{\beta}}(1-\bar{\beta}) > \phi$, and note that this constraint on ϕ is weaker, i.e., $(\bar{\beta}\mu - 1)\frac{1-\bar{\beta}}{\bar{\beta}}\left(\frac{\bar{\beta}^2\mu}{\bar{\beta}\mu-1}\right)^{\bar{\beta}\mu} > \frac{\mu\bar{\beta}-1}{\bar{\beta}}(1-\bar{\beta})$ (to see this, rewrite this inequality as $\frac{1}{\mu} > \bar{\beta}(1-\bar{\beta})$, and observe that this is true because $\frac{1}{\mu} \geq 1-\bar{\beta}$ and $\bar{\beta} < 1$).

When (A.42) holds, since oligarchs' utility U^E is an inverse U-shaped function of λ, there is a range $[\underline{\lambda}, \bar{\lambda}]$ for which entrepreneurs will support the hybrid leader. Moreover, as U^E is decreasing in ϕ, the range shrinks as ϕ grows. When (A.42) fails, the range is empty. ■

Proof of Proposition 10. Using (D.9) from Lemma D.9 and (A.19), U_k^E can be written as

$$U_k^E = (1-\phi)\frac{1-X}{1-\bar{\beta}}\left(\frac{X}{\bar{\beta}}\right)^{\bar{\beta}\mu-1}\Gamma$$

where X is given in (A.40) and Γ is defined in (A.20). Part 1 follows immediately from the fact that increasing A does not affect X and lowers Γ (the curves on Figure 2 shift down).

Taking the derivative w.r.t. X, it is easy to verify that U_k^E is increasing in X if and only if $1 - \frac{1}{\bar{\beta}\mu} > X$. Thus, $1 < \frac{1}{\bar{\beta}\mu}$ implies that U_k^E is decreasing in X. Because starting from no sanctions a marginal increase in B or C raises X, it follows that $\underline{\lambda}$ goes down in this case.

Suppose that $1 > \frac{1}{\bar{\beta}\mu}$ and consider the case with no sanctions initially. Plugging in the value of X, we obtain that in this case U_k^E is increasing in X if and only if $\frac{1}{\bar{\beta}\mu} > \lambda$. As a marginal increase in B or C raises X, it follows that $\underline{\lambda}$ and $\bar{\lambda}$ both go down in this case. These observations verify part 2 of the proposition.

Finally, for part 3 recall that an increase in D is equivalent to a reduction in ϕ. From (A.21), a reduction in ϕ shifts U_k^E up, so that $\underline{\lambda}$ falls while $\bar{\lambda}$ increases. ■

F Appendix: Productivity in Hybrid Regimes (proofs for Section 5.2)

We formally introduce an R&D sector in the tradition of Romer (1990). The sector develops new technologies that increase the profits of firms adopting them. Unlike Romer (1990) our R&D sector does not develop new varieties (as Ω_j is the leader's choice variable in our model), but improves the productivity of a firm producing a given variety, in the sense of decreasing the marginal cost of production at the expense of increasing the fixed cost. As De Ridder (2024) explains, this is a good description for the investment in intangible inputs: this type of investment in productivity tends to be firm-specific, implying limited positive spillovers to other firms.

F.1 The R&D sector

For each industry, we introduce a group of inventors of size ε and an ex-ante period. Inventors have the same utility function for consumption as workers. Every inventor can choose to leave her profession to be a worker, but no-one outside of this set of size ε can be an inventor (as they do not possess the talent for it).

In the ex-ante period, inventors can use their unit labor to produce a patent for a technology for the given industry. A technology is a pair of fixed cost and marginal cost (f_j, c_j). Inventors can choose to patent $\kappa_j \in \mathbb{R}^+$ giving the technology $f_j = f_j(\kappa_j)$ and $c_j = \frac{1}{\kappa_j^{\mu-1}}$, hence the function $f_j(\kappa_j)$ characterizes feasible technologies in the given industry.

After the leader allows a firm to operate, but before production begins, the firm has to obtain a patent. We assume that patents are sold for shares in the firm. If the inventor receives δ share in a firm for his patent, his income is $\delta(\pi_j - f_j)$. This is paid before profit is shared between entrepreneurs and the leader, so that entrepreneurs and the leader will only share $(1-\delta)(\pi_j - f_j)$. Each inventor is atomistic and takes δ as given. As one inventor can create only a single technology, each naturally picks the one which maximizes $\pi_j - f_j$. Thus, in equilibrium, each patent in a given industry will have the same content.

To close the model, we require that each inventor is indifferent between becoming a worker or an inventor i.e., that each inventor gets an income of 1. This implies

$$\varepsilon = \delta\Omega(\pi_j - f_j).$$

Clearly, apart from endogenizing c_j and f_j this extension does not change the logic of our economy. To simplify, we take the limit $\varepsilon \to 0$, implying $\delta \to 0$.

F.2 The choice of productivity

We assumed that technology describes the combination of fixed cost $f_j = f_j(\kappa)$ and productivity κ_j defined as $\kappa_j \equiv c_j^{\frac{-1}{\mu-1}}$. Here we derive the equilibrium technology patented, sold and used in industry j.

Suppose that the firm producing variety ω in industry j chooses the technology that yields marginal cost $c_j(\omega)$. Proceeding exactly as in Appendix A we obtain that the profit-maximizing price for each variety ω is

$$p_j(\omega) = \mu c_j(\omega), \tag{A.43}$$

and the price index is

$$P_j^{\frac{1}{\mu-1}} = \left(\int_0^{\Omega_j} p_j(\omega)^{\frac{1}{1-\mu}}\,d\omega\right)^{-1} = \mu^{\frac{1}{\mu-1}}\left(\int_0^{\Omega_j} c_j(\omega)^{\frac{1}{1-\mu}}\,d\omega\right)^{-1} \tag{A.44}$$

Therefore, produced quantities are

$$q_j(\omega) = \hat{\beta}_j Y(i) P_j^{\frac{1}{\mu-1}} p_j(\omega)^{\frac{\mu}{1-\mu}} = \\ \hat{\beta}_j Y \mu^{-1}\left(\int_0^{\Omega_j} c_j(\omega)^{\frac{1}{1-\mu}}\,d\omega\right)^{-1} c_j(\omega)^{\frac{\mu}{1-\mu}}$$

using (A.44).

Profit, disregarding the fixed cost is

$$\begin{aligned}\pi_j(\omega) &= q_j(\omega)\left(p_j(\omega) - c_j(\omega)\right) \\ &= \beta_j Y \left(\int_0^{\Omega_j} c_j(\omega)^{\frac{1}{1-\mu}}\, d\omega\right)^{-1} c_j(\omega)^{\frac{1}{1-\mu}} \\ &= \beta_j Y \frac{1}{K_j} \kappa_j(\omega)\end{aligned}$$

where $K_j \equiv \int_0^{\Omega_j} \kappa_j(\omega)\, d\omega$.

Assuming that each firm takes the average productivity in the sector, K_j, as given,

$$\frac{\partial \pi_j(\kappa_j(\omega))}{\partial \kappa_j(\omega)} = \beta_j Y \frac{1}{K_j} = f_j'(\kappa)$$

gives the productivity decision of each firm. Hence, in equilibrium, we have $K_j = \Omega_j \kappa_j^*$, and

$$\beta_j Y \frac{1}{\Omega_j f_j} = \kappa_j^* \frac{f_j'\left(\kappa_j^*\right)}{f_j\left(\kappa_j^*\right)} \tag{A.45}$$

determines the equilibrium choice of productivity κ_j^*.

F.3 Proof of Proposition 11

Lemma F.10 *Under the assumption that $\frac{\kappa}{f(\kappa)}$ is concave and single-peaked, $\frac{\kappa f'(\kappa)}{f(\kappa)}$ is monotonically increasing for $\kappa \geq \kappa^{**}$.*

Proof. Concavity of $\frac{\kappa}{f(\kappa)}$ implies $-\kappa f'' f^2 - 2ff'(f - \kappa f') < 0$, and hence $G \equiv \kappa f'' f + 2f'f - 2\kappa(f')^2 > 0$.

The derivative of $\frac{\kappa f'}{f}$ is proportional to $(f' + \kappa f'')f - \kappa(f')^2$, which can be written as

$$G + f'(\kappa f' - f).$$

But for $\kappa \geq \kappa^{**}$, we have $\kappa f' - f > 0$ from the single peakedness assumption, so both terms in this sum are positive and $\frac{\kappa f'(\kappa)}{f(\kappa)}$ is monotonically increasing. ■

Log social welfare is

$$\ln W(\Omega, w) = \ln\left(1 + \sum_j \Omega_j (w_j - f_j)\right) + \sum_j \beta_j \mu \left(\ln \Omega_j - \frac{\ln c_j}{\mu - 1}\right) \tag{A.46}$$

Using Proposition 1, when $\lambda = 0$, this is proportional to

$$\sum_j \beta_j \mu \left(\ln \frac{\beta_j}{f_j} - \frac{\ln c_j}{\mu - 1}\right) = \sum_j \beta_j \mu \left(\ln \beta_j - \ln \frac{f_j(\kappa_j)}{\kappa_j}\right)$$

Clearly, this expression is maximized by κ_j^{**}.

Next, consider $\lambda > 0$. For $\lambda < \lambda''$, $w_j = f_j$, therefore (A.46) can be written as

$$\sum_j \beta_j \mu \left(\ln \Omega_j f_j - \ln f_j - \frac{\ln c_j}{\mu - 1} \right) = \sum_j \beta_j \mu \left(\ln \Omega_j f_j - \ln \frac{f_j(\kappa_j)}{\kappa_j} \right)$$

From Proposition 2, $\Omega_j f_j$ does not depend on f_j or c_j (and hence on κ_j) in equilibrium. Hence, this expression is still maximized by κ_j^{**}.

For $\lambda > \lambda''$, $w_j = (1 - \phi)\, \pi_j$ implying that (A.46) can be written as

$$\ln \left(1 + \sum_j ((1-\phi)\, \Omega_j f_j \frac{\pi_j}{f_j} - \Omega_j f_j) \right) + \sum_j \beta_j \mu \left(\ln \Omega_j f_j - \ln \frac{f_j(\kappa_j)}{\kappa_j} \right)$$

From Proposition 3, neither $\Omega_j f_j$ nor π_j / f_j depends on f_j or c_j (and hence on κ_j). Hence, this expression is also maximized by κ_j^{**}. This concludes the proof of Part 1.

Next, note that

$$1 = \kappa_j^{**} \frac{f_j'(\kappa_j^{**})}{f_j(\kappa_j^{**})} \tag{A.47}$$

by definition.

For $\lambda = 0$, Proposition 1 showed that $Y = 1$, $\Omega_j = \frac{\beta_j}{f_j}$ and $w_j = f_j$. Substituting into (A.45), and comparing to (A.47) shows that $\kappa_j^* = \kappa_j^{**}$ in this case, as stated in Part 2(a).

For Part 2(b), recall that the left-hand side of (A.45) can be written as

$$\beta_j Y \frac{1}{\Omega_j f_j} = \frac{\beta_j}{1 - \bar{\beta}} \left(1 - \sum_{j=1}^{J} \Omega_j f_j \right) \frac{1}{\Omega_j f_j} \tag{A.48}$$

(see (A.9) and (A.4)). From Propositions 2 and 3, we know that $\Omega_j f_j$ is decreasing in λ, and therefore (A.48) is increasing in λ. Because the left-hand side of (A.45) is increasing in λ and (as shown above) is independent of κ_j, Lemma F.10 implies that κ_j^* must be increasing in λ.

Finally, note that, for a given λ, Lemma F.10 guarantees that the equilibrium κ_j^* given in (A.45) is unique.

G Appendix: Public procurement

G.1 Proofs for Section 5.3

Proof of Proposition 12. Here we provide a full solution for the case of $J = 1$. (See Appendix G.2 for the solution and additional results for the $J = 2$ case.) From (17), the quantity of the public good is

$$Q_J = q_J \Omega_J^{\mu} = \frac{\tau(Y - \Omega_J(\pi_J - w_J)) + \Delta}{m c_J} \Omega_J^{\mu - 1}. \tag{A.49}$$

and consumption of the numeraire good is

$$Q_0(i) = \frac{\hat{\beta}_0}{1-\hat{\beta}_J}(1-\tau)\,Y(i),$$

Substituting in Q_J and Q_0 into the welfare function gives the modified problem

$$\begin{gathered}\max_{\Omega_J, w_J, m, \tau} \lambda \ln\left(\Omega_J(\pi_J(\Omega_J) - w_J)\right) + \\ (1-\lambda)\left[\begin{array}{c}\left(1-\hat{\beta}_J\right)\ln(1-\tau)\left(1+\Omega_J(w_J-f_J)\right) + \hat{\beta}_J \ln\left(\tau\left(1+\Omega_J(w_J-f_J)\right)+\Delta\right) \\ +\hat{\beta}_J(\mu-1)\left(\ln\Omega_j - \frac{\ln c_j}{\mu-1}\right) - \hat{\beta}_J \ln m\end{array}\right]\end{gathered} \tag{A.50}$$

subject to

$$\pi_J(\Omega_J) = \left(\tau(1+\Omega_J(w_J-f_J))+\Delta\right)\frac{m-1}{m\Omega_J}$$

(8) and (9).

Following the argument for the baseline case in Proposition 2, consider first the case when only the PC (8) binds, hence $w_J = f_J$. Then, the first order conditions give

$$\begin{aligned}\tau &= 1-(1+\Delta)(1-\lambda)\frac{1-\hat{\beta}_J}{1+\hat{\beta}_J(\mu-1)(1-\lambda)} \\ \Omega_J &= (1+\Delta)\frac{(\mu-1)\hat{\beta}_J}{f_J}\frac{(1-\lambda)}{1+\hat{\beta}_J(\mu-1)(1-\lambda)} \\ m &= \frac{\lambda}{(1-\lambda)\hat{\beta}_J}+\mu.\end{aligned}$$

substituting in the above expressions into (A.49) gives

$$Q_J = \frac{f^{\mu-1}\left(\frac{(\Delta+1)(1-\lambda)(\mu-1)\hat{\beta}_J}{(1-\lambda)(\mu-1)\hat{\beta}_J+1}\right)^{\mu}}{\mu-1}. \tag{A.51}$$

For the EC to be slack, we need $\frac{\pi_J(\Omega_J)}{f_J} < \frac{1}{1-\phi}$, which yields

$$\lambda < \lambda^{PP\prime} = \frac{(\mu-1)\phi\hat{\beta}_J}{(\mu-1)\phi\hat{\beta}_J - \phi + 1}$$

Now consider the case when only the EC (9) binds, implying $w_J = (1-\phi)\,\pi_J$. The first order conditions give

$$\tau = 1 - \frac{(1+\Delta)\,\phi\left(1-\hat{\beta}_J\right)(1-\lambda)}{(\Delta+1)\lambda(1-\phi) - \Delta(1-\lambda)(\mu-1)\phi\hat{\beta}_J + \phi},$$
$$\Omega_J = \frac{(\mu-1)\hat{\beta}_J}{f_J}\frac{(\Delta+1)(1-\lambda)}{(1-\lambda)(\mu-1)\hat{\beta}_J+1},$$
$$m = \frac{\lambda}{(1-\lambda)\,\phi\hat{\beta}_J} + 1,$$

implying (A.51) again for Q_J.

For the PC to be slack, we need $\frac{\pi_J(\Omega_J)}{f_J} > \frac{1}{1-\phi}$ or $\lambda > \lambda^{PP''}$. We find that $\lambda^{PP''} = \lambda^{PP\prime} \equiv \lambda^{PP}$, that is, there is no "constrained industry capture" range corresponding to Proposition 2(ii).

Part 2 of the proposition is easily verified by differentiating the expressions above. ■

Proof of Proposition 13. The first statement follows from substituting in the solutions from the proof of Proposition 12 into W and Y_L and taking derivatives.

For the second statement, we solve problem (A.50) with the additional, binding constraint of $m = \bar{m}$ (and setting $\Delta = 0$). For small λ, (i.e. binding (8)), this gives

$$\tau = \frac{\lambda + (1-\lambda)\mu\hat{\beta}_J}{(1-\lambda)(\mu-1)\hat{\beta}_J+1}$$
$$\Omega_J = \frac{\bar{m}-1}{\bar{m}}\frac{1}{f_J}\frac{(1-\lambda)(\mu-1)\hat{\beta}_J\left((1-\lambda)\mu\hat{\beta}_J+\lambda\right)}{(\mu-1)\hat{\beta}_J(1-\lambda)\left((1-\lambda)(\mu-1)\hat{\beta}_J+(\lambda+1)\right)+\lambda}$$

It is easy to check that $\bar{m}$ binds iff

$$\hat{\beta}_J(\bar{m}-\mu) < \frac{\lambda}{1-\lambda}$$

Substituting into (A.49) for this case gives

$$Q_J = \left(\frac{(1-\lambda)\mu\hat{\beta}_J+\lambda}{\bar{m}}\right)^{\mu}\frac{\left(\frac{\bar{m}-1}{f}\frac{(1-\lambda)(\mu-1)\hat{\beta}_J}{(\mu-1)\beta_J\left((1-\lambda)^2(\mu-1)\hat{\beta}_J+1-\lambda^2\right)+\lambda}\right)^{\mu-1}}{(1-\lambda)(\mu-1)\hat{\beta}_J+1}. \tag{A.52}$$

For large λ, (i.e. binding (9)) we get

$$\tau = \frac{\bar{m}\left(\lambda + (1-\lambda)\hat{\beta}_J\right)}{1 + (\bar{m}-1)\left(\lambda + (1-\lambda)\left(\phi + (1-\phi)\,\hat{\beta}_J\right)\right)},$$

$$\Omega_J = \frac{(1-\lambda)(\mu-1)\hat{\beta}_J}{f_J\left((1-\lambda)(\mu-1)\hat{\beta}_J + 1\right)}$$

and $\bar{m}$ binds if and only if

$$\phi\hat{\beta}_J\,(\bar{m}-1) < \frac{\lambda}{1-\lambda}.$$

In this case, we have

$$Q_J = \frac{f^{\mu-1}\left(\lambda + (1-\lambda)\hat{\beta}_J\right)\left(\frac{(1-\lambda)(\mu-1)\hat{\beta}_J}{(1-\lambda)(\mu-1)\hat{\beta}_J+1}\right)^{\mu}}{(\mu-1)\,(1-\lambda)(1+(\bar{m}-1)\phi)\hat{\beta}_J}. \tag{A.53}$$

Then, substituting into W and Y_L and differentiating our expressions give the results. ■

G.2 Public procurement with multiple industries

In this appendix, first we set up the general model with public procurement in sector $J > 1$. Then, we present additional results for the $J = 2$ case, providing further insights on the effect of public procurement compared to our discussion in the main text.

For the general set up, observe that under our assumptions, since consumer i allocates his after-tax income across the goods of the $J-1$ private industries only, his consumption index for industry $j < J$, previously equation (4), changes to

$$Q_j(i) = \frac{\hat{\beta}_j}{1-\hat{\beta}_J}\frac{\Omega_j^{\mu-1}}{\mu c_j}\,(1-\tau)\,Y(i).$$

Consumption of the numeraire good is

$$Q_0(i) = \frac{\hat{\beta}_0}{1-\hat{\beta}_J}\,(1-\tau)\,Y(i),$$

and given the leader's choice, the quantity of the public good is

$$Q_J = q_J\Omega_J^{\mu} = \frac{\tau(Y - \sum_{j>0}\Omega_j(\pi_j - w_j))}{mc_J}\Omega_J^{\mu-1}. \tag{A.54}$$

The leader's problem is now

$$\max_{\Omega,w,m,\tau} \lambda \ln\left(\sum_j \Omega_j(\pi_j - w_j)\right) + (1-\lambda)\left[\begin{array}{c} \ln\left(1+\sum_j \Omega_j(w_j - f_j)\right) + \sum_{j>0}\beta_j\mu\left(\ln\Omega_j - \frac{\ln c_j}{\mu-1}\right) \\ +(1-\hat{\beta}_J)\ln(1-\tau) + \hat{\beta}_J(\ln\tau - \ln m) \end{array}\right] \tag{A.55}$$

subject to (6),(8),(9) and profit expressions that modify (5):

$$\pi_j = \frac{\beta_j}{\left(1-\hat{\beta}_J\right)\Omega_j}\left(\sum_{j>0}\Omega_j(\pi_j - w_j) + (1-\tau)(Y - \sum_{j>0}\Omega_j(\pi_j - w_j))\right) \tag{A.56}$$

$$\pi_J = \tau(Y - \sum_{j>0}\Omega_j(\pi_j - w_j))\frac{m-1}{m\Omega_J}. \tag{A.57}$$

The following propositions describe the benchmark case of a welfare-maximizing leader ($\lambda = 0$), the equilibrium when $\lambda > 0$, and finally the impact of external transfers. We first state the propositions, then prove them jointly.

Proposition G.3 *Suppose that $\lambda = 0$ and $J = 2$. The leader does not limit entry, $\pi_1 = f_1$, $\pi_J = f_J$, and does not distort the markup, $m = \mu$. However, the tax revenue is used to increase spending in industry J leading to more competition in industry J and less spending and less competition in every other industry:*

$$\Omega_J = \Omega_J^{PP} \equiv \frac{\beta_J}{f_J(1-\beta_0)} > \Omega_J^{FE}$$

and

$$\Omega_1 = \Omega_1^{PP} \equiv \frac{\beta_1}{f_1}\frac{\left(1-\beta_0-\hat{\beta}_J\right)}{(1-\beta_0)\left(1-\hat{\beta}_J\right)} < \Omega_1^{FE}$$

and $\tau = \tau \equiv \frac{\hat{\beta}_J}{1-\beta_0} > \hat{\beta}_J$.

Proposition G.4 *Suppose that $J = 2$, $\phi_j = \phi\ \forall j$. Then there exist thresholds λ'_P and λ''_P such that:*
1. When $\lambda \in (0, \lambda'_P)$, $w_1 = f_1$, $w_J = f_J$. In addition, for $\lambda < \min\left(\lambda'_P, \frac{(\mu-1)\hat{\beta}_0}{1+(\mu-1)\hat{\beta}_0}\right)$ the leader does not limit entry in industry 1, $\pi_1 = f_1$, therefore, he does not extract profit from this sector.
2. When $\lambda \in (\lambda''_P, 1)$, $w_1 > f_1$, $w_J > f_J$ and $\Omega_j = \frac{\beta_j\mu}{f_j}\frac{(1-\lambda)}{\mu\bar{\beta}(1-\lambda)+1}\ \forall j$. Still, as $m > \mu$, $\frac{\pi_J}{f_J} > \frac{\pi_1}{f_1} > 1$.
In both cases the leader overprices public goods, $m > \mu$, overspends on public procurement, $\tau > \hat{b}^{PP}$, and limits entry in the public good sector, $\Omega_J < \Omega_J^{PP}$, to generate profit $\pi_J > f_J$. Furthermore, as λ increases, each of these distortions increases in magnitude (i.e., m, τ and π_J increase while Ω_J decreases), and the quantity of public goods, Q_J, falls.

Proposition G.5 *Suppose that $J = 2$, $\phi_j = \phi$ and we are at an interior solution ($\lambda \in (0, \lambda'_P)$ or $\lambda \in (\lambda''_P, 1)$ where $\lambda'_P < \lambda''_P$ is determined in Proposition G.4). Let $\Omega_j(\Delta)$, $\pi_j(\Delta)$, $m(\Delta)$, etc. denote equilibrium outcomes for a given Δ.*

1. *Then markups, profits and the relative number of firms across industries are insensitive to Δ, i.e., $\pi_j(\Delta) = \pi_j(0)$, $m(\Delta) = m(0)$, and $\frac{\Omega_1(\Delta)}{\Omega_2(\Delta)} = \frac{\Omega_1(0)}{\Omega_2(0)}$.*

2. *The number of firms and each industry's revenue is proportional to $(1+\Delta)$, i.e.,*

$$\Omega_j(\Delta) = (1+\Delta)\,\Omega_j(0)$$
$$P_j(\Delta)Q_j(\Delta) = (1+\Delta)\,P_j(0)Q_j(0)$$

3. *Hence, a reduction in external funds reduce both welfare and the leader's income. The relative effect is given by*

$$\frac{\partial \ln W}{\partial \Delta} \Big/ \frac{\partial \ln Y_L}{\partial \Delta} = [\mu(1-\hat{\beta}_0) + \hat{\beta}_0] > 1.$$

Proof of Propositions G.3, G.4 and G.5. We solve the Lagrangian corresponding to problem (A.55) but with the budget constraint (17) that includes external funds. For Proposition G.4, set $\Delta = 0$ in the expressions below and for Proposition G.3, set $\lambda = 0$ as well.

Case 1. Assume that both PCs bind and both are ICs slack. That is $w_1 = f_1$, $w_J = f_J$, and $\frac{\pi_j}{f_j} < \frac{1}{1-\phi}$. Then there are two possible subcases.

(i) Suppose that $\pi_1 > f_1$. Then solving (A.55), we obtain

$$\Omega_j = (1+\Delta)\frac{\mu\beta_j}{f_j}\frac{1-\lambda}{\lambda + (1-\lambda)\mu(1-\beta_0)} \text{ for } j = 1, J$$

$$m = \frac{(\mu-1)\hat{\beta}_0\left(\left(1-\hat{\beta}_0\right)(\mu-1) + \hat{\beta}_J\right) + \left(1-\hat{\beta}_J\right)\mu\hat{\beta}_J}{\hat{\beta}_J\left(\hat{\beta}_0(\mu-1) + 1 - \hat{\beta}_J\right)} + \frac{\lambda}{(1-\lambda)\hat{\beta}_J}$$

$$\tau = 1 - \mu\left(1-\hat{\beta}_J\right)^2(1-\lambda)\frac{1+\Delta}{\left((\mu-1)\hat{\beta}_0 + 1 - \hat{\beta}_J\right)\left(\lambda + \left(\mu\left(1-\hat{\beta}_0\right) + \hat{\beta}_0\right)(1-\lambda)\right)}$$

$$\frac{\pi_1}{f_1} = \frac{(\lambda-1)\hat{\beta}_J + 1}{(1-\lambda)\left(\hat{\beta}_0(\mu-1) - \hat{\beta}_J + 1\right)}$$

$$\frac{\pi_J}{f_J} = \frac{\hat{\beta}_0\left(1-\hat{\beta}_0\right)(\mu-1) + \left(1-\hat{\beta}_J\right)\hat{\beta}_J}{\hat{\beta}_J\left(\hat{\beta}_0(\mu-1) + 1 - \hat{\beta}_J\right)} + \frac{\hat{\beta}_0(\mu-1)\lambda + \left(1-\hat{\beta}_J\right)\lambda}{(1-\lambda)(\mu-1)\hat{\beta}_J\left(\hat{\beta}_0(\mu-1) + 1 - \hat{\beta}_J\right)}$$

Then $\pi_1 > f_1$ is equivalent to $\frac{\hat{\beta}_0(\mu-1)}{\hat{\beta}_0(\mu-1)+1} < \lambda$.

(ii) Suppose that $\frac{\hat{\beta}_0(\mu-1)}{\hat{\beta}_0(\mu-1)+1} > \lambda$, in which case we must have $\pi_1 = f_1$. From (A.55), we then get

$$\Omega_1 = \frac{\beta_1}{f_1}\frac{(\Delta+1)\left((1-\lambda)\left(\mu\hat{\beta}_1+\hat{\beta}_0\right)+\lambda\right)}{\left(1-\hat{\beta}_J\right)\left((1-\lambda)\left(\mu\left(1-\hat{\beta}_0\right)+\hat{\beta}_0\right)+\lambda\right)}$$

$$\Omega_J = \frac{(\Delta+1)(1-\lambda)(\mu-1)\hat{\beta}_J}{f_J\left((1-\lambda)\left(\mu\left(1-\hat{\beta}_0\right)+\hat{\beta}_0\right)+\lambda\right)}$$

$$m = \frac{\lambda\left(\left(1-\hat{\beta}_0\right)(\lambda+\mu(1-\lambda))+\hat{\beta}_0\right)+(1-\lambda)^2\mu\left(1-\hat{\beta}_J\right)\hat{\beta}_J}{(1-\lambda)\hat{\beta}_J\left(1-(1-\lambda)\hat{\beta}_J\right)}$$

$$\tau = 1-\left(1-\hat{\beta}_J\right)(1-\lambda)(\Delta+1)\frac{\lambda+(1-\lambda)\left(\mu\hat{\beta}_1+\hat{\beta}_0\right)}{\left(1-\hat{\beta}_J(1-\lambda)\right)\left(\lambda+(1-\lambda)\left(\mu\left(1-\hat{\beta}_0\right)+\hat{\beta}_0\right)\right)}$$

$$\frac{\pi_J}{f_J} = \frac{(1-\lambda)\hat{\beta}_J\left(1-\hat{\beta}_J\right)-\hat{\beta}_0\lambda}{\hat{\beta}_J\left(1-(1-\lambda)\hat{\beta}_J\right)}+\lambda\frac{\lambda+\mu(1-\lambda)-\hat{\beta}_J(1-\lambda)}{\hat{\beta}_J(1-\lambda)\left(1-\hat{\beta}_J(1-\lambda)\right)(\mu-1)}$$

For Lemma G.3, set $\lambda = 0$ and $\Delta = 0$ in the expressions above. For the first and second parts of Proposition G.5, observe that both in cases (i) and (ii) π_1, π_J and m are independent of Δ while Ω_1, Ω_J are proportional to $(1+\Delta)$. This also implies that expenditures $P_1Q_1 = \mu c\Omega_1$ and $P_JQ_J = mc\Omega_J$ are proportional to $(1+\Delta)$.

We also have to find the threshold λ'_P such that for any $\lambda < \lambda'_P$ the IC constraints for both industries are slack as conjectured. Note that $\frac{\pi_1}{f_1} < \frac{\pi_J}{f_J}$, because $m > \mu$ and $\frac{\pi_J}{f_J}$ is monotonically increasing in λ. Therefore, we only need to ensure that $\lambda < \lambda'_P$ implies $\frac{\pi_J}{f_J} < \frac{1}{1-\phi}$. For $\lambda = 0$, we have $\frac{\pi_J}{f_J}|_{\lambda=0} = 1$ (from case (ii)). In addition, in both cases $\lim_{\lambda\to 1}\frac{\pi_J}{f_J} = \infty$. Therefore, by continuity, there must be a λ'_P for which $\frac{\pi_J}{f_J}|_{\lambda=\lambda'_P} = \frac{1}{1-\phi}$. At the threshold between cases (i) and (ii) we have

$$\frac{\pi_J}{f_J}\Big|_{\lambda=\frac{\hat{\beta}_0(\mu-1)}{\hat{\beta}_0(\mu-1)+1}} = \frac{\frac{m-1}{m}(\tau+\Delta)}{f_J\Omega_J}\Big|_{\lambda=\frac{\hat{\beta}_0(\mu-1)}{\hat{\beta}_0(\mu-1)+1}} = \frac{\hat{\beta}_0\left(\frac{\mu}{\hat{\beta}_J}-1\right)+1-\hat{\beta}_J}{1+(\mu-1)\hat{\beta}_0-\hat{\beta}_J}$$

therefore, if

$$\frac{\hat{\beta}_0\left(\frac{\mu}{\hat{\beta}_J}-1\right)+1-\hat{\beta}_J}{1+(\mu-1)\hat{\beta}_0-\hat{\beta}_J} < \frac{1}{1-\phi}$$

then λ'_P is given by the $\lambda \in (0,1)$ solving $\frac{\pi_J}{f_J} = \frac{1}{1-\phi}$ in the first subcase.

Case 2. Now assume that λ is sufficiently large that only the IC constraints bind, that is

$\frac{\pi_j}{f_j} > \frac{1}{1-\phi}$ for both $j = 1, J$. Then, from (A.55) we get

$$\begin{aligned}
\Omega_j &= (1+\Delta)\,\frac{\hat{\beta}_j(\mu-1)}{f_j}\frac{1-\lambda}{1+(1-\lambda)(\mu-1)\left(1-\hat{\beta}_0\right)} \\
m &= \frac{1}{\hat{\beta}_J(1-\lambda)}\frac{\left(1-\hat{\beta}_J\right)\left(\frac{\lambda}{\phi}+\hat{\beta}_J(1-\lambda)\right)-\beta_1}{1-\hat{\beta}_J-\beta_1} \\
\tau &= 1-\frac{\mu\phi\left(1-\hat{\beta}_J\right)^2(1-\lambda)(\Delta+1)}{\left(\left(1-\hat{\beta}_J\right)\left(\phi\left(1-\hat{\beta}_0+\mu\hat{\beta}_0\right)(1-\lambda)-\mu(\phi-\lambda)\right)-\phi\hat{\beta}_0(\mu-1)^2\left(1-\hat{\beta}_0\right)(1-\lambda)\right)\Delta+\left(\phi\hat{\beta}_0(\mu-1)+\left(1-\hat{\beta}_J\right)(\phi+\lambda\mu(1-\phi))\right)} \\
\frac{\pi_1}{f_1} &= \frac{1-(1-\lambda)\hat{\beta}_J}{(1-\lambda)\left(\hat{\beta}_0(\mu-1)+1-\hat{\beta}_J\right)} \\
\frac{\pi_J}{f_J} &= \frac{\left(1-\hat{\beta}_J\right)\lambda\mu-(\mu-1)\phi\left(1-(1-\lambda)\hat{\beta}_J\right)\left(1-\hat{\beta}_0-\hat{\beta}_J\right)}{(1-\lambda)(\mu-1)\phi\hat{\beta}_J\left(\hat{\beta}_0(\mu-1)+1-\hat{\beta}_J\right)}
\end{aligned} \tag{A.58}$$

As in case 1, for the first and second parts of Proposition G.5, observe that π_1, π_J and m are independent of Δ while Ω_1, Ω_J are proportional to $(1+\Delta)$. This also implies that expenditures $P_1Q_1 = \mu c\Omega_1$ and $P_JQ_J = mc\Omega_J$ are also proportional to $(1+\Delta)$.

We also have to find the threshold λ''_P such that for any $\lambda > \lambda''_P$ the IC constraints for both industries bind as conjectured. Note that $\frac{\pi_1}{f_1} < \frac{\pi_J}{f_J}$, because $m > \mu$ and that $\frac{\pi_1}{f_1}$ is monotonically increasing in λ. Therefore, we only need to ensure that $\lambda > \lambda''_P$ implies $\frac{\pi_1}{f_1} > \frac{1}{1-\phi}$. It is easy to see that

$$\lambda''_P \equiv 1 - \frac{1}{\left(\hat{\beta}_0(\mu-1)+1-\hat{\beta}_J\right)\frac{1}{1-\phi}+\hat{\beta}_J}$$

is the solution.

Differentiating the above expressions for m, τ, π_J, and Ω_J w.r.t. λ verifies the statements of Proposition G.4. Substituting them into the expressions for welfare and the leader's income Y_L, and directly calculating the elasticities yields Lemma G.5.

Finally, to verify that Q_J is decreasing in λ (Proposition G.4), we calculate Q_J using (A.54) and the expressions above for each case (with Δ set to 0). This yields

$$\begin{aligned}
Q_J &= \frac{\hat{\beta}_J^{\mu}}{c_J}\frac{1}{\frac{\lambda}{1-\lambda}+\mu(1-\hat{\beta}_0)+\hat{\beta}_0}\left(\frac{1}{f_J}\frac{\mu-1}{\frac{\lambda}{1-\lambda}+\mu(1-\hat{\beta}_0)}\right)^{\mu-1} \\
Q_J &= \frac{(\mu-1)^{\mu-1}}{c_J f_J^{\mu-1}}\left(\frac{\hat{\beta}_J}{\frac{1}{1-\lambda}+(1-\hat{\beta}_0)(\mu-1)}\right)^{\mu} \\
Q_J &= \frac{\hat{\beta}_J^{\mu}}{c_J}\frac{1}{\frac{\lambda}{1-\lambda}+\mu(1-\hat{\beta}_0)+\hat{\beta}_0}\left(\frac{\mu-1}{f_J}\frac{1}{\frac{1}{1-\lambda}+(\mu-1)\left(1-\hat{\beta}_0\right)}\right)^{\mu-1}
\end{aligned} \tag{A.59}$$

for cases 1(i), 1(ii), and 2, respectively. The welfare-maximizing benchmark, Q_J^{PP} is obtained by setting $\lambda = 0$ in (A.59). It can easily be verified that each of these expressions is smaller than Q_J^{PP} and decreasing in λ. ■