Equilibrium Administrations*

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Abstract

We develop a model of policy making with an endogenous bureaucracy. Parties choose platforms and ideologically differentiated citizens decide whether to enter the public sector anticipating the platforms that they may be asked to implement. Bureaucrats prefer to work on policies closer to their ideal, and voters judge the performance of an administration taking both politicians' and bureaucrats' actions into account. The model provides an equilibrium framework to study the emergence of partisan or neutral bureaucracies and their consequences for government performance. It shows how bureaucratic partisanship can develop in modern civil service systems; why political polarization and bureaucratic partisanship reinforce each other; why bureaucratic neutrality is associated with competitive elections; and why partisanship lowers government efficiency and increases output fluctuations. Our results yield a number of policy implications regarding political appointments, public sector wages, seniority benefits, and recruiting measures that raise the intrinsic motivation of bureaucrats.

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1 Introduction

Effective government requires motivated bureaucrats. For this reason, it is common for newly elected leaders to worry about whether the bureaucracy they inherit will be willing to implement and promote their policy agenda. In the Trump administration, this conflict was thrust into the public eye by the president's comments about the "deep state" on the one hand, and by career civil servants speaking out against the administration's policies on the other.[1]

The existence of such conflicts shows both that bureaucrats have their own ideas regarding policies, and that their willingness to faithfully implement policies far from their ideal cannot be taken for granted. This simple observation raises far-reaching questions about the operation of government. Which citizens self-select into the bureaucracy under different policies, and what impact does this have on the size and productivity of government? How do politicians respond to the heterogeneity of bureaucrats' preferences? What will the distribution of bureaucrats' preferences and effort look like under different levels of political competition, in systems with more political appointees, or if wages in the public sector increase relative to the private sector?

Existing models have limited applicability to these questions. Most previous studies of bureaucrat selection (e.g., François (2000); Besley and Ghatak (2005); Delfgaauw and Dur (2008, 2010); Dal Bó et al. (2013)) focus on *public service motivation* (PSM), a preference for serving in government regardless of who is in power. They do not model policy preferences and the resulting conflict with politicians. On the other hand, models of bureaucrat-politician interactions (e.g., Epstein and O'Halloran (1999); Ting (2003); Gailmard and Patty (2007); Fox and Jordan (2011); Ujhelyi (2014a); Forand (2019); Li et al. (2020); Sasso and Morelli (2020)) typically ignore the entry of bureaucrats.

In this paper, we combine these approaches in an equilibrium framework that features entry and effort by motivated bureaucrats, policy choices by elected politicians, and electoral choices by voters who understand that implemented policies are a function of both political and bureaucratic decisions. We call this framework an *equilibrium administration*, and show that it provides a useful vehicle for studying the above questions, and several others, regarding public sector personnel policies and the operation of governments.

In our model, a continuum of citizens and two policy-motivated parties all have preferences over a one-dimensional policy space. Policies are chosen by the party in power, but government outcomes also have an additional dimension, intensity or *output*, and this

[1] For example, see Baker et al. (2019) ("Trump's war on the "deep state" turns against him") or Newland (2020) ("I'm haunted by what I did as a lawyer at the Trump Justice Department").

depends on the implementation effort of the bureaucracy as a whole. Citizens act both as voters and, if they choose to join the public sector, as bureaucrats. All actors prefer policies with ideologies closer to their own ideal, and also value aggregate government output. For example, a voter who values environmental regulations prefers higher government output when the policy ideology is pro-environment than when it is pro-business. Bureaucrats value their own contribution (effort) towards government output, particularly when the policy is close to their ideal. For example, an environmentalist bureaucrat prefers to work harder at implementing pro-environment policies than pro-business policies. Apart from this policy-motivation, bureaucrats also have public service motivation and receive utility from contributing effort to government output whatever the policy chosen by the governing party.

Voters choose parties based on the outcomes delivered by their administration. Because these outcomes depend on policy output as well as ideology, the bureaucracy is central to voters' evaluation of government performance. Parties choose policy ideology taking into account the effort that bureaucrats will be willing to exert. Because this generates a tradeoff between preferred policies and higher output, both the bureaucracy's ideological composition and public service motivation matters to office holders. Citizens decide whether to enter government or work in the private sector. Because this choice is based on the private-public wage gap and on citizens' expectations about what policies they will be asked to implement by the parties, the bureaucracy responds to both economic and political factors. In an equilibrium administration, these interrelated decisions by voters, parties and bureaucrats must all be consistent with one another.

Our model generates several insights. First, bureaucratic neutrality (a bureaucracy willing to exert effort regardless of the party in power) or partisanship (bureaucrats supporting specific parties) can emerge as equilibrium outcomes from the same underlying primitives. We do not assume that bureaucrats are inherently neutral/partisan or motivated/lazy, nor that these behaviors are fixed by institutions (i.e., that partisanship is the result of political patronage while neutrality is ensured by a civil service system). Instead, we show how these phenomena can emerge in equilibrium from the simple incentives of citizens as policy makers, bureaucrats, and voters.

Second, the model explains why partisanship in the bureaucracy is skewed towards the electorally stronger party, and why it is associated with lower government output, larger bureaucracies, shirking bureaucrats, and political budget cycles. The mechanism behind these patterns does not rely on political patronage or machine politics - thus, our model can be used to understand the emergence and implications of bureaucratic partisanship in modern civil service systems. Because partisanship implies policy conflict between bureaucrats and politicians in equilibrium, the model can also be used to shed light on periods of pronounced conflict between the chief executive and administrative agencies.

Third, we show that policy polarization between parties and partisanship in the bureaucracy are complements. Policy polarization leads to partisanship because no bureaucrat is willing to exert effort for both parties when their platforms are far apart. In turn, more partisan bureaucrats make it possible for a party to choose a more extreme platform without losing bureaucrat effort. Equilibrium administrations in which the bureaucracy is fully neutral have the least policy polarization. Conversely, parties choose their ideal policies, and hence are maximally polarized, in equilibrium administrations with a fully partisan bureaucracy.

Fourth, we find that bureaucratic neutrality is associated with competitive elections. Because partisan bureaucrats only work when "their" party wins, elections that are expected to be close attract fewer partisans into the public sector. This in turn gives the winning party an incentive to choose a more moderate policy in order to maintain government output. For both of these reasons, elections that are expected to be closely contested are conducive to bureaucratic neutrality.

Finally, we derive a number of implications for public sector personnel policies. Reducing the number of political appointments available to winning parties encourages policy moderation, and this in turn leads to less partisanship in the permanent civil service. Increasing seniority benefits can lower partisanship by encouraging bureaucrats to stay on rather than quit when they disagree with the incoming party's policies. Measures to reduce shirking among bureaucrats also encourage neutrality, but this comes at the cost of reduced bureaucrat entry, which has negative consequences for government output. Higher public sector wages drive differential selection of citizens by ideology and can either decrease or increase partisanship in the bureaucracy. Recruitment measures that raise bureaucrats' PSM can have unintended political consequences because more motivated bureaucrats give parties a license to choose more extreme policies.

As we show below, various elements of our analysis find support in the empirical literature on bureaucracies, notably in recent papers documenting the implications of ideology misalignment between bureaucrats and political executives in the US federal bureaucracy (Spenkuch et al., 2021; Bolton et al., 2021).

2 Related Literature

Our paper provides a tractable model that features endogenous entry and output in the bureaucracy, political competition, and heterogenous voters. We are not aware of another model that combines all these features, but elements of our approach have antecedents in several different literatures.

Studying the implications of bureaucrats' preferences for the optimal design of the public sector has a long tradition in economics (e.g., Dewatripont et al. (1999); François (2000); Alesina and Tabellini (2007); Prendergast (2007)). Much of the recent literature has increasingly focused on the wider "general equilibrium" implications of these preferences. Macchiavello (2008), Delfgaauw and Dur (2008, 2010), and Jaimovich and Rud (2014) study equilibrium workforce composition and wages in both the public and private sectors when some agents have public service motivation (PSM). Aldashev et al. (2018) study equilibrium sorting of motivated agents between the non-profit and for-profit sectors, and the impact on donations to non-profits. Closer to our model, Besley and Ghatak (2005) study the matching of motivated workers to organizations with heterogenous missions, and present an extension where the organization's choice of mission is endogenous. In contrast to all of these approaches, we explicitly model the fact that the public sector is a *political* organization. In our model, the missions (i.e., policies) bureaucrats are tasked with implementing are the outcome of politicians' and voters' choices, and these missions are uncertain ex ante. Bureaucrats have both PSM and policy preferences, and we focus on understanding the implication of bureaucrat sorting in a political equilibrium.

Because our model features politicians, bureaucrats, and voters, it relates to the literature studying how agency relationships between politicians and bureaucrats affect voters' ability to screen and incentivize politicians (Fox and Jordan, 2011; Ujhelyi, 2014a; Vlaicu and Whalley, 2016; Forand, 2019; Forand and Ujhelyi, 2021; Li et al., 2020). To maintain tractability, these models tend to allow for limited heterogeneity, do not include PSM, and ignore the endogenous formation of bureaucracies. Related models focus on politicians' choice to delegate authority to bureaucrats but do not discuss elections or bureaucrat entry (Epstein and O'Halloran, 1999; Gailmard and Patty, 2007).[2]

More generally, our paper contributes to the literature studying the endogenous formation of government. While the citizen-candidate literature initiated by Osborne and Slivinski (1996) and Besley and Coate (1997) studies the entry of politicians in equilibrium, we focus on the entry of bureaucrats. Similarly, a growing literature on state capacity highlights the need to understand how the state's ability to implement various policies emerges endogenously (Besley and Persson, 2009; Acemoglu et al., 2011). Our model provides a possible approach to microfounding state capacity in an established democracy by focusing on heterogenous bureaucrats' choices regarding entry and effort.[3]

[2] Gailmard and Patty (2007) and Cameron and de Figueiredo (2020) allow a bureaucrat to quit in response to the executive's action.

[3] Acemoglu et al. (2011) show how the interaction between a rich elite and the bureaucracy can affect state capacity during transitions to democracy.

3 Model

3.1 Setup

Citizens, parties and government policy. There is a continuum of citizens whose ideology b is distributed uniformly over $[-I/2, I/2]$. There are two parties, L and R, with ideologies -1 and 1, respectively. Let p_L denote the probability that L is elected, and $p_R = 1 - p_L$ the probability that R is elected. Without loss of generality, we assume that $p_L \geq p_R$, so that party L has an (at least weak) electoral advantage. In many countries, some parties have enjoyed disproportionate access to government over long stretches of time, and our setting will address the effects of such partisan imbalances on the bureaucracy.

When in power, the parties implement policies with ideology $x_L, x_R \in [-1, 1]$, where $x_L \leq x_R$. The difference $\Delta x = x_R - x_L$ in the two parties' policies will be our measure of political polarization.[4] We assume that the dispersion in citizen ideologies, captured by I, is large relative to the bound of 1 on the policy space. This assumption simplifies our analysis by ensuring that any feasible policy has access to the same level of support from bureaucrats.

Once the policy ideology x has been chosen, bureaucrats will determine government output Q. This will capture the total amount of effort exerted by bureaucrats - for example, the intensity with which they implement the policy x. Citizens value both policy ideology x and output Q, with the crucial feature that they prefer higher levels of output when the policy is closer to their ideal point b. We specify the resulting utility as

$$Q \times (\alpha - |x - b|), \tag{1}$$

where $\alpha > 0$ measures citizens' value from output at their ideal policy ideology $x = b$.

Parties are policy motivated, and, given their own ideology, they receive the same utility (1) as citizens. We assume that both parties prefer higher output for all feasible policies: recalling that party L has ideal policy -1 and party R has ideal policy 1, this requires that $\alpha \geq 2$.

Citizens as bureaucrats. Citizens choose whether to join the public sector or work in the private sector instead. A *bureaucracy* $B \subseteq [-I/2, I/2]$ will consist of the citizens who chose to enter the public sector. If a citizen joins the public sector (i.e., becomes a bureaucrat), she decides how much effort to exert in the implementation of government policies. Specifically,

[4] Polarization in the parties' ideal policies (1 vs -1) is held fixed throughout. Differences in such underlying preferences are difficult to measure empirically (see Canen et al. (2020)).

she chooses her contribution q to government output, leading her to receive a payoff

$$q \times (\phi - |x - b|) \tag{2}$$

in addition to (1). Thus, in addition to her utility as a citizen, a bureaucrat also obtains direct utility from her contribution to government. As above, this utility decreases in the distance between her ideology b and the government's policy choice x: bureaucrats obtain lower utility from government work when asked to implement policies that are farther from their ideal. The parameter $\phi > 0$ is our measure of public service motivation: it captures bureaucrats' intrinsic utility from producing government output, regardless of the policy ideology. This may include a warm-glow utility from the act of contributing to a public good, net of any effort cost.

For simplicity, we assume that $q \in \{0, 1\}$, which we can interpret as bureaucrats deciding to either work ($q = 1$) or shirk ($q = 0$). Total government output Q will simply aggregate the effort decisions of individual bureaucrats, as we detail below. Public sector wages (or any other fixed benefit from entering the bureaucracy) are normalized to 0.

Citizens working in the private sector. If a citizen chooses to take a job in the private sector, she earns wage $w > 0$ (in addition to her payoff in (1)). Because all citizens have the option to enter government only to shirk, a negative private-public wage gap would lead to all citizens joining the bureaucracy. A positive wage gap would arise endogenously, for example, in a model in which citizens differ in their public sector motivation (as in Delfgaauw and Dur (2010)) or in their private sector productivity (as in bureaucratic analogues to Caselli and Morelli (2004) or Mattozzi and Merlo (2008) who focus on the selection of politicians). We abstract from these issues and focus only on ideological heterogeneity among potential bureaucrats, so that, given $w > 0$, only those citizens who most value the policies being implemented by governments will have incentives to become bureaucrats. The level of the private-public wage gap can reflect the power of public sector unions, differences in the relative productivity of the private and public sectors, or the presence of public sector rents delivered by politicians for redistributive or clientelistic reasons.

3.2 Equilibrium

Optimal bureaucracies. Our goal is to capture the fact that citizens choose to enter government against a forecast of the parties they will serve over their entire careers. We model a simple reduced form proxy for these richer dynamic expectations. Specifically, a *policy lottery* $\chi = (x_L, x_R; p_L, p_R)$ yields policy x_L with probability p_L and policy x_R with probability $p_R = 1 - p_L$. Bureaucrats' career decisions to join the government are made before the realization of policy lotteries, while their production decisions are made after they know which party holds office. One interpretation of the probabilities attached to the two parties' policies is that they represent the fraction of their careers that bureaucrats expect to spend in the service of each party.

Because government output is produced by a continuum of bureaucrats, each individual bureaucrat takes total output Q as given, so that her choices only depend on the direct utility from government work from (2). A bureaucrat with ideology b will work, rather than shirk, for a party on the implementation of policy x whenever her public service motivation (ϕ) overcomes her ideological gap with the government's policy ($|x-b|$). Formally, an *optimal production decision* for bureaucrat b is

$$q_b(x) = \begin{cases} 1 & \text{if } |x-b| \leq \phi, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Whether a citizen joins the bureaucracy depends on the value of her opportunities in the private sector. Given a policy lottery χ and a private sector wage w, an *optimal application decision* for citizen with ideology b is $a_b(\chi) \in \{0,1\}$, where $a_b(\chi) = 1$ denotes her decision to become a bureaucrat. We have that

$$a_b(\chi) = \begin{cases} 1 & \text{if } p_L q_b(x_L)(\phi - |x_L - b|) + p_R q_b(x_R)(\phi - |x_R - b|) \geq w, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Note that the production decision (3) is conditional on the policy x, while the career decision in (4) is conditional on the policy lottery χ. We assume that whenever they are indifferent, citizens choose to become bureaucrats, and bureaucrats choose to work, respectively. This is to simplify the exposition of our results, none of which depend on this selection of optimal decisions.

Finally, given a policy lottery χ and a private sector wage w, an *optimal bureaucracy* $B(\chi) = \{b : a_b(\chi) = 1\}$ simply aggregates the optimal application decisions of all citizens. Correspondingly, given a bureaucracy B and a policy x, the aggregate quantity of government output is $Q^B(x) = \int_B q_b(x)^{\mathrm{d}b}/_I$. Given some policy lottery χ, expected government output is $Q^B(\chi) = p_L Q^B(x_L) + p_R Q^B(x_R)$.

Optimal policies. Parties cannot commit to policies before the election: if elected, a party will choose policy x to maximize its payoff (1). In doing so, parties take the bureaucracy as given, but they understand that the level of effort exerted by bureaucrats will depend on the ideology of the policy they are asked to implement. Correspondingly, given some bureaucracy B, the set of *optimal policies* for party $P = L, R$ with ideal policy $b_P \in \{-1, 1\}$

is

$$x_P(B) = \text{argmax}_{x \in [-1,1]} Q^B(x)(\alpha - |x - b_P|). \tag{5}$$

Equilibrium administrations. Our equilibrium notion ties together bureaucrats' entry decisions, parties' policy choices and citizens' preferences over governments. An *equilibrium administration* consists of a policy lottery $\chi^* = (x_L^*, x_R^*; p_L^*, p_R^*)$ along with a bureaucracy B^* which satisfy:

(i) given policy lottery χ^*, B^* is an optimal bureaucracy, i.e., $B(\chi^*) = B^*$,

(ii) given the bureaucracy B^*, the policy x_P^* of party $P = L, R$ is an optimal policy, i.e., $x_P^* \in x_P(B^*)$,

(iii) given policies (x_L^*, x_R^*) and bureaucracy B^*, the fraction of voters who prefer party L to party R is p_L^*, i.e., p_L^* is the fraction of citizens in the set $\{b \in [-I/2, I/2] : Q^{B^*}(x_L^*)(\alpha - |x_L^* - b|) \geq Q^{B^*}(x_R^*)(\alpha - |x_R^* - b|)\}$.

An equilibrium administration describes bureaucratic and political outcomes in a unified framework: the bureaucrats who choose to enter government must correctly forecast the policies they will be asked to implement over their careers, parties must have incentives to choose the policies that attract exactly these bureaucrats, and the resulting government policy and output must enjoy a level of support among voters that is consistent with bureaucrats' beliefs about which parties will hold office.

3.3 Discussion of main modelling assumptions

Bureaucrats' motivations. We model individual bureaucrats as infinitesimal actors inside government, although in the aggregate their effort determines output. This is not meant to capture the incentives of senior bureaucrats who may have a significant impact not just on output, but also on policy ideology or the incumbent's electoral fortunes. Bureaucrats in our model are lower-level bureaucrats, and our goal is to capture the interactions of governing parties with the rank-and-file bureaucracy as a whole.

In line with a large literature in public administration, we assume that bureaucrats are motivated by the intrinsic value they derive from working in government. Part of this value comes from PSM. Although the importance of PSM is well established empirically, there are different approaches to modelling it in the literature. Like us, Delfgaauw and Dur (2008) model PSM as a value placed by the employee directly on effort. François (2000), Prendergast (2007) and Besley and Ghatak (2005) model PSM as a value placed on output, while Delfgaauw and Dur (2010) and Dal Bó et al. (2013) present models where PSM is a fixed utility from working in the public sector. Importantly, as another source of intrinsic motivation, we allow bureaucrats to have policy preferences. Recent empirical results by Spenkuch et al. (2021) show that policy preferences and the resulting conflict with politicians indeed matter for bureaucrats' performance.

Political control of the bureaucracy. Our setting is meant to capture a democracy with an established civil service, where explicit regulations and implicit norms limit incoming parties' ability to exert political control over the bureaucracy. As a result, parties anticipate that government performance depends on the support that their polices enjoy in the bureaucracy. There is extensive anecdotal evidence on this view of the bureaucracy as a constraint for politicians. As noted by Peters (2002) in the British context, political executives coming into power "have almost invariably reported overt or covert resistance by their civil servants and the existence of a "departmental view" about policy that limits the effectiveness of any political leader." (p222) Similarly, several US presidents are famous for viewing civil servants in the federal bureaucracy with distrust (Aberbach and Rockman, 1976; Rourke, 1992; Golden, 2000)

In our main results, we abstract from various managerial controls that politicians can leverage to increase bureaucratic output, but we return to two of them in extensions: in Section 7.1, we allow political appointments to the bureaucracy, and in Section 7.4 allow parties to impose penalties on shirking bureaucrats.

Bureaucrats' career decisions. Although our model is not dynamic, it is meant to capture the difference in time scales involved in potential bureaucrats' long-term career decisions and parties' within-term policy decisions. Citizens join the bureaucracy before policies are realized. Indeed, in many countries public service is considered to be a lifelong career choice and shifting between the public and private sectors is uncommon (Peters, 2002). In some cases (e.g., France), bureaucrats must attend specific schools, implying that their decision to work in government must be made years before they learn what policies they will implement.

Our concept of an equilibrium administration highlights the role of bureaucrats' beliefs regarding the political environment when choosing a career in the public sector. The three conditions in the definition impose increasing levels of sophistication on these beliefs. Condition (i) requires only that bureaucrats's career decisions are optimal against their expectations of both policy and electoral outcomes. Condition (ii) requires that bureaucrats' expectations of parties' policies be consistent with their entry decisions from (i), aggregated into B^*. Finally, condition (iii) requires that bureaucrats' expectations of voters' electoral choices be consistent with both their entry decisions from (i) and their expectations about parties' policies from (ii).

The assumption that the bureaucracy is fixed before elections rules out that bureaucrats may choose to quit instead of serving a government they disagree with. Although among career civil servants politically motivated quitting is infrequent empirically (Bolton et al., 2021; Spenkuch et al., 2021), Section 7.3 discusses an extension along these lines. Another way that the bureaucracy can change after elections is through political appointments, and we discuss this possibility in Section 7.1.

Voting. Voters' choices are guided by their forecast of the interactions of parties and bureaucrats. We do not specify an explicit model of electoral competition. Rather, we adopt a reduced form approach that only requires that parties' winning probabilities be responsive to their performance in government. That being said, our equilibrium results are consistent with the outcomes of a model with uncertainty about the median voter's ideology. To see this, suppose that in any election the median voter is drawn from a uniform distribution over citizen ideologies $[-I/2, I/2]$. In that case, the winning probability of party L, which is the probability that the median voter prefers L, is the same as the share of voters that prefer party L as described in part (iii) of our definition of an equilibrium administration.[5]

Size and composition of government. In our setting, the size, ideological composition and productivity of the government workforce are endogenous. In particular, the equilibrium number of bureaucrats is determined by the desirability of government jobs, which captures both political and economic factors. An alternative approach is to fix the supply of government jobs and let the private-public wage gap adjust to clear the labor market; we explore this extension, which preserves all the key incentives of our model, in Section 7.2.

We abstract from organizational divisions within government, assuming, in effect, that all bureaucrats are employed by the same agency or department. This is for simplicity; in Section 7.5 we show how all our results extend to a model in which bureaucrats are sorted into ideologically differentiated agencies.

4 Optimal bureaucracies

In this section, we treat parties' policy platforms and winning probabilities as parameters and describe bureaucrats' entry and effort decisions. Our goal is to study what these and other parameters imply for the composition of the bureaucracy and government output.

[5]We are implicitly assuming that bureaucrats' voting behavior is based on the same consideration as other citizens'. Allowing different motivations for bureaucrats as voters would be an interesting extension.

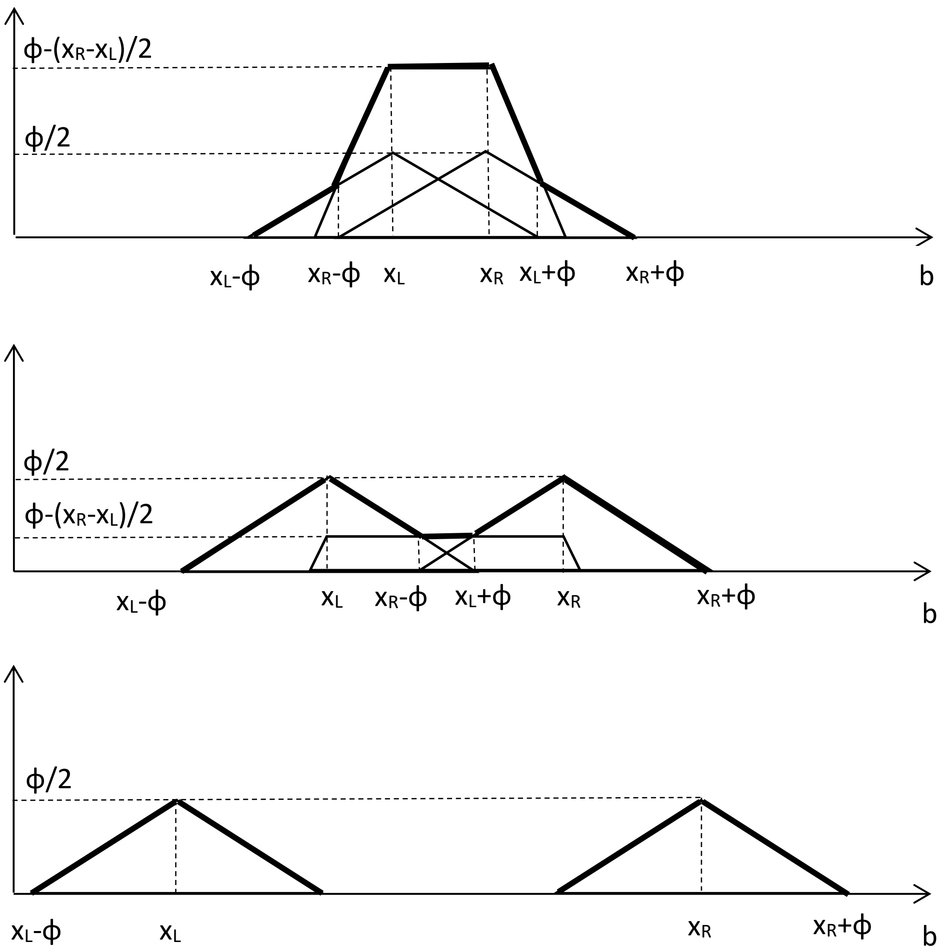

Figure 1: Polarization and bureaucratic effort. As polarization increases, the set of neutral bureaucrats shrinks: more bureaucrats exert effort for only one party.

4.1 Polarization, partisanship and neutrality

Given a policy lottery χ, let $U_b(\chi, L) = p_L(\phi - |x_L - b|)$ denote the expected payoff of a citizen with ideology b if she becomes a bureaucrat and works ($q = 1$) only if L wins, $U_b(\chi, R) = p_R(\phi - |x_R - b|)$ her payoff if she works only if R wins, and $U_b(\chi, LR) = \phi - p_L|x_L - b| - p_R|x_R - b|$ her payoff if she always works. Figure 1 illustrates these payoff functions for different policy pairs (x_L, x_R). For simplicity, we begin with the special case of $p_L = p_R = \frac{1}{2}$. The triangle on the left is $U_b(\chi, L)$, the one on the right is $U_b(\chi, R)$, and the trapezoid on the two upper graphs is $U_b(\chi, LR)$. The bureaucrat's optimal production decision (3) is determined by the maximum of these payoff functions, indicated in bold.

We can identify four groups of bureaucrats. On the first panel of Figure 1, bureaucrats with ideology $b < x_L - \phi$ or $b > x_R + \phi$ always shirk ($q = 0$). Bureaucrats with ideology $b \in$

$[x_L - \phi, x_R - \phi)$ work only if party L wins, while bureaucrats with ideology $b \in (x_L + \phi, x_R + \phi]$ work only if party R wins. We will refer to bureaucrats who only work for one of the parties as *partisans*. Finally, bureaucrats with ideology in $b \in [x_R - \phi, x_L + \phi]$ work regardless of which party wins. We will refer to bureaucrats who work for both parties as *neutral*. Note that here, partisanship or neutrality is not an inherent characteristic of bureaucrats. Rather, it is a *choice* that results from the simple interaction between bureaucrats' policy preferences and the policy platform of the party in power.[6]

Suppose that there is no political polarization (i.e., $\Delta x = x_R - x_L = 0$). Then clearly all bureaucrats who work under one party also work under the other party: all bureaucrats are neutral. As political polarization increases, some neutral bureaucrats become partisans. For these bureaucrats, the party farther from their preferred policy is now "too far": they prefer to shirk if this party wins. At the same time, each policy moves closer to some of the more extremist bureaucrats. Some of these bureaucrats previously shirked but now become partisans. The resulting pattern is illustrated on the top panel of Figure 1. Bureaucrats in the middle maximize their payoff on the trapezoid: these bureaucrats will be neutral. Bureaucrats on either side pick points on the triangles: these bureaucrats will be partisans. As political polarization increases, the triangles slide further apart, while the trapezoid is lowered. The resulting pattern is shown on the middle panel, where the number of neutral bureaucrats (on the trapezoid) has decreased while the number of partisans (on the triangles) has gone up. As polarization increases further, neutral bureaucrats disappear completely and all bureaucrats are now partisan (bottom panel).

While simple, the analysis so far yields an important lesson: political polarization reduces neutrality and creates partisanship in the bureaucracy. This is true even though political patronage and other forms of explicit political interference are absent from this model. Instead, partisanship emerges due to the incentives of policy-motivated bureaucrats: when polarization is high enough, there are simply no bureaucrats who would be willing to work for both parties.

To describe optimal bureaucracies, we must determine which citizens choose to apply for a public sector job. Recalling (4), this depends on the private-public wage gap. Graphically, this participation constraint limits the bureaucracy to those citizens with payoffs above a horizontal line at w. The effect of this is to screen some citizens out of the public sector - citizens who would have been partisan *or* neutral bureaucrats had they entered. One possibility is illustrated on the top panel of Figure 2, which follows up on the top panel of Figure 1. The optimal bureaucracy $B(\chi)$ consists of those bureaucrats with payoffs on the bold segment:

[6] Notice that partisanship vs. neutrality depends on, but is nevertheless distinct from, bureaucrats' "moderate" or "extreme" position on the ideological spectrum $[-I/2, I/2]$.

here, $w > 0$ screens out partisan bureaucrats. Intuitively, when polarization is relatively low, citizens who would become partisan bureaucrats choose to work in the private sector instead, and only citizens who will become neutral bureaucrats enter government. Thus, for relatively low polarization Δx, a higher w can make the bureaucracy more politically neutral.

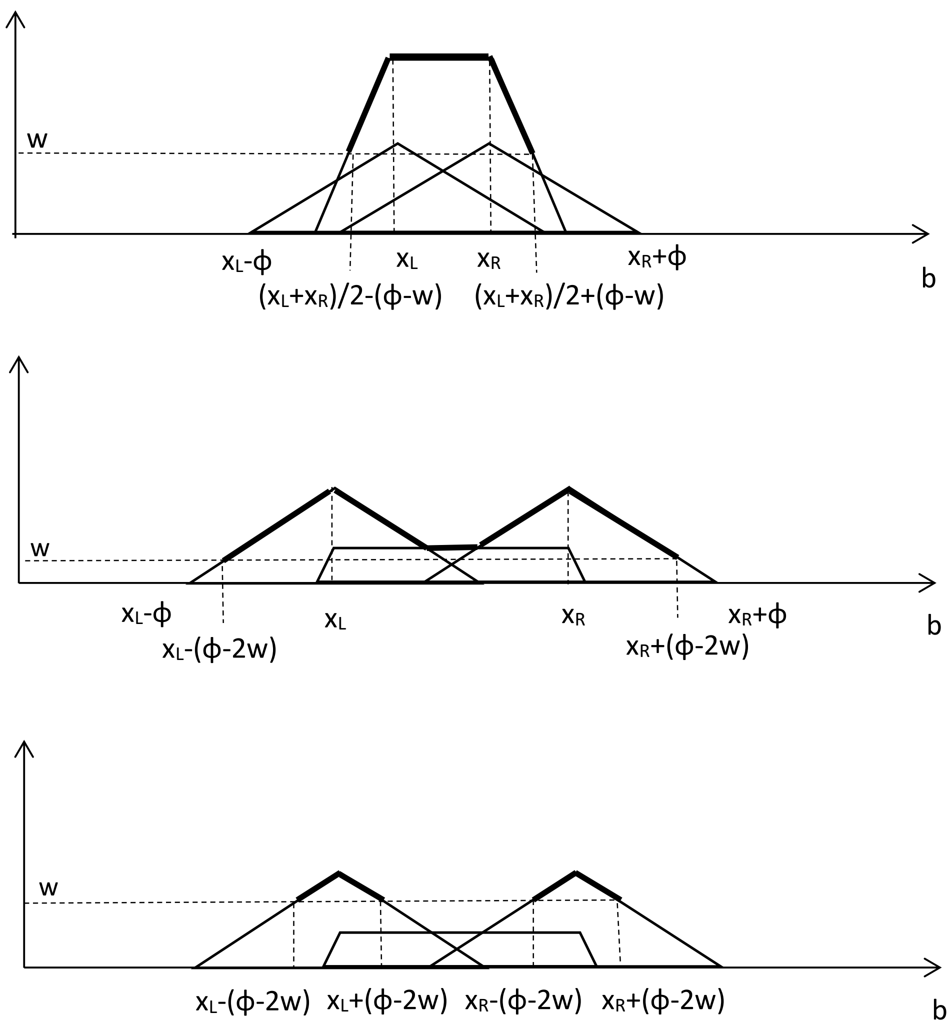

Figure 2: Wages and polarization. In the top panel, polarization is low and only centrist, neutral bureaucrats are willing to join the bureaucracy. The middle and bottom panels show that under high polarization, higher private sector wages can lead to partisanship in the bureaucracy. The top two panels are ideologically connected, while the bottom panel is ideologically disconnected.

A different possibility is illustrated by the two bottom panels of Figure 2, which correspond to the middle panel of Figure 1 but show two different levels of w. In the middle panel,

$w > 0$ screens out some partisan bureaucrats. However, as w increases further, it screens out neutral bureaucrats, as shown on the bottom panel. When polarization is high, neutral bureaucrats' payoffs are relatively low, and a higher private-public wage gap makes it more likely that these bureaucrats will choose the private sector. For relatively high polarization, a higher w makes the bureaucracy more partisan.

Figure 2 suggests three possible types of optimal bureaucracies: on the top panel, all entrants are neutral so the bureaucracy is *fully neutral (N)*; on the bottom panel all entrants are partisan so the bureaucracy is *fully partisan (P)*; and on the middle panel then entrant pool contains both partisans and neutrals, so the bureaucracy is *partially partisan (PP).*

The fact that in general party L may have an electoral advantage ($p_L \geq p_R$) creates another possibility, illustrated in Figure 3. In this case, the optimal bureaucracy is partially partisan but while all bureaucrats are willing to work for the stronger party L, not all bureaucrats are willing to work for the weaker party R. In other words, only the stronger party has partisan bureaucrats. This happens when polarization increases just above the level required for a fully neutral bureaucracy. As polarization moves into this range, the first partisan bureaucrats to enter will be those of the stronger party, L, since these bureaucrats' utility from working for only one party is larger. Here again, a feature commonly associated with party machines and political patronage can emerge endogenously: a stronger party may gain an administrative advantage by attracting partisan bureaucrats.

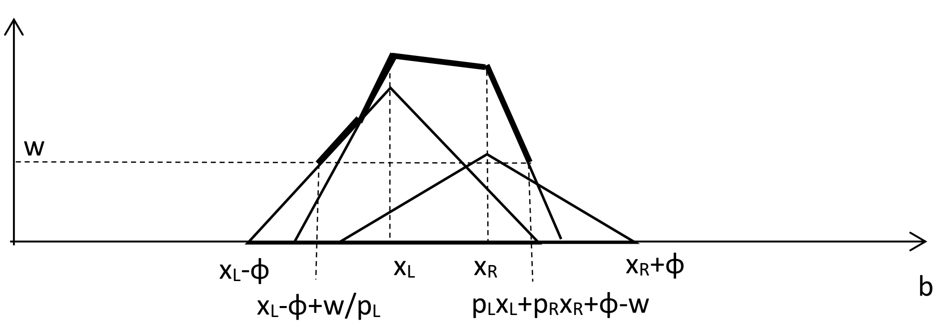

Figure 3: Asymmetric, partially polarized bureaucracy. Under medium polarization and a large party L electoral advantage, bureaucrats are either neutral or L partisans. This corresponds to case (PP3) of Proposition 1.

For future reference, note that the top two panels of Figure 2 and Figure 3 feature a bureaucracy that is *ideologically connected*, in that $B(\chi)$ is a convex set, while the bottom panel of Figure 2 show a bureaucracy that is *ideologically disconnected.*

The following proposition describes optimal bureaucracies in detail. To ensure that each party has at least some bureaucrats working for its policy (a feature that will always have to be true in equilibrium), we assume that $\phi > {}^{w}/_{p_R}$ (and hence also $\phi > {}^{w}/_{p_L}$).[7]

Proposition 1 *(P) The optimal bureaucracy is fully partisan if and only if party polarization is high ($\Delta x > 2\phi - {}^{w}/_{p_L}$). In this case*

$$B(\chi) = [x_L - (\phi - {}^{w}/_{p_L}), x_L + (\phi - {}^{w}/_{p_L})] \cup [x_R - (\phi - {}^{w}/_{p_R}), x_R + (\phi - {}^{w}/_{p_R})] \,.$$

(PP) The bureaucracy is partially partisan if and only if party polarization is medium (${}^{w}/_{p_L} < \Delta x \leq 2\phi - {}^{w}/_{p_L}$). If further

- *(PP1) $2\phi - {}^{w}/_{p_R} < \Delta x \leq 2\phi - {}^{w}/_{p_L}$, then both parties have partisan bureaucrats, and some neutral bureaucrats enter while some stay out of the public sector. In this case*

$$B(\chi) = [x_L - (\phi - {}^{w}/_{p_L}), {}^{1}/_{p_L - p_R}(p_L x_L - p_R x_R + (\phi - w))] \cup [x_R - (\phi - {}^{w}/_{p_R}), x_R + (\phi - {}^{w}/_{p_R})] \,.$$

- *(PP2) ${}^{w}/_{p_R} < \Delta x \leq 2\phi - {}^{w}/_{p_R}$, then both parties have partisan bureaucrats, and all neutral bureaucrats enter the public sector. In this case*

$$B(\chi) = [x_L - (\phi - {}^{w}/_{p_L}), x_R + (\phi - {}^{w}/_{p_R})].$$

- *(PP3) ${}^{w}/_{p_L} < \Delta x \leq {}^{w}/_{p_R}$, then only party L has partisan bureaucrats, and some neutral bureaucrats enter while others stay out of the public sector. In this case*

$$B(\chi) = [x_L - (\phi - {}^{w}/_{p_L}), p_L x_L + p_R x_R + (\phi - w)].$$

(N) The bureaucracy is fully neutral if and only if party polarization is low ($\Delta x \leq {}^{w}/_{p_L}$). In this case

$$B(\chi) = [p_L x_L + p_R x_R - (\phi - w), p_L x_L + p_R x_R + (\phi - w)].$$

To see the intuition behind the conditions for the different cases in Proposition 1, note that the incentives that constrain the existence of both fully partisan and fully neutral bureaucracies are those of the bureaucrats who prefer the advantaged party L. This is reflected in the fact that the conditions on polarization in both (P) and (N) cases in Proposition 1 involve only the winning probability p_L. This is because party L's electoral advantage draws more bureaucrats into government. Therefore, the key constraint on fully partisan bureaucracies is that party L might attract more centrist citizens who could be tempted to work for party R once in government. Correspondingly, the key constraint on fully neutral

[7] The proofs of all results are in the Appendix.

bureaucracies is that party L may attract more left wing citizens who could be tempted to become partisan once in government. The fact that the advantaged party L is more adept at attracting both partisan and neutral bureaucrats also explains our results on partially partisan bureaucracies. In particular, it is possible both for party L to have partisan bureaucrats while party R does not (relatively low polarization, case ($PP3$)), and for neutral bureaucrats closer to L to enter while those closer to R stay out (relatively high polarization, case ($PP1$)).

4.2 Implications

4.2.1 Public sector wages and bureaucratic neutrality

A sizeable literature investigates the impact of government wages on bureaucrats' motivations - specifically, whether higher wages lead to the selection of bureaucrats with higher or lower PSM (e.g., Macchiavello (2008); Delfgaauw and Dur (2010); Valasek (2018); Gibbs (2020); see Finan et al. (2017) for a review of the empirical literature). Our model highlights a complementary dimension of heterogeneity: bureaucrats' policy preferences.[8] Proposition 1 shows how public sector wage policies can influence the bureaucracy's ideological composition and hence its partisanship or neutrality.

We saw that higher polarization reduces neutrality and leads to more partisanship in the bureaucracy. Proposition 1 shows that a higher w makes full bureaucratic neutrality more resilient to a small degree of political polarization (partisanship in the bureaucracy only appears once $\Delta x > \frac{w}{p_L}$, and this condition becomes more stringent as w goes up). At the same time, as polarization increases, a higher wage gap w also speeds up the transition from a partially partisan to a fully partisan bureaucracy (in Proposition 1, the bureaucracy becomes fully partisan once $\Delta x > 2\phi - \frac{w}{p_L}$, and this is facilitated by a high w). In this sense, high-w environments are conducive to either a fully neutral or a fully partisan bureaucracy: low pay in the public sector restricts entry to those citizens who most value becoming bureaucrats, and these are neutral if polarization is low and partisan if polarization is high. On the other hand, low-w environments are more likely to have a mix of neutral and partisan bureaucrats. In particular, if $w \approx 0$ the opportunity cost of joining the bureaucracy just to work for their preferred party vanishes, so that any amount of polarization $\Delta x > 0$ will attract some partisan bureaucrats.

[8] There is an important conceptual difference between these two dimensions of heterogeneity. Changing a bureaucrat's PSM (holding all else constant) has a monotonic effect on a his utility from accepting a government job - a feature that is exploited by many papers in the literature. By contrast, changing a bureaucrat's ideology has an effect that is fundamentally nonmonotonic (as it depends on the location of the government policy x). Because of this, selection effects work differently in the two cases.

Note that, by definition, partisan bureaucrats will shirk with positive probability, so that bureaucrat "laziness" is endogenous in our model. Furthermore, the argument in the previous paragraph implies that the effect of government wages on shirking is ambiguous. Higher wages may lead to the entry of neutral bureaucrats, who never shirk, or to the entry of partisans, who sometimes do. Macchiavello (2008) shows that, empirically, there is a non-monotonic relationship between public sector wages and productivity across countries. Our model shows that this may be due in part to differences in policy polarization.

4.2.2 The size and output of the bureaucracy

We now discuss the implications of Proposition 1 for government size and production. To simplify notation, we let Q denote expected government output (replacing $Q^B(\chi)$) and let Q_L and Q_R denote the government output associated to parties L and R, respectively (replacing $Q^B(x_L)$ and $Q^B(x_R)$).

Corollary 1 *Expected government output Q and the size of the bureaucracy, $|B|$, are both increasing in PSM ϕ and decreasing in the private-public wage gap w. Expected output is decreasing in polarization Δx. It is highest under a fully neutral bureaucracy, lower under a partially partisan bureaucracy, and lowest under full partisanship. The size of the bureaucracy is increasing in polarization when $\Delta x < 2\phi - \frac{w}{p_R}$ and decreasing when $\Delta x > 2\phi - \frac{w}{p_R}$. Per-capita output ($Q/|B|$) is decreasing in polarization.*

Not surprisingly, expected output is higher when bureaucrats have higher public service motivation, and if the private-public wage gap is lower (since the latter helps staff the bureaucracy). Importantly, output is higher when political polarization is lower and the bureaucracy is more neutral. While a common view is that partisan bureaucracies produce less output because their bureaucrats are somehow "worse," note that we made no such assumption here. Instead, our results follow from the combination of policy motivation and selection. In our model when there is no selection ($w \approx 0$), expected output is independent of bureaucratic neutrality and partisanship.[9]

In our model, neutral and partisan bureaucrats represent two different "technologies" for producing output. Neutral bureaucrats always work, so they do not need be as numerous, while partisans only work some of the time, so there needs to be more of them to produce the same level of expected output. Thus, neutral bureaucrats can be substituted with more partisan bureaucrats to some extent. For example, when $\Delta x < 2\phi - \frac{w}{p_R}$, output Q is constant in Δx, but partially partisan bureaucracies ($\Delta x > \frac{w}{p_L}$) are larger than neutral bureaucracies ($\Delta x < \frac{w}{p_L}$).

[9] Corollary 5 in the Appendix shows that when $w = 0$, $Q(\chi, w) = 2\phi/I$ regardless of Δx.

These two technologies respond differently to selection effects (i.e., w). In particular, because partisan bureaucrats only get utility under one of the parties, they will be more sensitive to changes in w than neutrals. Consider a small increase in the private-public wage gap from $w \approx 0$. In a fully neutral bureaucracy, the marginal bureaucrats who stay out of government are neutral, while in a partially or fully partisan bureaucracy they are partisan. Comparing the fully neutral and the partially partisan cases, the substitution effect described above prevails: although an increase in w causes more partisan bureaucrats to stay out of government, these bureaucrats only contributed to output some of the time, so the drop in output is the same in both regimes. What distinguishes full partisanship from both of these cases is that not only the most extreme (relative to the party platforms) but also the most moderate bureaucrats are marginal, and stay out of government for a small increase in w. This makes fully partisan bureaucracies particularly sensitive to selection effects, which in turn explains why these bureaucracies produce lower output than the other two regimes when there is any selection ($w > 0$).

4.2.3 Political budget cycles

In this model, government output will be either Q_L or Q_R depending on whether party L or R wins the election. Thus, there may be a political cycle in government output associated with changes in the party in power. The magnitude of this cycle is captured by $\Delta Q \equiv Q_L - Q_R$, where $\Delta Q \geq 0$.

Corollary 2 *When winning probabilities are equal ($p_L = p_R$), there is no political cycle in government output ($\Delta Q = 0$). If $p_L > p_R$, then $\Delta Q \geq 0$ and the cycle is increasing in political polarization.*

While it is natural that a government staffed with partisan bureaucrats will result in political cycles in the policy ideology x, the above result shows that cycles can also appear in the quantity of output. This is because with asymmetric winning probabilities, citizens who have chosen to enter the public sector will be biased towards the stronger party. When that party wins, more bureaucrats who have chosen to enter will be willing to work on implementing its policy platform x_L. In this sense, more ideological polarization (Δx) creates output fluctuations by affecting the composition of the bureaucracy. Furthermore, because bureaucratic partisanship is increasing in ideological polarization, it follows that fully partisan bureaucracies have the largest output cycle. For their part, fully neutral bureaucracies have no cycle because all bureaucrats work regardless of who is elected.[10]

[10] Bostashvili and Ujhelyi (2019) find significant differences in political budget cycles under patronage vs. civil service. Our results indicate that such differences could also be present across civil service systems as

4.2.4 The impact of political competitiveness

Finally, our model can be used to analyze how the competitiveness of elections affects bureaucratic partisanship and government output. Since $p_L \geq p_R$, an increase in the competitiveness of elections corresponds to a reduction in p_L and an increase in p_R.

Corollary 3 *An increase in the competitiveness of elections (i) increases the set of parameter values for which the bureaucracy is fully neutral; (ii) increases expected government output Q if* $\Delta x \leq 2(\phi - w)$ *and has an inverse U-shaped effect on Q if* $\Delta x > 2(\phi - w)$.

For parameters that admit a fully neutral bureaucracy, an increase in the competitiveness of elections is more likely to lead to a fully neutral bureaucracy and to a bureaucracy producing higher expected output. These findings provide an interesting perspective on the incentives for institutional reforms that led to the establishment of the modern civil service. It has been argued that increased political competition was a driver for these reforms because incumbents could use them to remove patronage from the political toolkit of future opponents (Ruhil and Camões, 2003; Ting et al., 2013) and to make future policy changes more difficult (Hanssen, 2004). Our analysis above provides a complementary, efficiency rationale for why political competitiveness should matter for reform. Civil service systems created under different political conditions give rise to different incentives for bureaucrat entry and effort. When there is less political competition, citizens who choose to enter the public sector under a civil service system will tend to be more partisan. As shown above, this will cause expected government output to be lower. Civil service systems established in a more competitive environment are the most likely to result in full bureaucratic neutrality *and* maximize expected government output. In this sense, political competitiveness maximizes the potential benefits from civil service reform.[11]

The impact of winning probabilities in the model can also be used to explain why conflicts between the chief executive and the bureaucracy are more likely when a strong incumbent is replaced. When in 1953 Eisenhower's election ended the longest streak of a party's control over the presidency in modern history, the new president and his appointees "were reluctant to trust the career bureaucracy built during 20 years of Democratic rule." (Maranto, 1993, p681). In Sweden, when in 1976 the first non-socialist government in over 40 years was elected, incoming officials were faced with a "forest of red needles" (a badge worn by Social

a function of political polarization.

[11] Interestingly when polarization is high (so that full neutrality is not possible), closer elections can lead to neutral bureaucrats staying out of the public sector and move the bureaucracy from partial to full partisanship. This can happen in case ($PP1$) in Proposition 1, where the marginal neutral entrant is moderate but close to party L, and she will choose the private sector if p_L falls. As $p_L \searrow 1/2$, this case disappears.

Democrats) and corresponding resistance among civil servants toward the new administration's policies (Pierre, 2004). Because in each case the conflict is with career civil servants rather than simply the previous administration's political appointees, episodes like this are not easily explained by political patronage. Instead, the source of conflict was likely the policy preferences of bureaucrats who self-selected into government. Our model is consistent with this: it predicts that bureaucrats who enter government at a time when party L is electorally stronger are likely to shirk under party R ex post.[12]

5 Optimal policies

In the last section, we determined how potential bureaucrats' expectations about political outcomes shaped their decision to enter government. In this section, we investigate how parties respond to the bureaucracy by characterizing their optimal policies given a fixed bureaucracy.

5.1 Parties' ideology-output tradeoff

From Section 4, we know that an optimal bureaucracy must take one of two forms: either $B = [\underline{b}, \overline{b}]$ (ideologically connected), or $B = [\underline{b}_L, \overline{b}_L] \cup [\underline{b}_R, \overline{b}_R]$ where $\overline{b}_L < \underline{b}_R$ (ideologically disconnected). We will first characterize parties' optimal policies when served by ideologically connected bureaucracies, and then use these results to study ideologically disconnected bureaucracies. From Proposition 1, we know that an ideologically connected bureaucracy must have $\underline{b} \geq x_L - \phi$ and $\overline{b} \leq x_R + \phi$ because any bureaucrat who is so far to the left (right) that she would not be willing to work for policy x_L (x_R) would never join the bureaucracy in the first place. This implies that we can restrict attention to bureaucracies that satisfy $\underline{b} \geq -1 - \phi$ and $\overline{b} \leq 1 + \phi$. In the following proposition, we characterize the optimal policies of party L only, which is without loss of generality. Because optimal policies are unique, we abuse notation and let $x_L(B)$ denote that policy directly.

Proposition 2 *Suppose that the bureaucracy is ideologically connected, and let* $\hat{x} = 1/2\,(\alpha - 1 + \underline{b} - \phi)$.

[12]In the US, an alternative explanation sometimes suggested for Republican administrations' conflicts with the bureaucracy is that bureaucrats are inherently liberal, but according to some observers this view may be too simplistic (Maranto, 1993; Michaels, 2017; Rothman and Lichter, 1983). To the extent that bureaucrats in the US federal government *are* relatively liberal, this begs an explanation. Our model offers one possibility, pointing to self-selection in an era when bureaucrats would likely be tasked with implementing liberal policies.

- *If the bureaucracy is small ($\overline{b} - \underline{b} \leq 2\phi$), then party L's optimal policy is*

$$x_L(B) = \begin{cases} -1 & \textit{if } \hat{x} < \max\{\underline{b} - \phi, -1\}, \\ \hat{x} & \textit{if } \max\{\underline{b} - \phi, -1\} \leq \hat{x} \leq \max\{\overline{b} - \phi, -1\}, \\ \max\{\overline{b} - \phi, -1\} & \textit{if } \hat{x} > \max\{\overline{b} - \phi, -1\}. \end{cases} \quad (6)$$

- *If the bureaucracy is large ($\overline{b} - \underline{b} > 2\phi$), then party L's optimal policy is*

$$x_L(B) = \begin{cases} -1 & \textit{if } \hat{x} < \max\{\underline{b} - \phi, -1\}, \\ \hat{x} & \textit{if } \max\{\underline{b} - \phi, -1\} \leq \hat{x} \leq \underline{b} + \phi, \\ \underline{b} + \phi & \textit{if } \hat{x} > \underline{b} + \phi. \end{cases} \quad (7)$$

Because implementing more left wing policies will tend to induce more right wing and centrist bureaucrats to shirk, party L's optimal policy resolves a tradeoff between more extreme, ideologically preferred policies and more moderate policies which yield higher output. How this tradeoff is resolved depends on the size of the bureaucracy. If the bureaucracy is small ($\overline{b} - \underline{b} \leq 2\phi$, as on the top panel of Figure 2), then party L will at most moderate its policy until bureaucrat $\overline{b}$ is willing to work, which is policy $\max\{\overline{b} - \phi, -1\}$. Because the bureaucracy is small, party L still has the support of bureaucrat $\underline{b}$ when implementing this policy. Put differently, it is the rightmost bureaucrat who constrains the policy choices of party L. However, party L need not have incentives to choose the moderate policy $\max\{\overline{b} - \phi, -1\}$ and guarantee that all bureaucrats choose to work. By moving left from this policy, party L loses on production but gains on policy ideology. Clearly, party L will never choose a policy so extreme that it loses the production of the leftmost bureaucrat $\underline{b}$. This is policy $\max\{\underline{b} - \phi, -1\}$. It is straightforward to verify that party L's utility in this range is strictly concave, so that it admits a unique global maximizer, which is the policy $\hat{x}$ in the statement of Proposition 2. Therefore, if $\hat{x} < \max\{\underline{b} - \phi, -1\}$, party L prefers ideology to production and chooses its ideal policy, whereas if $\hat{x} > \max\{\overline{b} - \phi, -1\}$, party L prefers production and chooses policy $\max\{\overline{b} - \phi, -1\}$. If instead $\max\{\underline{b} - \phi, -1\} \leq \hat{x} \leq \max\{\overline{b} - \phi, -1\}$, then party L's optimal policy choice is interior and, at the margin, incentives for ideology and production are balanced.

If the bureaucracy is large ($\overline{b} - \underline{b} > 2\phi$, as might be the case on the middle panel of Figure 2), then party L's optimal policy choice is similar to the case of a small bureaucracy. The key difference is that party L cannot provide incentives for bureaucrat $\overline{b}$ to work without inducing bureaucrat $\underline{b}$ to shirk. Put differently, if the bureaucracy is sufficiently large then no government can induce all bureaucrats to work. Therefore, in this case the rightmost bureaucrat plays no role in party L's policy decision. Instead, the most right wing policy that party L will choose is constrained by the preferences of the leftmost bureaucrat.

In our results on equilibrium administrations below, we require an analogue of Proposition 2 that characterizes parties' optimal policies when bureaucracies are ideologically disconnected. While this result is obtained by a simple adaptation of the result from Proposition 2, its statement is cumbersome. Therefore, we only briefly discuss the result here and relegate its statement and a detailed discussion to Appendix A.2. The key issue is that when the bureaucracy is ideologically disconnected, party L faces the choice of which section of the bureaucracy to try to motivate with its policy choice. This new tradeoff can provide incentives for policy extremism. For example, if the left section of the bureaucracy is large, then party L can choose to ignore right wing bureaucrats. Alternatively, if there are few left wing bureaucrats and the right section of the bureaucracy contains enough moderates, then party L can choose a policy which leads all left wing bureaucrats to shirk.[13] Finally, if the gap between the two sections of the bureaucracy is small, then party L can choose a policy that induces both some left and right wing bureaucrats to work. In this case, an ideologically disconnected bureaucracy provides incentives for policy moderation: because the bureaucracy has a missing moderate section, party L must move further away from its ideal policy in order to increase production.

5.2 Implications

The following corollary describes how party L's optimal policy is affected by the parameters of the model.

Corollary 4 *Suppose that the bureaucracy is ideologically connected. Party L chooses a more moderate policy when it values output more (higher α), the bureaucracy is more right-wing (higher $\underline{b}$ or $\overline{b}$), and when bureaucrats' PSM ϕ is lower.*

Party L's policy choice balances the tradeoff between output and ideology, and thus naturally favors the former when α is high. When either $\underline{b}$ or $\overline{b}$ is higher, the bureaucracy contains more right-wing bureaucrats. For a given policy x_L, this increases the conflict between party L and the bureaucracy: more bureaucrats shirk. To restore output, L has to choose a more moderate policy. These tradeoffs are consistent with anecdotal evidence on newly elected leaders' conflicts with the incumbent bureaucracy (see Section 3.3) and

[13] In both these cases, we can characterize party L's optimal policy by applying the results of Proposition 2 to the left and right sections of the bureaucracy separately.

empirical evidence on politicians' attempts to circumvent the career civil service (Ujhelyi, 2014b; Moreira and Perez, 2020).

Interestingly, our result on the PSM parameter ϕ shows that public-spirited bureaucrats can allow parties to choose more extreme policies. Since the literature on the implications of bureaucrats' PSM tends to ignore politics, higher PSM is typically viewed as unambiguously beneficial. Our model shows that when political parties take PSM into account, a higher PSM can have offsetting negative effects by making it easier for politicians to choose policies closer to their own ideal. There exist both anecdotal and empirical evidence consistent with this mechanism. Newland (2020) describes the decision of high-PSM career bureaucrats to remain in the Trump administration. She notes that this allowed the president to successfully pursue policies that these bureaucrats did not agree with: "We may have been victims of the system, but we were also its instruments." In a different context, observers often note the contribution of motivated bureaucrats to extreme policies under dictatorship. For example, empirical work by Heldring (2019) shows how an efficient Prussian bureaucracy also contributed to a more "efficient" deportation of Jews in Nazi Germany.

On the one hand, public service motivated bureaucrats join the government and exert effort reliably because they attach high value to their contribution to government output. On the other hand, politicians can then exploit these bureaucrats' relative insensitivity to ideology in order to implement more extreme policies.

6 Equilibrium administrations

So far, our results have limited the interactions of the political and bureaucratic components of our model. In Section 4, we fixed parties' policies and electoral prospects and studied the formation and composition of the bureaucracy. In Section 5, we held the bureaucracy fixed and studied the policies that resolved parties' ideology-output tradeoff. In this section, we flesh out the implications of these results when both the politics and the bureaucracy are treated as endogenous. To do this, we study equilibrium administrations (as defined in Section 3.2). We focus in turn on equilibrium administrations supporting the three forms of bureaucracy: fully partisan, fully neutral, and partially partisan. Our results below do not provide complete characterizations of these equilibria. Rather, we provide the conditions under which these equilibria exist and focus on their implications regarding parties' policy choices and electoral prospects.

6.1 Equilibria with fully partisan bureaucracies

Proposition 3 *In any equilibrium administration with a fully partisan bureaucracy, both parties choose their ideal policies ($x_L^* = -1$ and $x_R^* = 1$). If such an equilibrium exists where L has a strict electoral advantage ($p_L^* > p_R^*$), then there also exists an equilibrium administration in which neither party has an electoral advantage ($p_L^* = p_R^* = 1/2$).*

There exists $1 \leq \tilde{\phi} < 1 + w$ such that an equilibrium administration with a fully partisan bureaucracy exists if and only if $2w \leq \phi < \tilde{\phi}$.

The proposition makes sharp predictions about equilibrium polarization under fully partisan bureaucracies: parties must implement their ideal policies. The reason for this result is that under full partisanship, each bureaucrat only works for one party, and with $w > 0$ all entering bureaucrats must derive a strictly positive payoff from doing so. But then party L can always reduce x_L slightly and move the policy closer to its ideal policy without sacrificing any production. Therefore, equilibrium policies are maximally polarized. As we will see below, policy moderation can only be supported in equilibrium when parties cater to bureaucrats with ideologies farther away from their own, i.e., when the bureaucracy has at least some neutral bureaucrats. This is never the case in fully partisan bureaucracies, where each party has monopoly power over its bureaucrats.

The result that in equilibrium fully partisan bureaucracies imply maximal policy polarization extends the finding from Section 4 where we saw that more polarization leads to more partisanship. In our model, partisanship in the bureaucracy and polarization in parties' policy platforms are complements. Under high polarization, no bureaucrat is willing to work for both parties, and all entering bureaucrats must be partisan. In turn, parties can exploit partisan bureaucrats and increase polarization. While the idea that partisanship and polarization are complements would be natural in a patronage system where politicians can select bureaucrats who will be loyal supporters of their policies, our model most closely resembles a modern civil service system where bureaucrats are free from such political interference. Yet, we find that one possible equilibrium of such an administration looks very much like a patronage system, with parties choosing polarized policies that are implemented by partisan bureaucrats.

Note that even if parties' equilibrium policies are symmetric, it does not follow that their winning probabilities will be the same. Asymmetric winning probabilities can persist in equilibrium because of different output levels $Q^{B^*}(x_L^*)$ and $Q^{B^*}(x_R^*)$ arising from self-reinforcing expectations. For example, if party L is expected to have an electoral advantage, then it will draw in more bureaucrats, which leads to higher output and hence confirms party L's electoral advantage. As the proposition shows, this can yield an equilibrium with

$p^*_L > p^*_R$ alongside the equilibrium with $p^*_L = p^*_R = 1/2$. Furthermore, the equilibrium with equal winning probabilities exists whenever the equilibrium with uneven probabilities exists.[14]

Under what conditions are symmetric fully partisan bureaucracies supported in equilibrium? First, w must be low enough relative to bureaucrats' public service motivation for them to enter and work for one party only. This requires that $\phi \geq w/p^*_R = w/p^*_L = 2w$. At the same time, bureaucrats' public service motivation cannot be too strong relative to w, because otherwise the moderate bureaucrats in both sections of the bureaucracy would choose to work for the opposite party ex post: this requires that $2 = x^*_R - x^*_L > 2\phi - w/p^*_L = 2(\phi - w)$, or $\phi < 1 + w$. Finally, parties cannot have incentives to moderate in order to increase production. This will be satisfied if the two sections of the bureaucracy are sufficiently far apart (which imposes the upper bound $\tilde{\phi}$ on bureaucrats' public service motivation).

6.2 Equilibria with fully neutral bureaucracies

Proposition 4 *In any equilibrium administration in which the bureaucracy is fully neutral, parties' policies are symmetric with $x^*_L = \max\{-1, -w\} = -x^*_R$, and neither party has an electoral advantage ($p^*_L = p^*_R = 1/2$).*

Such an equilibrium exists only if $\phi \geq w$. If furthermore $w \geq 1$, then an equilibrium exists, whereas if $w < 1$ an equilibrium exists if and only if $\alpha - 1 - 2\phi + 3w \geq 0$.

The proposition shows that in equilibrium administrations some policy polarization is unavoidable. Even full bureaucratic neutrality, which results in the lowest level of polarization, yields $x^*_R - x^*_L > 0$ as long as $w > 0$. The reason for this is straightforward: since a neutral bureaucracy exerts effort for both parties, there is a range of policy ideologies where parties can move away from each other without sacrificing any production, but moving closer to their favorite policies.

In an equilibrium with a fully neutral bureaucracy, each party optimally chooses the most extreme policy which still induces the bureaucrats most opposed to them to work. Interestingly, this ensures that no party has an electoral advantage in equilibrium. Intuitively, the reason is that electoral advantages, through citizens' selection in the bureaucracy, would generate bureaucratic advantages that undermine parties' equilibrium incentives. More specifically, if party L had an electoral advantage, then the most right wing bureaucrat in a fully

[14] In an equilibrium with a fully partisan bureaucracy, parties must prefer their ideal policy to any more moderate policy which could induce some bureaucrats from the opposite section to work. Given this, the most stringent incentive constraint is that of the electorally disadvantaged party, who has the most to gain from moderation. Hence, if party R has an incentive to choose policy 1 in an equilibrium with a winning probability less than $1/2$, then both parties have incentives to choose their ideal policy in an equilibrium with winning probabilities equal to $1/2$.

neutral bureaucracy would strictly prefer working for party L to shirking. But this would be incompatible with party L's incentives, because it could choose a more extreme policy while ensuring that all bureaucrats still worked.

The result that in equilibrium fully neutral bureaucracies must have maximum political competition extends the finding in Corollary 3 showing that more competition was conducive to full neutrality (at least for levels of polarization that allowed for a fully neutral bureaucracy). Since equilibria with electoral imbalances are possible with either fully or partially partisan bureaucracies (see Proposition 3 above and 5 below), our model makes the prediction that full bureaucratic neutrality will be correlated with more competitive elections.

By definition, an equilibrium with a fully neutral bureaucracy must have all parties provide the same level of output Q, and combined with the fact that $p_L^* = p_R^*$ it follows that $x_L^* = -x_R^*$ (so the voter with ideology 0 is indifferent between the two parties). Equilibrium policies depend on the size of the private-public wage gap w. If the wage gap is small, then the equilibrium bureaucracy is large, so that both parties need to moderate to motivate the bureaucrat most opposed to them to work. If the wage gap is large, then both parties choose their ideal policies.

To see the intuition for the conditions for existence of fully neutral equilibria, note that parties must not have incentives to choose more extreme policies. When $w \geq 1$, parties implement their ideal policies, so this is not a concern. Otherwise, parties prefer moderate policies if their value from output (α) is high, or if the left wing of the bureaucracy is not too large. The latter occurs if public service motivation ϕ is low or if the wage gap w is high.

When combined with Proposition 3, Proposition 4 shows that our model can explain the emergence of fully neutral bureaucracies as a result of the same equilibrium forces that can give rise to fully partisan bureaucracies. Neither type of bureaucracy depends on institutional choices regarding bureaucratic regulations, nor on assumptions that bureaucrats are either apolitical technocrats or subservient to party machines. In fact, the same underlying parameters can be consistent with both types of equilibrium administrations: for example, it can be verified that all the conditions of both Propositions 3 and 4 are satisfied if $\phi = 3/20$, $w = 1/20$ and $\alpha = 43/20$. This suggests that equilibrium coordination can have a role to play in explaining the politicization and performance of government bureaucracies. Given the same fundamentals, one set of expectations can lead to a partisan bureaucracy and polarized politics, whereas different expectations can lead to a neutral bureaucracy which acts as a barrier to polarization.

6.3 Equilibria with partially partisan bureaucracies

To study partially partisan bureaucracies, we focus on the case of an ideologically connected bureaucracy where both parties have partisan bureaucrats (case ($PP2$) in Proposition 1).

Proposition 5 *Consider an equilibrium administration with a partially partisan, ideologically connected bureaucracy in which both parties have partisan bureaucrats. Such an equilibrium exists only if $\phi \geq 2w$. If further $\alpha \geq 2(\phi - w)$, then such an equilibrium exists if and only if $1/2(\alpha - 1 + 3w) \leq \phi \leq \alpha - 1 + w$. If instead $\alpha < 2(\phi - w)$, then such an equilibrium exists if and only if $w \leq 1$.*

If such an equilibrium with $p_L^* > 1/2$ exists, then an equilibrium also exists for any lower level of electoral advantage for party L ($1/2 \leq p_L < p_L^*$). Equilibria in which party L has a lower electoral advantage have more party polarization ($x_R^* - x_L^*$).

In the equilibria considered in Proposition 5, parties face an ideologically connected bureaucracy which contains partisans for both L and R (as well as neutral bureaucrats). Parties' trade off choosing more extreme policies by relying on their own partisans or moderating further to attract their opponent's partisans. Therefore, in the typical equilibrium party L balances these objectives by choosing the interior policy $\hat{x}$ from Proposition 2. This is the case when $\alpha \geq 2(\phi - w)$, while if $\alpha < 2(\phi - w)$, then party L chooses its ideal policy. The other conditions in the statement ensure that, as required for a partially partisan bureaucracy based on Proposition 1, equilibrium party polarization $x_R^* - x_L^*$ is intermediate.

Equilibrium policies depend on the ideological composition of the bureaucracy, which in turn depends on the electoral prospects of the parties. In any equilibrium, the electorally advantaged party L exploits the bureaucracy's bias in its favour by choosing more extreme policies than the electorally disadvantaged party R. We addressed the potential for multiple equilibria supported by self-fulfilling production expectations in our results for fully partisan bureaucracies (Proposition 3). In that case, parties always implemented their ideal policies, which limited the number of potential equilibria to two. In the case of partially partisan bureaucracies, parties' policies adjust continuously to changes in electoral advantage, and equilibrium multiplicity is endemic. In particular, we show that equilibrium constraints are easier to satisfy when parties' electoral prospects are more even, so that given an equilibrium with $p_L^* > 1/2$, there also exist equilibria supporting all winning probabilities $1/2 \leq p_L < p_L^*$ for party L.

Interestingly, party polarization is *higher* in equilibria in which political competition is more intense and party L has less of an electoral advantage. This is in contrast to our discussion in Section 5, which emphasized that a lower p_L shrinks the number of bureaucrats working for L, and hence gives L an incentive to moderate its policy platform. An offsetting effect arises because L's reduced advantage also leads to the entry of more partisan bureaucrats willing to work for R only. This gives an incentive for R to choose a more extreme policy, which more than offsets L's moderation and leads to increased polarization. Thus, the endogenous formation of the bureaucracy can explain why parties' platforms might fail to converge even when elections are expected to be close.[15]

7 Applications and extensions

7.1 The impact of political appointees

Our model of the bureaucracy has focused on the "permanent" civil service. In reality, even in civil service systems, elected politicians can make a number of political appointments to the bureaucracy. The purpose of these appointments is to provide politicians with trusted allies who are willing to work on implementing their policies in the different agencies. The empirical literature has documented the direct effects of political appointees on agency performance and career bureaucrats (Lewis, 2008; Richardson, 2019). Our model can be used to understand some of the equilibrium effects of these appointments.

To this end, suppose that parties can make a mass $\rho > 0$ of appointments after they are elected. Assume that parties have access to separate pools of citizens that they can call upon to staff these political positions, and that these bureaucrats always work (choose $q = 1$) for the parties that appoint them. Therefore, given a policy x and a permanent bureaucracy B, government output becomes $Q^B(x) + \rho$.

Notice that in this setting, the presence of partisan appointees has no direct effect on the permanent bureaucracy, i.e., given a policy lottery χ the optimal permanent bureaucracy is again determined through the optimal application decisions from (4). Thus, we abstract away from many subtleties of team production, such as whether permanent bureaucrats dislike working with political appointees. Yet, we find that political appointments will still have an *indirect* effect on the permanent bureaucracy. Because they influence parties' optimal policy choices, political appointments will impact permanent bureaucrats' equilibrium entry and production decisions.

Proposition 6 *Given a mass ρ of political appointees, an equilibrium administration with a fully neutral bureaucracy exists only if $\phi \geq w$. If furthermore $w \geq 1$, then an equilibrium*

[15] Recall that we also found a lack of policy convergence with close elections in equilibria with a fully neutral bureaucracy (Proposition 4). However, in that case close elections were the *only* possible equilibrium. With partial partisanship, we obtain a range of equilibria with different levels of election closeness and corresponding polarization, and our model predicts a positive correlation between these two features.

exists, whereas if $w < 1$ an equilibrium exists if and only if $\alpha - 1 - 2\phi + 3w \geq \rho$. Equilibrium policies and winning probabilities are given by $x_L^* = \max\{-1, -w\} = -x_R^*$ and $p_L^* = p_R^* = 1/2$.

This proposition shows that, holding other parameters constant, a larger mass of political appointees ρ makes it more difficult to have a fully neutral bureaucracy in equilibrium. With political appointees, the optimal policies for party $P = L, R$ from (5) are solutions to $\max_{x\in[-1,1]}[Q^B(x) + \rho][\alpha - |x - b_P|]$. Given a permanent bureaucracy B, parties' optimal policies will be more extreme when ρ is higher. This is because political appointees contribute to output regardless of the policy ideology x, and this tilts parties' tradeoff between preferred policies and higher output towards more extreme policies. But we know from Proposition 1 that more policy polarization makes it less likely that the equilibrium bureaucracy will be fully neutral. Thus, a higher ρ can destroy the equilibrium with a fully neutral bureaucracy.[16] For the same reason, more political appointees expand the range of fully partisan equilibria.[17]

In this way, political appointments can have a spillover effect on the permanent bureaucracy. Even without any direct conflict between political and career appointees, a system that provides elected governments with more political appointees may be incompatible with neutrality, and lead to partisanship, in the permanent bureaucracy.

The discussion so far has treated the mass of political appointments ρ made by parties as exogenous. This is equivalent to assuming that making appointments is costless but subject to a limit ρ, since then a party will always use all its available appointments to increase production. Suppose now that making appointments has some cost that is independent of the policy x. Since the marginal utility of output is higher when the policy x is closer to the party's ideal, parties will have more incentive to increase ρ in equilibria with more polarized policies. From Proposition 6, this will in turn reduce neutrality in the permanent bureaucracy. This creates another channel through which polarization leads to partisanship in the bureaucracy.

7.2 Equilibrium wages

In our model, the number of jobs available in the bureaucracy adjusts to meet the supply of workers applying to work for the government. This allows our model to address how differences in government size are related to electoral politics and policy polarization. In reality, the number of bureaucratic positions may be constrained by a number of fiscal,

[16] This can happen when $w < 1$. When $w \geq 1$, parties choose their ideal policies so that polarization is already maximized.

[17] Specifically, proceeding as in the proof of Proposition 3, one can show that a larger ρ raises the threshold $\tilde{\phi}$ required for a fully partisan equilibrium.

legislative or other political factors. Here, we assume that the government has a fixed number of positions denoted $\overline{B} > 0$.

A first note is that our main analysis is preserved if w is exogenous and, when $|B(\chi)| > \overline{B}$, the bureaucracy is staffed randomly from the applicant pool. In this case a fraction $\overline{B}/|B(\chi)|$ of each type of applicant from $B(\chi)$ is employed. Clearly, this would affect neither bureaucrats' production decisions nor citizens' decisions to apply for a government job. Given any policy x, government output is $Q^B(x) = \overline{B}/|B(\chi)| Q^{B(\chi)}(x)$. Therefore, parties' optimal policies are the same as in our main model, and hence any equilibrium administrations from Section 6 would be preserved.

Suppose now that the private-public wage gap adjusts to balance the demand and supply of government jobs. In this setting, we can explore our model's joint predictions on bureaucratic compensation and partisanship. To this end, we denote the optimal bureaucracy associated to policy lottery χ as $B(\chi, w)$, to highlight the private-public pay gap in citizens' incentives to join the bureaucracy. Given a policy lottery χ, we say that w^* is an *equilibrium wage* if it clears the market for bureaucrats, i.e., if $|B(\chi, w^*)| = \overline{B}$.

In the following proposition, we characterize equilibrium wages under two simplifying assumptions. First, we restrict attention to policy lotteries with symmetric winning probabilities, i.e., with $p_L = p_R = 1/2$. This condenses our results on optimal bureaucracies from Proposition 1: only cases (P), $(PP2)$ (in which the two parties have the same number of partisan bureaucrats) and (N) remain. Second, we assume that $\overline{B} < 2\phi/I$. This ensures that the government's hiring target is low enough that, given any policy lottery χ, the target can be exceeded by private-public wage gaps that are low enough (i.e., $\lim_{w \to 0} |B(\chi, w)| > \overline{B}$).

Proposition 7 *Fix policy lottery χ with $p_L = p_R = 1/2$, and suppose that $\overline{B} < 2\phi/I$.*

An equilibrium wage supporting a fully partisan bureaucracy (P) exists if and only if $\Delta x > \phi + \overline{B}I/4$, and this wage is $w^* = 1/2 \left(\phi - \overline{B}I/4\right)$.

An equilibrium wage supporting a partially partisan bureaucracy (PP) exists if and only if $\overline{B} \geq \phi/I$ and $2(\phi - \overline{B}I/2) \leq \Delta x \leq 2/3 \left(\phi + \overline{B}I/2\right)$, and this wage is $w^* = 1/2 \left(\phi - \overline{B}I/2 + \Delta x/2\right)$.

An equilibrium wage supporting a fully neutral bureaucracy (N) exists if and only if $\Delta x \leq 2\left(\phi - \overline{B}I/2\right)$, and this wage is $w^* = \phi - \overline{B}I/2$.

In all types of bureaucracies (i.e., whether (P), (PP) or (N)), the equilibrium private-public wage gap (i) grows in PSM ϕ and (ii) declines in public sector labor demand $\overline{B}$. These types of effects also appear in previous studies of public sector wages that emphasize PSM (e.g., Besley and Ghatak (2005); Macchiavello (2008)). However, our model shows that the scale of such effects depends on the degree of partisanship in the bureaucracy. The reason for (i) is that workers with high PSM require more compensation to select into the private sector rather than becoming bureaucrats. The utility of public sector employment is most sensitive to PSM for neutral bureaucrats, and correspondingly the equilibrium wage gap grows fastest in PSM in a fully neutral bureaucracy (N). The reason for (*ii*) is that staffing a larger public sector requires higher relative wages for bureaucrats (i.e., a lower w). As discussed following Corollary 1, partisan bureaucrats are more sensitive to wages than neutral bureaucrats because they only obtain positive payoffs in the public sector under one of the parties. Therefore, the equilibrium wage gap shrinks most slowly in $\overline{B}$ in a fully partisan bureaucracy (P).

Our model can also be used to analyze the relationship between wages and bureaucrats' policy motivations. Due to the countervailing effects (*i*) and (*ii*), our model predicts a non-monotonic relationship between equilibrium wages and bureaucratic partisanship or policy polarization. Public sector wages can be lower (i.e., higher w^*) under full partisanship than under full neutrality if $\overline{B}$ is high enough (by computation, if $\overline{B} > {}^{4\phi}/_{3I}$).[18] The lowest public sector wage is achieved when the bureaucracy is partially partisan (as can be verified by computation, in particular because equilibrium wages increase in Δx under (PP)). This is due to the fact that medium levels of polarization maximize the number of citizens that prefer becoming a bureaucrat (whether as neutral or partisan) to joining the private sector. In equilibrium, this increased labor supply leads to lower relative wages in the public sector. In this way, political polarization may affect equilibrium public sector wages by changing the attractiveness of bureaucratic careers.[19]

7.3 Politically-motivated quits and seniority benefits

Our model implicitly assumes that bureaucrats prefer to stay in government and shirk, rather than quit, when having to serve a party they disagree with. This assumption is not out of line with available evidence. In the US, turnover among career civil servants is low. Bolton et al. (2021) report that total annual turnover in the federal civil service over the period 1988-2011 was between 4.4%-8% on average (depending on the category of employees). They estimate that, on average, only 0.4 percent of employees quit due to a mismatch with the ideology of the incoming administration. Doherty et al. (2018) find that bureaucrats who decide to leave

[18] However, our previous results tell us that wage savings would be a poor measure of government effectiveness, as fully partisan bureaucracies produce less output than fully neutral bureaucracies that have the same size (here, expected outputs are ${}^{\overline{B}}/_2$ and $\overline{B}$, respectively).

[19] In Besley and Ghatak (2005), it is possible to have a perfect match between a worker's and an organization's mission, and this always lowers equilibrium wages. In our case, the match between parties and the bureaucracy can never be perfect because (i) parties always need to work with heterogenous bureaucrats, and (ii) bureaucrats who enter the public sector do not know which party they will be matched with (i.e., who will win the election). In our case, better matching, as occurs under low polarization and bureaucratic neutrality, does not necessarily lead to lower wages.

government are more likely to do so *before* an election rather than after, which is consistent with our assumption that career decisions are made based on expected rather than realized policies.

Without disputing these empirical patterns, we now investigate what happens if bureaucrats can quit after an election, once x is realized. To make this problem interesting, let $0 \leq \delta \leq 1$ denote the rate at which bureaucrats' earnings potential decreases in the private sector relative to the public sector due to their time in government. That is, a bureaucrat who joins and then leaves the public sector earns $(1-\delta)w$. The "depreciation rate" δ could capture a combination of seniority benefits in the public sector and time preference. Notice that our main model had $\delta = 1$, whereas $\delta = 0$ would mean that the same relative private sector wage is always available to bureaucrats.

Clearly, any bureaucrat prefers (at least weakly) to leave the public sector rather than shirk. Therefore, in this model, partisan bureaucrats are those whose transitory stints in public office are tied to the political fortunes of a particular party, whereas neutral bureaucrats form the permanent part of the government workforce.

Proposition 8 *Suppose that bureaucrats have the option to quit and that the relative private sector wage depreciates at a rate* $0 \leq \delta \leq 1$. *The optimal bureaucracy is fully partisan if and only if* $\Delta x > 2\phi - (1+(1-\delta)(2p_L - 1))^{w}/_{p_L}$, *and it is fully neutral if and only if* $\Delta x \leq {}^{\delta w}/_{p_L}$.

An option to quit has two effects on selection into the bureaucracy. First, by increasing the value of shirking ex post (which now involves returning to the private sector) this option increases the incentive for entry ex ante. Second, by reducing the incentive to work for their less preferred party, it also shifts the boundary between neutral (permanent) and partisan (transitory) bureaucrats towards the latter. Correspondingly, Proposition 8 shows that a lower δ increases the range of polarization for which the bureaucracy is fully partisan, and decreases the range of polarization for which the bureaucracy is fully neutral. (Notice that the expressions above reduce to those of Proposition 1 when $\delta = 1$). In the limiting case in which bureaucrats can always return to the private sector at no cost ($\delta = 0$), the bureaucracy can be fully neutral only if the parties converge: otherwise, any degree of polarization will attract some transitory partisan bureaucrats.[20]

The depreciation rate δ of the relative private sector wage captures the relative benefits of a career in government conditional on having become a bureaucrat. It is common for government bureaucracies to provide various benefits tied to employee seniority, such as increasing

[20] Proposition 8 takes parties' policies as given but recall that, by Proposition 4, equilibrium administrations with fully neutral bureaucracies must have positive levels of polarization. Therefore, it follows that the limiting model with $\delta = 0$ cannot admit such equilibrium administrations.

pay scales, government-specific training programs and promotion ladders. A higher value of δ can represent these benefits or other measures that help public sector retention. Proposition 8 then suggests a connection between improving career prospects inside government and discouraging bureaucratic partisanship. By forcing potential bureaucrats to anticipate the likelihood of serving many parties, making long careers in government more attractive promotes the selection of neutral bureaucrats.

7.4 The effect of disciplining bureaucrats

Our model assumes that bureaucrats are able to shirk when asked to implement policies that they disagree with. Several studies deal with the effect of monitoring and incentives in the public sector (see Finan et al. (2017) for a review). How would our results be affected if politicians had the power to better monitor and discipline bureaucrats' production decisions?

A simple way to include this in our model is to assume that bureaucrats bear a cost κ from shirking, which could capture various penalties for poor performance or the effort required for bureaucrats to evade their overseers' monitoring. Our model has $\kappa = 0$, whereas the limiting model with $\kappa \to \infty$ would capture the case in which all bureaucrats must work for all policies.[21]

When existing bureaucrats choose their level of effort, discipline acts like an increase in public service motivation. To see this, notice that optimal production decisions from (3) can be adapted so that $q_b(x) = 1$ iff and only if $|x - b| \leq \phi + \kappa$, so that bureaucrats are willing to work for policies that are further away from them ideologically. Therefore, *for a fixed bureaucracy*, increased discipline promotes output.

However, in our model the prospect of being constrained to work more once in government will also affect the selection of citizens into the bureaucracy. Furthermore, discipline affects potential bureaucrats differentially depending on their ideology. Increased discipline has no impact on neutral bureaucrats, since these bureaucrats will never shirk. Discipline lowers the payoff of partisan bureaucrats: given a policy lottery χ, the payoff of an L-partisan becomes $U_b(\chi, L) = p_L(\phi - |p_L - b|) - p_R\kappa$ and the payoff of an R-partisan $U_b(\chi, R) = p_R(\phi - |p_R - b|) - p_L\kappa$. On Figure 2, the triangles representing these payoffs shift down. This has two implications for entry. First, some partisan bureaucrats who would previously enter may now choose the private sector instead. Second, some bureaucrats who previously entered as partisans may now enter as neutrals. These two effects have opposing implications for output: the first, "extensive margin" effect is negative, while the second, "intensive margin" effect is positive. Moreover, because $p_L \neq p_R$, κ affects the payoffs of L-partisans and R-

[21] Because ex post parties have an incentive to make the punishment for shirking as large as possible, κ can be interpreted as the highest possible level of punishment that the civil service system allows.

partisans differently, and therefore both the extensive and intensive margin effects differ for the two parties. The following result describes the consequences of increased discipline on output in optimal bureaucracies in the different cases of Proposition 1.

Proposition 9 *If an optimal bureaucracy is fully neutral (N), then increased discipline* κ *has no effect on output* Q*. Under full partisanship (P) or partial partisanship (PP), discipline reduces output.* Q_L *increases in case (PP2) and decreases in cases (P), (PP1), and (PP3).* Q_R *increases in case (PP3) and decreases in cases (P), (PP1), and (PP2).*

As described above, discipline can only have positive effects for output on the intensive margin, when one or both parties gain neutral bureaucrats (bureaucrats who would previously enter as partisans but now enter as neutrals). Because under full partisanship there are no neutrals to gain, it follows immediately that the effect of κ on Q, Q_L and Q_R must be negative in this case.

The cases under partial partisanship are more subtle. There are two key observations. First, a party's output cannot increase unless *the other party* has partisan bureaucrats who can be converted into neutrals. This explains why Q_L must decrease when only L has partisans (case (PP3)) and when the marginal R-partisan prefers the private sector to becoming neutral (case (PP1)): in neither of these cases are there any R-partisans to convert. This also explains how Q_R can increase when R has no partisans (case (PP3)): here there is no effect on the extensive margin, and only the positive effect from converting L's partisans remains. Second, because $p_L > p_R$, for party R the extensive margin effect (if nonzero) is always larger in absolute value than the intensive margin effect (if nonzero), while the opposite is true for party L. This explains why Q_R is reduced in cases (PP1) and (PP2) and why Q_L is increased in case (PP2): in all of these cases, there are both intensive and extensive margin effects, and the former dominate for L, while the latter dominate for R.

Proposition 9 shows that increasing politicians' ability to discipline bureaucrats can be costly for output when policy-motivated bureaucrats can respond not just through effort, but also through their entry decisions. This has some interesting implications for choices regarding managerial control over bureaucrats. For example, because κ always raises output ex post but has negative implications ex ante, both parties may have an incentive to commit to a low κ (giving up the possibility to discipline) by agreeing to institutional constraints.

We also see that parties with different electoral prospects may have opposing preferences regarding discipline, with either party L or R sometimes benefitting from a higher κ. Recent empirical work has explored the implications of political competition on politicians' incentives to discipline bureaucrats (Rogger, 2014; Nath, 2016). Our results highlight that these incentives can also both depend on and shape the ideological composition of the bureaucracy.

Finally, Proposition 9 illustrates how incentives provided to public sector workers can generate different outcomes than incentives in the private sector (François, 2000; Dixit, 2002; Glazer, 2004; Delfgaauw and Dur, 2008). In our case, the ability to shirk provides partisan bureaucrats with some insurance against ideological variation in the policies they are tasked with implementing. Stronger work incentives can lead such bureaucrats to select out of government employment, and thereby lower expected output. This suggests a novel role for the low-powered incentives typically observed in the public sector.

7.5 Multiple agencies

There is well-documented ideological heterogeneity across agencies in the US government (Clinton and Lewis, 2008; Clinton et al., 2012), and observers have noted that the match between agency ideology and the ideology of the party in power is likely to matter for government productivity. As noted by Maranto (1993), "we can expect future conservative administrations to continue to have difficult relations with liberal bureaucracies; future liberal administrations will almost certainly have difficult relations with mainly conservative organizations." (p.697).

Our model can be readily used to think about agency performance as a function of the match between agency ideology and policies. A simple approach is to fix election probabilities (as in Sections 4 and 5), and view our model as that of an agency. This has the immediate implication that conflict with the executive is more likely in agencies responsible for implementing more polarized policies. For example, conflict may be less likely with Highway Safety Administration bureaucrats than with Immigration and Customs Enforcement officials. It is also immediate that more left-wing agencies will have higher productivity, and less conflict, under left-wing administrations, while the opposite is true for more right-wing agencies.

For some applications, it may be interesting to explicitly allow for a government with multiple agencies. One of these is evaluating personnel practices that determine the allocation of bureaucrats to agencies. Inter-agency mobility is rare in Scandinavian countries and in the U.S., but more common in the U.K. and in France (Peters, 2002, p224).[22] Bureaucrats in the Indian Administrative Service are allocated randomly to work in different Indian states (Bertrand et al., 2020).

To allow for multiple agencies, suppose that the government consists of N agencies. Assume that the policy x chosen by the party in power applies to all agencies. This can be

[22] Attempts have been made in the US to promote cross-agency mobility of higher-level civil servants. One example is the creation of the Senior Executive Service by the Civil Service Reform Act of 1978, which apparently failed to fulfill this goal (Harper, 1993).

interpreted as the intensity of the ideological direction given to all agencies by the government, which is then mapped into each agency's specific policy domain. Citizens who apply for jobs in the bureaucracy are then sorted into its different agencies. To model this in reduced form, given any measurable set $B \subseteq [-I/2, I/2]$ of entering bureaucrat ideologies, let $\mu(B)$ denote the measure of B. An *agency sorting rule* assigns measures $\nu_1^B, ..., \nu_N^B$ to all such B, which satisfy the adding-up constraint $\sum_{i=1}^N \nu_i^B(B') = \mu(B')$ for all measurable $B' \subseteq B$. Notice that while the distribution of ideologies in the bureaucracy B is uniform by assumption, the distribution of ideologies in agency i generated by ν_i^B need not be. Analogously to our main model, assume that output in agency i is obtained by aggregating the effort decisions of bureaucrats in the agency: $Q_i^B(x) = \int_B q_b(x) d\nu_i(b)$.

Because the ideological slant of parties' policies is applied to all agencies, a citizen's decision to enter the bureaucracy is independent of the agency sorting rule. However, this rule determines the distribution of productivity across agencies, as well as the relationship of an agency's productivity to the identity of the governing party. For example, to capture the case in which entering bureaucrats in B are distributed uniformly across agencies, we let $N\nu_i^B(B') = \mu(B')$ for all agencies i and all $B' \subseteq B$. In this case, the distribution of bureaucrats' ideologies in any agency is identical to the ideological distribution in the bureaucracy as a whole. Measuring agency productivity by output per capita, then in this case all agencies are equally productive. As another example, consider a sorting rule in which agencies with lower indexes tend to attract more left-wing bureaucrats.[23] In this case, because parties' policies are such that $x_L \leq x_R$, agencies' productivities are also ranked: agencies with lower indexes are more productive when party L is in power and less productive when party R is in power.

While beyond the scope of this paper, our framework could be used to study the impacts of different agency sorting rules on government performance, which would depend on how agency outputs $Q_1^B(x), ..., Q_N^B(x)$ are aggregated into government output $Q^B(x)$.

8 Conclusion

In most countries, public sector organizations employ a sizable proportion of the labor force. Since politicians use bureaucracies to achieve contested policy ends, it is natural to ask how bureaucrats select into public service, when they are motivated to exert effort, and finally how policy platforms anticipate these choices. The "citizen bureaucrats" in our model provide

[23] Formally, we can order the measures $\nu_1^B, ..., \nu_N^B$ by an analogue of first-order stochastic dominance: $\frac{\nu_i^B([-I/2,b])}{\nu_i^B(B)} \geq \frac{\nu_j^B([-I/2,b])}{\nu_j^B(B)}$ for all agencies $i < j$ and all ideologies $b \in B$. In words, given any ideology $b \in B$, a greater proportion of bureaucrats in agency i are to the left of b than in agency j.

the crucial link between the policies promised by parties and their implementation.

Our theory of equilibrium administration is centered around endogenous entry into government service by ideologically heterogeneous citizens. Citizens have some public service motivation and trade off the policies they expect to execute with private sector wages. In turn, parties trade off policies that they like with their implementation by motivated bureaucrats. In equilibrium, parties' winning probabilities reflect voters' valuation of their policies *and* their implementation. To our knowledge, this combination of endogenous bureaucracy, policy making, and elections is unique in the literature.

Our model explains the emergence of full bureaucratic neutrality, full partisanship, or the partially partisan cases in between, as equilibrium outcomes. We believe this provides a useful perspective for understanding how different types of bureaucracies might develop across modern democracies, or across different agencies within the same country. Bureaucratic partisanship is more likely when political polarization is high and elections are less competitive. In equilibrium, partisan bureaucracies have shirking bureaucrats, lower levels of government productivity, and political output cycles. We have cited various pieces of evidence consistent with such effects, but there is clearly a scope for empirical work to test some of these results further.

The model also yields a number of implications regarding public sector personnel policies. Reducing political appointments and increasing seniority benefits can lower partisanship in the permanent civil service. Policies to reduce shirking have a similar effect, but these can have unintended negative consequences for output. Wage policies have nonmonotonic effects that are conditional on political polarization and bureaucrats' PSM. Finally, recruitment policies that increase PSM may have unintended consequences by allowing parties to choose more extreme policy platforms.

By endogenizing the bureaucratic side of policy making, the model suggests numerous paths for future work. First, we consider only a rudimentary agency problem between politicians and bureaucrats. The "technology" of policy making could be enriched by including more subtle interaction between politicians, career bureaucrats, and, as in one of our extensions, political appointees. Our results suggest that features of this technology may be important for understanding who becomes a bureaucrat and how they perform on the job. Second, the electoral side of our model could be expanded to include more politics, different electoral institutions, and a variety of voter behaviors. Again, our approach suggests that these details may be relevant in determining bureaucrats' career choices. Finally, it would be interesting to include other sources of heterogeneity between citizens (such as private sector productivity). Relatively little is known about how characteristics of the people who choose to work in government shape the policy agendas of elected politicians, and one contribution of our paper is to offer a framework that is well-suited for studying this question further.

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A Appendix

A.1 Proofs of results stated in the text

Proof of Proposition 1. (P) Suppose that the bureaucracy is fully partisan, as in the bottom panel of Figure 2. It follows that those bureaucrats who apply to work for party L only must be those with ideologies in the set $\{b : U_b(\chi, L) \geq w\} = [x_L - (\phi - {}^{w}/_{p_L}), x_L + (\phi - {}^{w}/_{p_L})]$. Similarly, those bureaucrats who apply to work for party R only must be those with ideologies in the set $\{b : U_b(\chi, R) \geq w\} = [x_R - (\phi - {}^{w}/_{p_R}), x_R + (\phi - {}^{w}/_{p_R})]$. It must be that the most right wing bureaucrat entering to work for party L only is not willing to work for party R, or that $x_L + [\phi - {}^{w}/_{p_L}] < x_R - \phi$. This reduces to $x_R - x_L > 2\phi - {}^{w}/_{p_L}$. Similarly, it must be that the most left wing bureaucrat entering to work for party R is not willing to work for party L, or that $x_R - (\phi - {}^{w}/_{p_R}) > x_L + \phi$. This reduces to $x_R - x_L > 2\phi - {}^{w}/_{p_R}$. Because $p_L \geq p_R$, the bureaucracy is fully partisan if and only if $x_R - x_L > 2\phi - {}^{w}/_{p_L}$. Bureaucrats willing to work for party $P = L, R$ are those with $b \in [x_P - \phi + \frac{w}{p_P}, x_P + \phi - \frac{w}{p_P}]$, which yields the Q's given in the proposition.

(N). Suppose that the bureaucracy is fully neutral, as on the top panel of Figure 2. It follows that the set of applying bureaucrats must be those with ideologies in $\{b : U_b(\chi, LR) \geq w\} = [\underline{b}, \bar{b}]$. Under the assumption that $\phi > {}^{w}/_{p_R} \geq {}^{w}/_{p_L}$, it follows that $U_{x_L}(\chi, L) > w$ and $U_{x_R}(\chi, R) > w$, so that we must have that $\underline{b} < x_L \leq x_R < \bar{b}$. Using the expression for $U_b(\chi, LR)$, we then have that $\underline{b} = p_L x_L + p_R x_R - (\phi - w)$ and $\bar{b} = p_L x_L + p_R x_R + (\phi - w)$. It must be that the most left wing bureaucrat entering to work for both parties would not benefit from entering to work for party L only, or that $p_L x_L + p_R x_R - (\phi - w) \leq x_L - (\phi - {}^{w}/_{p_L})$. This reduces to $x_R - x_L \leq {}^{w}/_{p_L}$. Similarly, it must be that the most right wing bureaucrat entering to work for both parties would not benefit from entering to work for party R only, or that $p_L x_L + p_R x_R + (\phi - w) \geq x_R + (\phi - {}^{w}/_{p_R})$. This reduces to $x_R - x_L \leq {}^{w}/_{p_R}$. Because $p_L \geq p_R$, the bureaucracy is fully neutral if and only if $x_R - x_L \leq {}^{w}/_{p_L}$. All bureaucrats $b \in B(\chi)$ work for both parties.

(PP) Finally, suppose that the bureaucracy is partially partisan. First, it must be the case that some citizens join the applicant pool to produce for one party only. The most left wing citizen willing to enter to produce for party L only is not willing to produce for party R if $x_L - (\phi - {}^{w}/_{p_L}) < x_R - \phi$, which reduces to $x_R - x_L > {}^{w}/_{p_L}$. Similarly, the most right wing citizen willing to enter to produce for party R only is not willing to produce for party L if $x_R + (\phi - {}^{w}/_{p_R}) > x_L + \phi$, which reduces to $x_R - x_L > {}^{w}/_{p_R}$. Second, it must be the case that some citizens join the applicant pool to produce for both parties. The most right wing citizen willing to enter to produce for party L only is willing to produce for party R if $x_L + (\phi - {}^{w}/_{p_L}) \geq x_R - \phi$, which reduces to $x_R - x_L \leq 2\phi - {}^{w}/_{p_L}$. Similarly, the most left

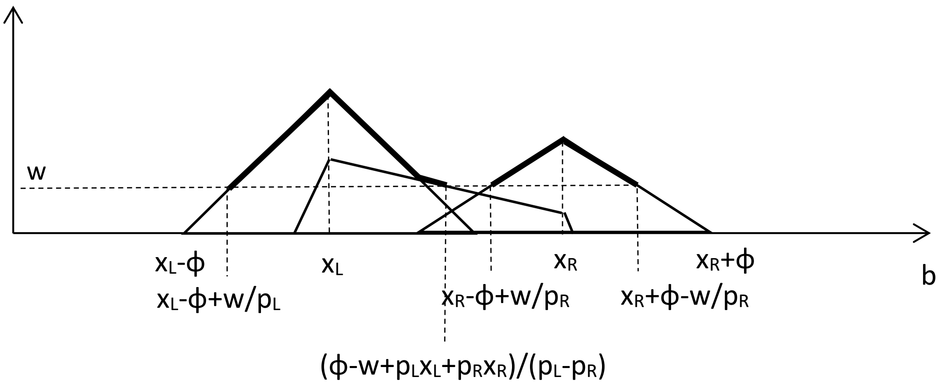

Figure 4: PP1. Both parties have partisan bureaucrats, and neutral bureaucrats are closer to the electorally favored party.

wing citizen willing to enter to produce for party R only is willing to produce for party L if $x_R - (\phi - {}^w/_{p_R}) \leq x_L + \phi$, which reduces to $x_R - x_R \leq 2\phi - {}^w/_{p_R}$.

Collecting the results from the previous paragraph, it follows that the bureaucracy is partially partisan if and only if ${}^w/_{p_L} < x_R - x_L \leq 2\phi - {}^w/_{p_L}$. There are three subcases. PP3. First, if ${}^w/_{p_L} < x_R - x_L \leq {}^w/_{p_R}$, then the bureaucracy is $B(\chi) = [x_L - [\phi - {}^w/_{p_L}], p_L x_L + p_R x_R + (\phi - w)]$, and only party L has partisan bureaucrats. This case is illustrated in Figure 3. Some neutral bureaucrats to the right of party R do not enter. Bureaucrats willing to work for party L are those with $b \in [x_L - \phi + \frac{w}{p_L}, p_L x_L + p_R x_R + \phi - w]$, and bureaucrats willing to work for R are those with $b \in [x_R - \phi, p_L x_L + p_R x_R + \phi - w]$.

PP2. Second, if ${}^w/_{p_R} \leq x_R - x_L \leq 2\phi - {}^w/_{p_R}$, then the bureaucracy is $B(\chi) = [x_L - (\phi - {}^w/_{p_L}), x_R + (\phi - {}^w/_{p_R})]$. This case is illustrated on the middle panel of Figure 2. Both parties have partisan bureaucrats, and all neutral bureaucrats enter the public sector. Bureaucrats willing to work for party L are those with $b \in [x_L - \phi + \frac{w}{p_L}, x_L + \phi]$, and bureaucrats willing to work for R are those with $b \in [x_R - \phi, x_R + \phi - \frac{w}{p_R}]$.

PP1. Third, if $2\phi - {}^w/_{p_R} < x_R - x_L \leq 2\phi - {}^w/_{p_L}$, then the bureaucracy is $B(\chi) = [x_L - (\phi - {}^w/_{p_L}), {}^1/_{p_L - p_R}(p_L x_L - p_R x_R + \phi - w)] \cup [x_R - (\phi - {}^w/_{p_R}), x_R + (\phi - {}^w/_{p_R})]$ and it is not convex. This case is illustrated on Figure 4. In this case, both parties have partisan bureaucrats, and there are neutral bureaucrats in the middle. However, some neutral bureaucrats closer to R prefer to stay out of the public sector. Bureaucrats willing to work for party L are those with $b \in [x_L - \phi + \frac{w}{p_L}, \frac{p_L x_L - p_R x_R + \phi - w}{p_L - p_R}]$. Bureaucrats willing to work for party R are those with $b \in \left[x_R - \phi, \frac{p_L x_L - p_R x_R + \phi - w}{p_L - p_R}\right] \cup [x_R - \phi + \frac{w}{p_R}, x_R + \phi - \frac{w}{p_R}]$.

■

We compute government output, either expected or ex post, under the cases covered by

Proposition 1. For ease of reference we group these expressions in the following corollary.

Corollary 5 *Government output is as follows:*
In case (P), $Q_L = {}^2/_I(\phi - \frac{w}{p_L})$, $Q_R = {}^2/_I(\phi - \frac{w}{p_R})$ *and* $Q = {}^2/_I(\phi - 2w)$.
In case (PP1), $Q_L = {}^1/_I\left(\frac{p_L x_L - p_R x_R + \phi - w}{p_L - p_R} + \phi - {}^w/_{p_L} - x_L\right)$, $Q_R = {}^1/_I\left(\frac{p_L x_L - p_R x_R + \phi - w}{p_L - p_R} + 3\phi - 2{}^w/_{p_R} - x_R\right)$ *and* $Q = {}^2/_I\left(\phi - 2w + \frac{p_L p_R}{p_L - p_R}\left(2\phi - {}^w/_{p_L} - (x_R - x_L)\right)\right)$.
In case (PP2), $Q_L = {}^1/_I(2\phi - \frac{w}{p_L})$, $Q_R = {}^1/_I(2\phi - \frac{w}{p_R})$ *and* $Q = {}^2/_I\,(\phi - w)$.
In case (PP3), $Q_L = {}^1/_I\,(2\phi - w - {}^w/_{p_L} + p_R(x_R - x_L))$, $Q_R = {}^1/_I\,(2\phi - w - p_L(x_R - x_L))$ *and* $Q = {}^2/_I\,(\phi - w)$.
In case (N), $Q_L = Q_R = Q = {}^2/_I\,(\phi - w)$.

Proof of Corollary 1. From Corollary 5, expected government production is

$$Q^{B(\chi)}(\chi) = \begin{cases} {}^2/_I\,[\phi - 2w] & \text{if } \Delta x > 2\phi - {}^w/_{p_L}, \\ {}^2/_I\left[\phi - 2w + \frac{p_L p_R}{p_L - p_R}\left[2\phi - {}^w/_{p_L} - \Delta x\right]\right] & \text{if } 2\phi - {}^w/_{p_R} < \Delta x \le 2\phi - {}^w/_{p_L}, \\ {}^2/_I\,[\phi - w] & \text{if } \Delta x \le 2\phi - {}^w/_{p_R}. \end{cases} \tag{8}$$

It is easily verified that the expressions for $Q = Q^{B(\chi)}(\chi)$ in (8) are in increasing order: fully partisan bureaucracies yield the lowest expected output, partially partisan bureaucracies yield weakly more, and fully neutral bureaucracies yield weakly more than partially partisan bureaucracies. The results regarding the effects of ϕ, w, and Δx on Q are immediate.

From Proposition 1, the size of the bureaucracy is given by

$$|B(\chi)| = \begin{cases} {}^2/_I\left[2\phi - \frac{w}{p_L p_R}\right] & \text{if } \Delta x > 2\phi - {}^w/_{p_L}, \\ {}^2/_I\left[2\phi - \frac{w}{p_L p_R}\right] + \frac{1}{I}\frac{p_R}{p_L - p_R}\left[2\phi - {}^w/_{p_L} - \Delta x\right] & \text{if } 2\phi - {}^w/_{p_R} < \Delta x \le 2\phi - {}^w/_{p_L}, \\ {}^1/_I\left[\Delta x + 2\phi - \frac{w}{p_L p_R}\right] & \text{if } {}^w/_{p_R} < \Delta x \le 2\phi - {}^w/_{p_R}, \\ {}^1/_I\left[\Delta x + 2\phi - \frac{w}{p_L p_R}\right] + {}^{p_L}/_I\left[{}^w/_{p_R} - \Delta x\right] & \text{if } {}^w/_{p_L} < \Delta x \le {}^w/_{p_R}, \\ {}^2/_I[\phi - w] & \text{if } \Delta x \le {}^w/_{p_L}. \end{cases} \tag{9}$$

With some algebra, it is easily verified that $|B(\chi)|$ weakly increases in Δx for $\Delta x < 2\phi - {}^w/_{p_R}$ and decreases for $\Delta x > 2\phi - {}^w/_{p_R}$.

If the bureaucracy is fully neutral ($\Delta x \le {}^w/_{p_L}$), then per-capita production $\frac{Q}{|B(\chi)|}$ is 1. For ${}^w/_{p_L} < \Delta x \le 2\phi - {}^w/_{p_R}$, production remains constant while the bureaucracy expands, so per-capita production falls below 1. When $2\phi - {}^w/_{p_R} < \Delta x \le 2\phi - {}^w/_{p_L}$ both production and the bureaucracy are decreasing in polarization. We can write per-capita production as

$$\frac{Q}{|B(\chi)|} = \frac{a - b\Delta x}{c - d\Delta x},$$

where $b = \frac{p_L p_R}{p_L - p_R}$ and $d = \frac{p_R}{2(p_L - p_R)}$. Therefore, it can be computed that $\frac{\partial}{\partial(\Delta x)} \frac{Q}{|B(\chi)|} \leq 0$ as long as $\frac{Q}{|B(\chi)|} \leq \frac{b}{d} = 2p_L$. However, because per-capita production is no greater than 1 and $p_L \geq {}^1\!/_2$, this is always satisfied. Thus, in this range, per-capita production is lowest when $\Delta x = 2\phi - {}^w\!/_{p_L}$.

Finally, if the bureaucracy is fully partisan ($\Delta x \geq 2\phi - {}^w\!/_{p_L}$), then per-capita production is

$$\frac{Q}{|B(\chi)|} = \frac{\phi - 2w}{2\phi - {}^w\!/_{p_L p_R}},$$

which is independent of party platforms. ■

Proof of Corollary 2. From Corollary 5, we have

$$\Delta Q = Q^{B(\chi)}(x_L) - Q^{B(\chi)}(x_R) = \begin{cases} 0 & \text{if } \Delta x < {}^w\!/_{p_L} \\ \Delta x - {}^w\!/_{p_L} & \text{if } {}^w\!/_{p_L} < \Delta x < {}^w\!/_{p_R} \\ {}^w\!/_{p_R} - {}^w\!/_{p_L} & \text{if } {}^w\!/_{p_R} < \Delta x < 2\phi - {}^w\!/_{p_R} \\ \Delta x - 2\phi - {}^w\!/_{p_L} + 2{}^w\!/_{p_R} & \text{if } 2\phi - {}^w\!/_{p_R} < \Delta x < 2\phi - {}^w\!/_{p_L} \\ 2({}^w\!/_{p_R} - {}^w\!/_{p_L}) & \text{if } 2\phi - {}^w\!/_{p_L} < \Delta x, \end{cases}$$

which immediately implies the statements in the corollary. ■

Proof of Corollary 3. (i) From Proposition 1, the condition for full neutrality is $\Delta x \leq {}^w\!/_{p_L}$, and this is relaxed when p_L goes down. (ii) Expected government production is given in (8). For $\Delta x < 2(\phi - w)$, a decrease in p_L can lead to a transition from the second case ($\Delta x \leq 2\phi - {}^w\!/_{p_R}$) to the third case ($2\phi - {}^w\!/_{p_R} < \Delta x \leq 2\phi - {}^w\!/_{p_L}$), which we already showed increases output. To establish that $\frac{\partial Q}{\partial p_L} \leq 0$ for all $\Delta x \leq 2(\phi - w)$ in the second case, the derivative can be shown to equal $\frac{1}{(2p-1)^2}(-2p^2 + 2p - 1 + \frac{w}{2\phi - \Delta x})$. This is negative for all p if and only if $\frac{w}{2\phi - \Delta x} \leq \frac{1}{2}$ which yields the stated condition. For $\Delta x > 2(\phi - w)$, $\frac{\partial Q(\chi,w)}{\partial p_L}$ has an inverse U-shape maximized at $p = {}^1\!/_2[1 + (2\frac{w}{2\phi - \Delta x} - 1)^{1/2}]$. ■

Proof of Proposition 2. Suppose the optimal policy of party L is $x_L = \text{argmax}_{x \in [-1,1]} Q^B(x)(\alpha - 1 - x)$. First, note that it cannot be that $x_L < \max\{\underline{b} - \phi, -1\}$. If we have that $\underline{b} - \phi \leq -1$, then such a policy is not feasible. If instead $\underline{b} - \phi > -1$, then $Q^B(x_L) = 0$. Because there exists $x'_L > x_L$ with $Q^B(x'_L) > 0$ and, by assumption, we have that $\alpha - 1 - x > 0$ for any feasible policy x, party L strictly prefers setting policy x'_L, contradicting the optimality of x_L.

Second, note that it cannot be that $x_L > \max\{\bar{b} - \phi, -1\}$ (notice that, by assumption, $\bar{b} - \phi < 1$, so that some such policy is always feasible). If this was the case, then we would have that $x_L + \phi > \bar{b}$. Therefore, we would have that $Q^B(x_L^*) \leq Q^B(\max\{\bar{b} - \phi, -1\})$, so that party L could choose a policy closer to its ideal without sacrificing output. This would contradict the optimality of x_L.

Third, note that it cannot be that $x_L > \max\{\underline{b} + \phi, -1\}$. If this was the case, then we would have that $x_L - \phi > \underline{b}$. Therefore, we would have that $Q^B(x_L) \leq Q^B(\max\{\underline{b} + \phi, -1\})$, contradicting the optimality of x_L. Also, recall that, by assumption, $\underline{b} + \phi > -1$, so that this step implies that $x_L \leq \underline{b} + \phi$.

Therefore, it follows that $\max\{\underline{b} - \phi, -1\} \leq x_L^* \leq \min\{\max\{\bar{b} - \phi, -1\}, \underline{b} + \phi\}$, and for any such policy we have that $Q^B(x_L) = x_L + \phi - \underline{b}$. Notice that if $\bar{b} - \underline{b} \leq 2\phi$, then $\max\{\bar{b} - \phi, -1\} \leq \max\{\underline{b} + \phi, -1\} = \underline{b} + \phi$, while if $\bar{b} - \underline{b} > 2\phi$, then $\max\{\bar{b} - \phi, -1\} \geq \underline{b} + \phi$. Correspondingly, if $\bar{b} - \underline{b} \leq 2\phi$, then x_L is a solution to

$$\max_{x \in [\max\{\underline{b} - \phi, -1\}, \max\{\bar{b} - \phi, -1\}]} (x + \phi - \underline{b})(\alpha - 1 - x),$$

whereas if $\bar{b} - \underline{b} > 2\phi$, then x_L is a solution to

$$\max_{x \in [\max\{\underline{b} - \phi, -1\}, \underline{b} + \phi]} (x + \phi - \underline{b})(\alpha - 1 - x).$$

These problems share the same objective function, which is strictly concave and has an unconstrained maximizer at $\hat{x} = {}^1\!/_2[\alpha - 1 + \underline{b} - \phi]$. From this the expressions for optimal policies in (6)and (7) follow. The only thing to note is that if $\hat{x} < \max\{\underline{b} - \phi, -1\}$, then it must be the case that $\underline{b} - \phi < -1$, and hence that $x_L^* = -1$. To see this, note that if $\hat{x} < \max\{\underline{b} - \phi, -1\} = \underline{b} - \phi$, then it follows that $\alpha - 1 - (\underline{b} - \phi) < 0$. But, because $\underline{b} - \phi \geq -1$ is a feasible policy, this contradicts the assumption that $\alpha > 2$. ■

Proof of Corollary 4. Immediate from the expressions for optimal policies in Proposition 2. ■

Proof of Proposition 3. *Step 0.* First we verify that, given any equilibrium policies (x_L^*, x_R^*) and any equilibrium bureaucracy B^*, there exists a cutoff voter type $i_L^* \in [-{}^I\!/_2, {}^I\!/_2]$, such that voter with ideology i supports party L only if $i \leq i_L^*$. The only reason this property may be in doubt is that parties differ in policy quality, which complements policy ideology for voters. Note that the fraction of voters that prefer party L to party R are those voters with ideology i such that

$$\frac{Q^{B^*}(x_L^*)}{Q^{B^*}(x_R^*)} \geq \frac{\alpha - |x_R^* - i|}{\alpha - |x_L^* - i|}. \tag{10}$$

Assume that $x_L^* < x_R^*$, as this will be true of all equilibrium policies. It can be verified that the right-hand side of the inequality in (10)is strictly increasing in i (this can be verified by taking derivatives separately for the three cases in which $i < x_L^*$, $x_L^* < i < x_R^*$ and $i > x_R^*$, respectively). This establishes the existence of marginal supporter i_L^* for party L, Correspondingly, the equilibrium mass of supporters of party L is $p_L^* = {}^1\!/_I(i_L^* + {}^I\!/_2)$.

Step 1. Returning to the proof of Proposition 3, consider an equilibrium in which the bureaucracy is fully partisan. From Proposition 1, we have that $x_R^* - x_L^* > 2\phi - {}^w\!/_{p_L^*}$ and $B^* = B(\chi^*) = [x_L^* - [\phi - {}^w\!/_{p_L^*}], x_L^* + [\phi - {}^w\!/_{p_L^*}]] \cup [x_R^* - [\phi - {}^w\!/_{p_R^*}], x_R^* + [\phi - {}^w\!/_{p_R^*}]]$. The bureaucracy is ideologically disconnected; the left-wing section works for party L only, and the right-wing section works for party R only.

We first show that we must have $x_L^* = -1$. To see this, let $[\underline{b}_L^*, \overline{b}_L^*]$ denote the left wing section of the equilibrium bureaucracy B^*, and note that $\overline{b}_L^* - \underline{b}_L^* < 2\phi$ because $w, p_L^* > 0$. From Proposition 10, the policy x_L^* must satisfy (6). If $x_L^* > -1$, then because $x_L^* \leq \max\{\overline{b}_L^* - \phi, -1\}$ by (6), we must have that $x_L^* \leq \overline{b}_L^* - \phi$. When combined with the fact that $x_L^* = {}^1\!/_2[\overline{b}_L^* + \underline{b}_L^*]$, we obtain that $\overline{b}_L^* - \underline{b}_L^* \geq 2\phi$, a contradiction. The symmetric argument applies to party R, so that we must have $x_R^* = 1$.

Step 2. Next, given any $p_L \geq {}^1\!/_2$, we can evaluate the preferences over parties of the marginal voter $i_L = I[p_L - {}^1\!/_2]$ implied by that winning probability, taking as given the facts that $x_L^* = -1$ and $x_R^* = 1$. The marginal voter i_L prefers party L if

$$\begin{aligned} Q^{B^*}(x_L^*)[\alpha - |x_L^* - i_L|] &= {}^2\!/_I[\phi - {}^w\!/_{p_L}][\alpha - (i_L + 1)] \\ &\geq {}^2\!/_I[\phi - {}^w\!/_{p_R}][\alpha - (1 - i_L)] \\ &= Q^{B^*}(x_R^*)[\alpha - |x_R^* - i_L|], \end{aligned}$$

where the expressions for Q^B follow from Corollary 5. This reduces to

$$\frac{4\phi(p_L - {}^1\!/_2)}{p_L(1 - p_L)} \left[\frac{w}{\phi I}(\alpha - 1 + {}^I\!/_2) - p_L(1 - p_L)\right] \geq 0. \tag{11}$$

If winning probability p_L^* is part of an equilibrium, then (11)must hold as an equality when evaluated at p_L^*. Therefore, there are two candidates for equilibrium, $p_L^* = {}^1\!/_2$ or $p_L^* > {}^1\!/_2$ such that $p_L^*(1 - p_L^*) = \frac{w}{\phi I}(\alpha - 1 + {}^I\!/_2)$. This second candidate is only well-defined if $\frac{w}{\phi I}(\alpha - 1 + {}^I\!/_2) < {}^1\!/_4$.

Step 3. We show that whenever the equilibrium with $p_L^* > {}^1\!/_2$ exists, then so does the equilibrium with $p_L^* = 1/2$. To see this, given any $0 < p_L < 1$, let $\underline{b_L}(p_L) = -1 - [\phi - w/p_L]$, $\overline{b}_L(p_L) = -1 + [\phi - w/p_L]$, $\underline{b_R}(p_L) = 1 - [\phi - w/1-p_L]$ and $\overline{b}_R(p_L) = 1 + [\phi - w/1-p_L]$. Given any measurable $B \subseteq I$, let $\mu(B)$ be the measure of B, and, given any policy x_L, let $U_L(x_L : p_L)$ denote the utility of party L when it chooses x_L facing the bureaucracy determined by p_L. Using the fact that the bureaucracy is fully partisan, we can write

$$U_L(x_L : p_L) = \left[\mu\left([\underline{b}_L, \overline{b_L}] \cap [x_L - \phi, x_L + \phi]\right) + \mu\left([\underline{b}_R, \overline{b_R}] \cap [x_L - \phi, x_L + \phi]\right)\right](\alpha - 1 - x_L),$$

which is continuous and, for fixed x_L, differentiable at almost every p_L. Given $x_L > -1$, at points of differentiability, we have that

$$\begin{aligned}
\frac{\partial}{\partial p_L}\left[U_L(-1 : p_L) - U_L(x_L : p_L)\right] &= \frac{2w}{Ip_L^2}\alpha - \frac{\partial}{\partial p_L}[\mu\left([\underline{b}_L, \overline{b}_L] \cap [x_L - \phi, x_L + \phi]\right) \\
&\quad + \mu\left([\underline{b}_R, \overline{b}_R] \cap [x_L - \phi, x_L + \phi]\right)][\alpha - 1 - x_L] \\
&\geq \frac{2w}{Ip_L^2}\alpha - \frac{\partial}{\partial p_L}\left[\mu\left([\underline{b}_L, \overline{b}_L] \cap [x_L - \phi, x_L + \phi]\right)\right][\alpha - 1 - x_L] \\
&\geq \frac{2w}{Ip_L^2}[1 + x_L] \\
&> 0,
\end{aligned}$$

where the first equality uses the fact that $[\underline{b}_L(p_L), \overline{b}_L(p_L)] \cap [-1-\phi, -1+\phi] = [\underline{b}_L(p_L), \overline{b}_L(p_L)]$, the first inequality follows because $\partial/\partial p_L \underline{b_R}(p_L) > 0$ and $\partial/\partial p_L \overline{b_R}(p_L) < 0$, the second inequality follows because $\partial/\partial p_L \underline{b_L}(p_L) < 0$ and $\partial/\partial p_L \overline{b_L}(p_L) > 0$, and the final inequality follows because $x_L > -1$. It follows that if party L prefers setting policy -1 to policy x_L under p_L, then it also prefers setting policy -1 to policy $x_L > -1$ when $p_L' > p_L$.

Now suppose that an equilibrium with $p_L^* > 1/2$ exists. It follows that party R prefers setting policy 1 to setting any policy $x_R < 1$ under p_L^*. By a symmetric argument to the one above, it follows that party R must prefer setting policy 1 to setting any policy x_R under $p_L = 1/2$. Because the parties are symmetric when $p_R = p_L = 1/2$ and they both choose their ideal policies, this also establishes that party L prefers setting policy -1 to setting any policy $x_L > -1$ under $p_L = 1/2$. Therefore, the equilibrium with $p_L^* = 1/2$ must exist, as desired.

Step 4. Based on Step 3, the conditions for existence of an equilibrium with $p_L^* = 1/2$ provide necessary and sufficient conditions for the existence of *any* fully partisan equilibria. To obtain these conditions, recall first that, from Proposition 1, partisan bureaucrats have incentives to enter only if $\phi \geq \min\{w/p_L^*, w/p_R^*\} = 2w$. In addition, from Step 1 above, the constraint that $x_R^* - x_L^* > 2\phi - w/p_L^*$ reduces to $\phi < 1 + w$. Thus, we must have $2w \leq \phi < 1 + w$.

It remains to be verified that parties have incentives to choose their ideal policies. Note that $p_L^* = 1/2$ implies that $\overline{b}_L^* - \underline{b}_L^* = \overline{b}_R^* - \underline{b}_R^*$. Suppose further that $\underline{b}_R^* - \underline{b}_L^* \geq 2\phi$. This corresponds to the case of a "large distance" between the two sections of the bureaucracy from Proposition 10, and that result shows that the optimal policy of party L is the same as the one identified by Proposition 2 (i.e., as though the right section of the bureaucracy was absent). Furthermore, from Step 1 we know that $\overline{b}_L^* - \underline{b}_L^* < 2\phi$, so that this corresponds to the case of a "small bureaucracy" from Proposition 2. In that result, if we substitute $\max\{\underline{b}_L^* - \phi, -1\} = \max\{\overline{b}_L^* - \phi, -1\} = -1$, as is the case here, then it follows that $x_L^* = -1$ is indeed the optimal policy. The condition $\underline{b}_R^* - \underline{b}_L^* \geq 2\phi$ reduces to $\phi \leq 1$.

Finally, suppose that $1 < \phi < 1 + w$ (so that $\underline{b}_R^* - \underline{b}_L^* < 2\phi$). It must be that party L does not have the incentive to choose policies more moderate than -1 in order to increase government production. The most moderate policy that party L can choose which attracts some right wing bureaucrats is $\underline{b}_R^* - \phi > -1$ and, by Proposition 10, the most extreme such policy that party L can choose is $\overline{b}_R^* - \phi$. For any $\underline{b}_R^* - \phi \leq x_L \leq \overline{b}_R^* - \phi$, party L's payoff is $Q^{B^*}(x_L)(\alpha - 1 - x_L) = 1/I \left[\overline{b}_L^* - \underline{b}_L^* + (x_L + \phi - \underline{b}_R^*)\right] (\alpha - 1 - x_L)$. It can be computed that

$$\frac{\partial}{\partial x_L} Q^{B^*}(x_L)(\alpha - 1 - x_L)|_{x_L = \underline{b}_R^* - \phi} = 1/I\, [\alpha - 2(1 - w)] > 0. \tag{12}$$

The inequality follows because $2w \leq \phi < 1 + w$ implies $w < 1$, and $\alpha > 2$ by assumption.

Notice that if $\phi \approx 1 + w$, then $\underline{b}_R^* - \phi \approx -1$. Thus, from (12), increasing x_L slightly above -1 will increase party L's payoff. Therefore, no equilibrium exists in this case. If, on the other hand, $\phi \approx 1$, then $\underline{b}_R^* - \phi \approx \underline{b}_L^* + \phi \approx -1 + 2w \gg -1$. In words, to attract bureaucrats from the right section, party L must choose a policy strictly more moderate than -1. However, $Q^{B^*}(x_L) \approx Q^{B^*}(-1)$ for any policy $\underline{b}_R^* - \phi \leq x_L \leq \underline{b}_L^* + \phi$. In words, by moderating, party L can attract very few bureaucrats from the right section without starting to lose bureaucrats from the left section. In this case a deviation to a moderate policy cannot be beneficial for party L, therefore, the equilibrium exists.

Therefore, there exists $1 < \tilde{\phi} < 1 + w$ such that the equilibrium exists if and only if $\phi \leq \tilde{\phi}$. (For the case $\underline{b}_R^* - \underline{b}_L^* \geq 2\phi$, we saw that this holds for $\tilde{\phi} = 1$.) ■

Proof of Proposition 4. Consider an equilibrium in which the bureaucracy is fully neutral. From Proposition 1, we have that $x_R^* - x_L^* \leq w/p_L^*$ and $B^* = B(\chi^*) = [p_L^* x_L^* + p_R^* x_R^* - (\phi - w), p_L^* x_L^* + p_R^* x_R^* + (\phi - w)]$. It can be verified that $x_L^* - \phi < \underline{b}^*$. Therefore, party L can never induce more left wing bureaucrats to work by choosing more left wing policies, but it also does not lose left wing bureaucrats by choosing policies slightly more moderate than x_L^*. Notice that any such equilibrium requires that $\phi \geq w$.

Suppose that we have $x_L^* > -1$. From Proposition 2, we know that $x_L^* \leq \max\{\overline{b}^* - \phi, -1\}$, which reduces to $x_R^* - x_L^* \geq {}^{w}/_{1-p_L^*}$. It follows that $x_R^* - x_L^* \leq {}^{w}/_{p_L^*} \leq {}^{w}/_{1-p_L^*} \leq x_R^* - x_L^*$, which implies that $p_L^* = {}^1/_2$ and $x_R^* - x_L^* = 2w$. Because the bureaucracy is fully neutral, we have that $Q^{B^*}(x_L^*) = Q^{B^*}(x_R^*)$, so that combined with $p_L^* = {}^1/_2$ we must have that $x_L^* = -x_R^*$, and therefore that $x_L^* = -w > -1$. Finally, it must be the case that party L does not have incentives to choose more extreme policies, which would decrease output but improve its ideological payoff. By Proposition 2, this occurs if $x_L^* \leq \hat{x}$, or if $-w \leq {}^1/_2[\alpha - 1 + \underline{b}^* - \phi]$, which reduces to $\alpha - 1 - 2\phi + 3w \geq 0$.

Using the symmetry of the parties' ideal policies, the previous argument can be repeated to show that $x_R^* < 1$ implies $x_L^* = -x_R^* > -1$. Therefore, the only possibility left is an equilibrium with $x_L^* = -x_R^* = -1$. Because $Q^{B^*}(x_L^*) = Q^{B^*}(x_R^*)$ under a fully neutral bureaucracy, this again implies $p_L^* = {}^1/_2$. The condition that $x_R^* - x_L^* \leq {}^{w}/_{p_L^*}$ then reduces to $w \geq 1$. In this case, no deviations by parties to more extreme policies are feasible. ■

Proof of Proposition 5. *Step 1.* Consider an equilibrium in which the bureaucracy is partially partisan and in which both parties have partisan bureaucrats. From Proposition 1, we have that ${}^{w}/_{1-p_L^*} \leq x_R^* - x_L^* \leq 2\phi - {}^{w}/_{1-p_L^*}$ and $B^* = B(\chi^*) = [x_L^* - (\phi - {}^{w}/_{p_L^*}), x_R^* + (\phi - {}^{w}/_{1-p_L^*})]$. It can be verified that $x_L^* - \phi < \underline{b}^*$ and that $x_L^* > \overline{b}^* - \phi$, so that if party L chooses more moderate policies then all left wing bureaucrats still work and some right wing bureaucrats choose to start working. From Proposition 2, it follows that $x_L^* \geq \hat{x} = {}^1/_2[\alpha - 1 + \underline{b}^* - \phi]$, which reduces to $x_L^* \geq \alpha - 1 - 2\phi + {}^{w}/_{p_L^*}$. Furthermore, if $x_L^* > -1$, then party L's policy choice is interior and we must have that $x_L^* = \alpha - 1 - 2\phi + {}^{w}/_{p_L^*}$. For party R, Proposition 2 yields that $x_R^* \leq {}^1/_2[\overline{b}^* + \phi - \alpha + 1]$, which reduces to $x_R^* \leq 1 - \alpha + 2\phi - {}^{w}/_{1-p_L^*}$. Also, if $x_R^* < 1$, then we must have that $x_R^* = 1 - \alpha + 2\phi - {}^{w}/_{1-p_L^*}$.

Step 2. We show that $p_L^* \geq {}^1/_2$ implies that party R must choose weakly more moderate policies than party L in any equilibrium: i.e., that $x_L^* + 1 \leq 1 - x_R^*$. To see this, note that, from Step 1,

$$\begin{aligned} x_L^* + 1 &= \max\{\alpha - 1 - 2\phi + {}^{w}/_{p_L^*}, -1\} + 1 \\ &= \max\{\alpha - 2\phi + {}^{w}/_{p_L^*}, 0\}. \end{aligned}$$

Similarly, we have that

$$\begin{aligned} 1 - x_R^* &= 1 - \min\{1 - \alpha + 2\phi - {}^{w}/_{1-p_L^*}, 1\} \\ &= \max\{\alpha - 2\phi + {}^{w}/_{1-p_L^*}, 0\}, \end{aligned}$$

so that $p_L^* \geq {}^1/_2$ implies that $x_L^* + 1 \leq 1 - x_R^*$, as desired. In words, this says (using

Proposition 2) that a disadvantaged party has higher incentives to choose moderate policies in order to stimulate government production.

Step 3. Now, we show that if there exists an equilibrium with $p_L^* > 1/2$, then there must also exist an equilibrium for all $1/2 \leq p_L < p_L^*$. First, from Step 2, given any equilibrium with $p_L^* > 1/2$, we cannot have that $x_L^* = -1$ and $x_R^* = 1$. Therefore, we have that $x_R^* = 1 - \alpha + 2\phi - w/1-p_L^*$ and $x_L^* = \max\{\alpha - 1 - 2\phi + w/p_L^*, -1\}$. Now consider any $1/2 < p_L < p_L^*$. Let $x_R = 1 - \alpha + 2\phi - w/1-p_L$ and $x_L = \max\{\alpha - 1 - 2\phi + w/p_L, -1\}$, which would be the parties' optimal policies if the bureaucracy was partially partisan with party L having winning probability p_L. It follows that

$$\begin{aligned}
x_R - x_L &= \min\left\{2(1-\alpha) + 4\phi - \frac{w}{p_L(1-p_L)}, 2 - \alpha + 2\phi - \frac{w}{1-p_L}\right\} \\
&> \min\left\{2(1-\alpha) + 4\phi - \frac{w}{p_L^*(1-p_L^*)}, 2 - \alpha + 2\phi - \frac{w}{1-p_L^*}\right\} \\
&= x_R^* - x_L^* \\
&\geq \frac{w}{1-p_L^*} \\
&> \frac{w}{1-p_L}.
\end{aligned}$$

Similarly, we have that

$$\begin{aligned}
2\phi - \frac{w}{1-p_L} - [x_R - x_L] &= \alpha - 1 + \max\left\{\alpha - 1 - 2\phi + \frac{w}{p_L}, -1\right\} \\
&\geq \alpha - 1 + \max\left\{\alpha - 1 - 2\phi + \frac{w}{p_L^*}, -1\right\} \\
&= 2\phi - \frac{w}{1-p_L} - [x_R^* - x_L^*] \\
&\geq 0.
\end{aligned}$$

Therefore, probability p_L, policies (x_R, x_L) and bureaucracy $B = B(\chi)$ form an equilibrium with a partially partisan bureaucracy. Notice that, by taking limits and invoking continuity, there is also such an equilibrium with $p_L = 1/2$.

Step 4. Now we characterize equilibria with $p_L^* = 1/2$. This is case PP2 in Proposition 1, and it immediately implies that such an equilibrium can only exist if $2w \leq \phi$. Recall, that $Q^{B^*}(x_L^*) = 1/I[2\phi - w/p_L^*]$ and $Q^{B^*}(x_R^*) = 1/I[2\phi - w/1-p_L^*]$ for any equilibrium p_L^*. Therefore, if $p_L^* = 1/2$, we have that $Q^{B^*}(x_L^*) = Q^{B^*}(x_R^*)$, which in turn implies that $x_R^* = -x_L^*$. First, suppose that $-1 < x_L^* = \alpha - 1 - 2(\phi - w)$, or $\alpha \geq 2(\phi - w)$. From Proposition 1, the necessary and sufficient condition for the existence of a PP equilibrium is now $2w \leq 2(1-\alpha) + 4\phi - 4w \leq$

$2(\phi - w)$. This can be rewritten as ${}^1\!/\!_2(\alpha-1+3w) \le \phi \le \alpha - 1 + w$. Second, suppose instead that $\alpha < 2(\phi - w)$ and therefore $x_L^* = -1$. From Proposition 1, such an equilibrium exists if $2w \le 2 \le 2(\phi - w)$. Given $2w \le \phi$, this reduces to $w \le 1$. Because of Step 3, these conditions are necessary and sufficient for the existence of PP equilibria. ■

Proof of Proposition 6. Since ρ does not affect the optimal bureaucracy or bureaucrats' optimal production choices, Proposition 1 still holds. Optimal policies for party $P = L, R$ are now solutions to $\max_{x \in [-1,1]}(Q^B(x) + \rho)(\alpha - |x - b_P|)$. Proceeding exactly as in the proof of Proposition 2, it can be verified that the proposition holds as written, with the interior optimal policy $\hat{x}$ changed to $\hat{x}_\rho = {}^1\!/\!_2\left(\alpha - 1 + \underline{b} - \phi - \rho\right)$. Note that this policy becomes more extreme as ρ goes up, and this in turn expands the parameter ranges for a more extreme optimal policy $x_L(B)$ when ρ is higher.

Consider an administrative equilibrium with a fully neutral bureaucracy. Using the previous observations, one can proceed exactly as in the proof of Proposition 4 to establish the stated results. In particular, nothing changes when $x_L^* = -x_R^* = -1$. When $x_L^* > -1$, the condition for L not to have an incentive to choose more extreme policies becomes $-w \le \hat{x}_\rho$, or $\alpha - 1 - 2\phi + 3w \ge \rho$. ■

Proof of Proposition 7. Fix policy lottery χ with $p_L = {}^1\!/\!_2$. Suppose that there exists an equilibrium wage w^* such that the corresponding bureaucracy is fully partisan. Recall that (9)computes the size of optimal applicant pools, and substituting $p_L = p_R = {}^1\!/\!_2$ it follows that w^* must solve ${}^4\!/\!_I\,(\phi - 2w^*) = \overline{B}$, so that $w^* = {}^1\!/\!_2\left(\phi - {}^{\overline{B}I}\!/\!_4\right) > 0$, where the inequality follows from $\overline{B} < {}^{2\phi}\!/\!_I$. Furthermore, for the bureaucracy to be fully partisan, Proposition 1 requires that $\Delta x > 2\,(\phi - w^*)$, which reduces to $\Delta x > \phi + {}^{\overline{B}I}\!/\!_4$.

Now suppose that there exists an equilibrium wage w^* such that the corresponding bureaucracy is fully neutral. From (9), it follows that w^* must solve ${}^2\!/\!_I\,[\phi - w^*] = \overline{B}$, so that $w^* = \phi - {}^{\overline{B}I}\!/\!_2 > 0$, where the inequality follows from $\overline{B} < {}^{2\phi}\!/\!_I$. Furthermore, for the bureaucracy to be fully neutral, Proposition 1 requires that $\Delta x \le 2w^*$, which reduces to $\Delta x \le 2\left[\phi - {}^{\overline{B}I}\!/\!_2\right]$.

Now suppose that there exists an equilibrium wage w^* such that the corresponding bureaucracy is partially partisan. From (9), it follows that w^* must solve ${}^1\!/\!_I\,(\Delta x + 2(\phi - 2w^*)) = \overline{B}$, so that $w^* = {}^1\!/\!_2\left(\phi - {}^{\overline{B}I}\!/\!_2 + {}^{\Delta x}\!/\!_2\right) > 0$, where the inequality follows from $\Delta x \ge 0$ and $\overline{B} < {}^{2\phi}\!/\!_I$. Furthermore, for the bureaucracy to be fully neutral, Proposition 1 requires that $2w^* \le \Delta x \le 2(\phi - w^*)$, which reduces to the inequalities $2\left(\phi - {}^{\overline{B}I}\!/\!_2\right) \le \Delta x \le {}^2\!/\!_3\left(\phi + {}^{\overline{B}I}\!/\!_2\right)$. However, these inequalities are well-defined only if $2\left(\phi - {}^{\overline{B}I}\!/\!_2\right) \le {}^2\!/\!_3\left(\phi + {}^{\overline{B}I}\!/\!_2\right)$, which reduces to $\overline{B} \ge {}^{\phi}\!/\!_I$. ■

Proof of Proposition 8. When bureaucrats have the option to quit, optimal production decisions from (3), which are now interpreted as optimal decisions to remain a bureaucrat, are such that $q_b(x) = 1$ if and only if $|x - b| \leq \phi - (1 - \delta)w$. Notice that the expression for $U_b(\chi, LR)$, which is the payoff to a bureaucrat with ideology b of joining the bureaucracy and working for both parties, is the same as in our main model. On the other hand, the payoff to this bureaucrat of joining the bureaucracy to work for party L only is now $U_b(\chi, L) = p_L(\phi - |x_L - b|) + p_R(1 - \delta)w$ (and symmetrically for $U_b(\chi, R)$). Given these substitutions, the proof of Proposition 8 is identical to the proof of Proposition 1 for cases (P) and (N). ■

Proof of Proposition 9. Compared to Section 4, the production decision becomes $q_b(x) = 1$ iff and only if $|x - b| \leq \phi + \kappa$. The payoff to L-partisans is now $U_b(\chi, L) = p_L(\phi - |x_L - b|) - p_R\kappa$, the payoff to R-partisans's is $U_b(\chi, R) = p_R(\phi - |x_R - b|) - p_L\kappa$, while the payoff to neutrals is unchanged: $U_b(\chi, LR) = \phi - p_L|x_L - b| - p_R|x_R - b|$. The results for cases (N) and (P) are immediate. In particular, in case (P) we have $Q_L = {}^2\!/_I(\phi - \frac{w+p_R\kappa}{p_L})$ and $Q_R = {}^2\!/_I(\phi - \frac{w+p_L\kappa}{p_R})$, both of which decrease in κ.

In case (PP1), the leftmost bureaucrat to enter has $p_L(\phi + b - x_L) - p_R\kappa = w$ or $b = x_L - \phi + \frac{w+p_R\kappa}{p_L}$ while the rightmost bureaucrat to work for L, $\bar{b}_L$, is neutral and therefore unchanged. We have $Q_L = {}^1\!/_I[\bar{b}_L - (x_L - \phi + \frac{w+p_R\kappa}{p_L})]$ and $\frac{\partial Q_L}{\partial \kappa} < 0$. Party R now gains some neutral bureaucrats: neutral bureaucrats with ideology $b \in [x_R - \phi - \kappa, \bar{b}_L]$ will work for R. Party R also loses some partisan bureaucrats: only partisan bureaucrats in $[x_R - (\phi - \frac{w+p_L\kappa}{p_R}), x_R + (\phi - \frac{w+p_L\kappa}{p_R})]$ will work for R. In sum, $Q_R = {}^1\!/_I(\bar{b}_L - x_R + 3\phi + \kappa - \frac{w+p_L\kappa}{p_R})$, and ${}^{\partial Q_R}\!/_{\partial \kappa} < 0$ since $\frac{p_L}{p_R} > 1$.

In case (PP2), both parties gain neutrals and lose partisans. Bureaucrats working for L are in $[x_L - (\phi - \frac{w+p_R\kappa}{p_L}), x_L + \phi + \kappa]$, so that $Q_L = {}^1\!/_I(2\phi + \kappa - \frac{w+p_R\kappa}{p_L})$ and $\frac{\partial Q_L}{\partial \kappa} > 0$ because $\frac{p_R}{p_L} < 1$. Bureaucrats working for R are in $[x_R - \phi - \kappa, x_R - (\phi - \frac{w+p_L\kappa}{p_R})]$, so that $Q_R = {}^1\!/_I(2\phi + \kappa - \frac{w+p_L\kappa}{p_R})$ and $\frac{\partial Q_R}{\partial \kappa} < 0$. We also have $p_L\frac{\partial Q_L}{\partial \kappa} + p_R\frac{\partial Q_R}{\partial \kappa} = 0$, so that Q is unaffected by κ.

Finally, in case (PP3) party L loses partisans without any gains, while party R gains some neutral "converts" without experiencing any losses. Bureaucrats working for L are in $[x_L - (\phi - \frac{w+p_R\kappa}{p_L}), \bar{b}]$, where $\bar{b}$ is the rightmost (neutral) bureaucrat to enter. We have $Q_L = \frac{1}{I}[2\phi - w - \frac{w+p_R\kappa}{p_L} + p_R(x_R - x_L)]$ and $\frac{\partial Q_L}{\partial \kappa} < 0$. Bureaucrats working for R are in $[x_R - \phi - \kappa, \bar{b}]$, so $Q_R = {}^1\!/_I[2\phi - w + \kappa - p_L(x_R - x_L)]$ and $\frac{\partial Q_R}{\partial \kappa} > 0$. We again have $p_L\frac{\partial Q_L}{\partial \kappa} + p_R\frac{\partial Q_R}{\partial \kappa} = 0$. ■

A.2 Optimal policies with disconnected bureaucracies

Consider a bureaucracy that is ideologically disconnected, i.e., such that $B = [\underline{b}_L, \overline{b}_L] \cup [\underline{b}_R, \overline{b}_R]$, where $\overline{b}_L < \underline{b}_R$. Analogously to the case of an ideologically connected bureaucracy, we assume that the leftmost bureaucrat is willing to work for policy -1 and the rightmost bureaucrat is willing to work for policy 1: here, this implies $\underline{b}_L > -1-\phi$ and $\overline{b}_R < 1+\phi$. We can describe optimal policies for party L in this case through our results from Proposition 2. There are several cases, depending on (i) the relative size of the left and right sections of the bureaucracy ($[\underline{b}_L, \overline{b}_L]$ vs. $[\underline{b}_R, \overline{b}_R]$), and (ii) the distance between them.

Proposition 10 *Suppose that the bureaucracy is ideologically disconnected, and let* $\hat{y} = {}^1\!/_2 \left[\rho + \underline{b}_R - \phi - [\overline{b}_L - \underline{b}_L]\right]$. *Let* x_L^B *denote L's optimal policy if only bureaucrats from the left section of the bureaucracy work,* z_L^B *if only bureaucrats from the right section work, and* y_L^B *if bureaucrats from both sections work. Policy* x_L^B *is obtained from Proposition 2 by substituting* $\overline{b} = \overline{b}_L$ *and* $\underline{b} = \underline{b}_L$. *Policy* z_L^B *is obtained from Proposition 2 by substituting* $\overline{b} = \overline{b}_R$ *and* $\underline{b} = \underline{b}_R$.

- *Suppose the left section of the bureaucracy is larger than the right section* ($\overline{b}_L - \underline{b}_L \geq \overline{b}_R - \underline{b}_R$).
  - *If the distance between the two sections of the bureaucracy is small* ($\overline{b}_R - \underline{b}_L \leq 2\phi$), *then L's optimal policy is either* x_L^B, *or* y_L^B *which is obtained from* (6) *by substituting* $\overline{b} = \overline{b}_R$, $\underline{b} = \underline{b}_L$ *and* $\hat{x} = \hat{y}$.
  - *If the distance between the two sections of the bureaucracy is intermediate* ($\underline{b}_R - \underline{b}_L \leq 2\phi < \overline{b}_R - \underline{b}_L$), *then L's optimal policy is either* x_L^B, *or* y_L^B *which is obtained from* (7) *by substituting* $\overline{b} = \overline{b}_R$, $\underline{b} = \underline{b}_L$ *and* $\hat{x} = \hat{y}$.
  - *If the distance between the two sections of the bureaucracy is large* ($\underline{b}_R - \underline{b}_L \geq 2\phi$), *then L's optimal policy is* x_L^B.
- *Suppose the left section of the bureaucracy is smaller than the right section* ($\overline{b}_L - \underline{b}_L < \overline{b}_R - \underline{b}_R$), *then L's optimal policy is either* x_L^B, z_L^B, *or* y_L^B *which is obtained from Proposition 2 by substituting* $\overline{b} = \overline{b}_R$ *and* $\underline{b} = \underline{b}_R$.[24]

Proof of Proposition 10. There are three cases for an optimal policy $x_L(B)$: *(i)* it could be that only bureaucrats in $[\underline{b}_L, \overline{b}_L]$ work for party L, which occurs if $x_L(B) + \phi < \underline{b}_R$; *(ii)*

[24] Notice that Proposition 10 is restricted to describing the three possible candidates for optimal policies (x_L^B, y_L^B and z_L^B, respectively). Which of these policies is actually optimal would need to be verified through computing the payoffs of party L.

it could be that bureaucrats in both $[\underline{b}_L, \bar{b}_L]$ and $[\underline{b}_R, \bar{b}_R]$ work for party L, which occurs if $x_L(B) + \phi \geq \underline{b}_R$ and $x_L(B) - \phi \geq \bar{b}_L$; and finally (iii) it could be that only bureaucrats in $[\underline{b}_R, \bar{b}_R]$ work for party L, which occurs if $x_L(B) - \phi > \bar{b}_L$. In all three cases, strict concavity of the payoff of party L ensures that any such policy is unique.

If a policy x_L^B from case (i) is optimal, then this policy must correspond to the policy from Proposition 2 with the substitutions $\underline{b} = \underline{b}_L$ and $\bar{b} = \bar{b}_L$. Similarly, if a policy z_L^B from case (iii) is optimal, then this policy must correspond to the policy from Proposition 2 with the substitutions $\bar{b} = \bar{b}_R$ and $\underline{b} = \underline{b}_R$. However, note that policy z_L^B can never be optimal if there are at least as many left wing as right wing bureaucrats, i.e., if $\bar{b}_L - \underline{b}_L \geq \bar{b}_R - \underline{b}_R$. To see this, recall that, by assumption, $-1 < \underline{b}_L + \phi$, so that $Q^B(\underline{b}_L + \phi) = \min\{\bar{b}_L - \underline{b}_L, 2\phi\}$. Also, it must be the case that $\underline{b}_L + \phi < z_L^B$. Finally, because $Q^B(z_L^B) \leq \min\{\bar{b}_R - \underline{b}_R, 2\phi\} \leq Q^B(\underline{b}_L + \phi)$, we obtain a contradiction to the optimality of z_L^B, as desired.

Now suppose that a policy y_L^B from case (ii) is optimal. First, notice that if $\underline{b}_R - \underline{b}_L > 2\phi$, which can be rewritten as $\underline{b}_L + \phi \leq \underline{b}_R - \phi$, party L cannot choose a policy which induces right wing bureaucrats to work without inducing some left wing bureaucrats to shirk. Because $Q^B(x_L) = Q^B(\underline{b} + \phi)$ for any policy $\underline{b}_L + \phi < x_L \leq \bar{b}_L + \phi$, no such policy x_L can be optimal. Therefore, assume that $\underline{b}_R - \underline{b}_L \leq 2\phi$ in the following. Now, if $\bar{b}_R - \underline{b}_L \leq 2\phi$, then from arguments mirroring those of Proposition 2, any such solution must be such that $\max\{\bar{b}_R - \phi, -1\} \leq y_L^B \leq \max\{\bar{b}_R - \phi, -1\}$. Similarly, if $\bar{b}_R - \underline{b}_L > 2\phi$, any such solution must be such that $\max\{\underline{b}_R - \phi, -1\} \leq y_L^B \leq \underline{b}_L + \phi$. Correspondingly, if $\bar{b}_R - \underline{b}_L \leq 2\phi$, then y_L^B is a solution to

$$\max_{x_L \in [\max\{\underline{b}_R - \phi, -1\}, \max\{\bar{b}_R - \phi, -1\}]} [x_L + \phi - \underline{b}_R + [\bar{b}_L - \underline{b}_L]][\alpha - 1 - x_L],$$

whereas if $\bar{b}_R - \underline{b}_L > 2\phi$, then y_L^B is a solution to

$$\max_{x_L \in [\max\{\underline{b}_R - \phi, -1\}, \underline{b}_L + \phi]} [x_L + \phi - \underline{b}_R + [\bar{b}_L - \underline{b}_L]][\alpha - 1 - x_L].$$

These problems share the same objective function, which is strictly concave and has an unconstrained maximizer at $\hat{y} = 1/2 \left[\alpha - 1 + \underline{b}_R - \phi - [\bar{b}_L - \underline{b}_L]\right]$. From this the expressions for optimal policies from (6)and (7) follow, after the appropriate substitutions. ■

When the bureaucracy is ideologically disconnected, party L faces the choice of which section of the bureaucracy to try to motivate with its policy choice. Party L can choose to ignore right wing bureaucrats, and in this case we can characterize its optimal policy (x_L^B) by applying the result of Proposition 2 to bureaucracy $[\underline{b}_L, \bar{b}_L]$. Similarly, party L could also choose to ignore left wing bureaucrats. In this case, the optimal policy (z_L^B) is obtained by applying the result of Proposition 2 to bureaucracy $[\underline{b}_R, \overline{b}_R]$. How could party L have incentives to ignore left wing bureaucrats? This can happen because if there are more right wing bureaucrats (i.e., if $\overline{b}_L - \underline{b}_L < \overline{b}_R - \underline{b}_R$), then choosing right wing policies can yield levels of government production that left wing policies cannot. No such policy can be optimal if instead there are more left wing bureaucrats, because any level of government production achieved by right wing bureaucrats only could be mimicked by a suitably chosen left wing policy. From Section 4, this means that only a politically disadvantaged party, which draws fewer partisan bureaucrats, can face incentives to ignore its section of the bureaucracy.[25]

Proposition 10 also allows for the possibility that party L's policy choice induces bureaucrats from both sections of the bureaucracy to work. This can happen when the gap between the two sections of the bureaucracy is small enough.[26] In this case, we can characterize this optimal policy (y_L^B) by applying the result of Proposition 2 to bureaucracy $[\underline{b}_L, \overline{b}_R]$, after having modified government production to account for the fact that there are no bureaucrats with ideology in $(\overline{b}_L, \underline{b}_R)$. To see how this affects optimal policies, suppose that $\overline{b}_R - \underline{b}_L \leq 2\phi$. Ignoring the gap between the two sections of the bureaucracy, this case is analogous to a small ideologically connected bureaucracy from Proposition 2. Suppose that $\max\{\underline{b}_L - \phi, -1\} < \hat{x} < \max\{\overline{b}_R - \phi, -1\}$ so that policy $\hat{x}$ would be optimal if the bureaucracy was ideologically connected. With an ideologically disconnected bureaucracy, party L's objective has a unique global maximizer, which is $\hat{y}$. Comparing the expressions for $\hat{x}$ and $\hat{y}$ shows that $\hat{y} > \hat{x}$: conditional on wanting to induce bureaucrats from both sections of the bureaucracy to work, disconnected bureaucracies provide incentives for moderation. Intuitively, if the bureaucracy is connected, then party L can increase production by inducing moderate right wing bureaucrats to exert effort. However, if the bureaucracy is disconnected, then increasing production requires additional policy moderation.

The following corollary verifies this last observation and establishes other comparative statics.

Corollary 6 *Suppose that the bureaucracy is ideologically disconnected and that party L finds it optimal to induce bureaucrats from both sections of the bureaucracy to work. Then x_L^* is increasing in ρ, $\underline{b}_R - \overline{b}_L$ and $\overline{b}_R$, and decreasing in ϕ and $\overline{b}_L - \underline{b}_L$.*

Proof of Corollary 6. Immediate from the expressions in Proposition 10. ■

[25] Note that because all bureaucrats in the section of the bureaucracy being ignored by both parties would strictly prefer to join the private sector, this eventuality is ruled out by equilibrium administrations.

[26] If the gap between the two sections of the bureaucracy is large enough ($\underline{b}_R - \underline{b}_L \geq 2\phi$), then party L cannot induce right wing bureaucrats to work without sacrificing the effort of leftmost bureaucrats. But then no policy garnering the support of both sections of the bureaucracy can be optimal, because choosing a more left wing policy would keep production constant but increase party L's ideological payoff.

These comparative statics results are analogous to those of Corollary 4, with two additional insights. First, party L's policy is decreasing in the size of its own section of the bureaucracy. A party with a strong position in the bureaucracy substitutes away from catering to bureaucrats from the opposite section. In Section 4, we saw that a party can acquire a strong position in the bureaucracy when it is electorally stronger. Together with Corollary 6, this suggests a novel mechanism through which a decrease in electoral competition can reduce policy convergence. In contrast to standard models where the incentive for more extreme policies comes from the increase in voter support, here this is the result of the large set of bureaucrats induced to work for the favored party. Thus, bureaucrats' behavior can reinforce the polarizing effect of decreasing electoral competition.

The second new insight of Corollary 6 is that, as described above, increased polarization in the bureaucracy can favor policy moderation by parties. This is a countervailing force compared to Section 4, where we showed that party polarization favors polarization in the bureaucracy (making a disconnected bureaucracy more likely).