Chapter 7

The Cost of Production

Topics to be Discussed

- Measuring Cost: Which Costs Matter?
- Costs in the Short Run & Long Run
- Long-Run Versus Short-Run Cost Curves
- Production with Two Outputs-- Economies of Scope

Introduction

- The production function measures the relationship between input and output.
- Given the production technology, managers must choose how to produce. (I.e., how many units ?)

Introduction

- To determine the optimal level of output and the input combinations, we must convert from the unit measurements of the production function to dollar measurements or costs.

Measuring Cost: Which Cost Matter?

- Accounting Cost
  - Consider only *explicit cost*, the out of pocket cost for such items as wages, salaries, materials, and property rentals

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Measuring Cost: Which Cost Matter?

- Economic Cost
  - Considers explicit and opportunity cost.
    - Opportunity cost is the cost associated with opportunities that are foregone by not putting resources in their highest valued use.
- Sunk Cost
  - An expenditure that has been made and cannot be recovered--they should not influence a firm's decisions.

Cost in the Short Run

- Total output is a function of variable inputs and fixed inputs.

- Therefore, the total cost of production equals the fixed cost (the cost of the fixed inputs) plus the variable cost (the cost of the variable inputs), or

$$TC = FC + VC$$

Cost in the Short Run

- Marginal Cost (*MC*) is the cost of expanding output by one unit. Since fixed cost have no impact on marginal cost, it can be written as:

$$MC = \frac{\Delta VC}{\Delta Q} = \frac{\Delta TC}{\Delta Q}$$

Cost in the Short Run

- Average Total Cost (ATC) is the cost per unit of output, or average fixed cost (AFC) plus average variable cost (AVC). This can be written:

$$ATC = \frac{TFC}{Q} + \frac{TVC}{Q}$$

$$ATC = AFC + AVC \quad \text{or} \quad \frac{TC}{Q}$$

Cost in the Short Run

- The Determinants of Short-Run Cost
  - *The relationship between the production function and cost* can be exemplified by either increasing returns and cost or decreasing returns and cost.

Cost in the Short Run

- The Determinants of Short-Run Cost
  - Increasing returns and cost
    - With increasing returns, output is increasing relative to input and variable cost and total cost will fall relative to output.
  - Decreasing returns and cost
    - With decreasing returns, output is decreasing relative to input and variable cost and total cost will rise relative to output.

Cost in the Short Run

- For Example: Assume the wage rate (w) is fixed relative to the number of workers hired. Then:

$$MC = \frac{\Delta VC}{\Delta Q}$$

$$VC = wL$$

Cost in the Short Run

- Continuing:

$$\Delta VC = w\Delta L$$

$$MC = \frac{w\Delta L}{\Delta Q}$$

Cost in the Short Run

- Continuing:

$$\Delta MP_L = \frac{\Delta Q}{\Delta L}$$

$$\Delta L \text{ for a 1 unit } \Delta Q = \frac{\Delta L}{\Delta Q} = \frac{1}{\Delta MP_L}$$

Cost in the Short Run

- In conclusion:

$$MC = \frac{w}{MP_L}$$

- ...and a low marginal product (MP) leads to a high marginal cost (MC) and vise versa.

Cost in the Short Run

- Consequently (from the table):
  - MC decreases initially with increasing returns
    - 0 through 4 units of output
  - MC increases with decreasing returns
    - 5 through 11 units of output

Cost in the Short Run

- AVC and the Production Function

$$AVC = \frac{VC}{Q}$$

$$VC = wL$$

$$AVC = \frac{wL}{Q}$$

Cost in the Short Run

- AVC and the Production Function

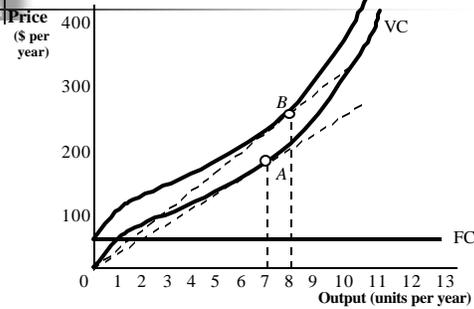$$AP_L = \frac{Q}{L}$$

$$AVC = \frac{w}{AP_L}$$

Cost in the Short Run

- Observations
  - If a firm is experiencing increasing returns, AP is increasing and AVC will decrease.
  - If a firms is experiencing decreasing returns, AP is decreasing and AVC will increase.
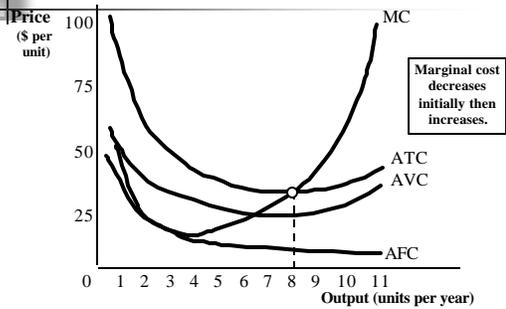
Cost in the Short Run

- Summary
  - The production function (MP & AP) shows the relationship between inputs and output.
  - The cost measurements show the impact of the production function in dollar terms.
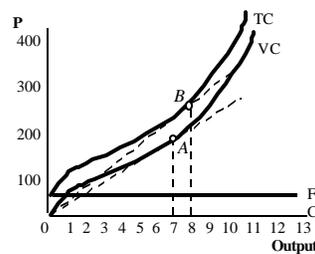
Cost Curves for a Firm

Price
($ per year)

400
300
200
100

TC
VC

B
A

FC

0 1 2 3 4 5 6 7 8 9 10 11 12 13
Output (units per year)

Cost Curves for a Firm

Price
($ per unit)

100
75
50
25

MC

Marginal cost decreases initially then increases.

ATC
AVC

AFC

0 1 2 3 4 5 6 7 8 9 10 11
**Output (units per year)**

Cost Curves for a Firm

- The line drawn from the origin to the tangent of the variable cost curve:
  - Its slope equals AVC
  - The slope of a point on VC equals MC
  - Therefore, MC = AVC at 7 units of output (point A)

P

400
300
200
100

TC
VC

B
A

FC

0 1 2 3 4 5 6 7 8 9 10 11 12 13
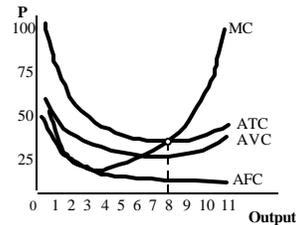**Output**

Cost Curves for a Firm

- The ray drawn from the origin to the tangent of the total cost curve:
  - The slope of a tangent equals the slope of the point.
 - ATC at 8 units = MC
  - Output = 8 units.

P

400
300
200
100

TC
VC

B
A

FC

0 1 2 3 4 5 6 7 8 9 10 11 12 13
Output
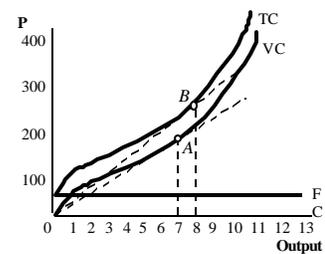
Cost Curves for a Firm

- Unit Costs
  - AFC falls continuously
  - When MC < AVC or MC < ATC, AVC & ATC decrease
  - When MC > AVC or MC > ATC, AVC & ATC increase

P

100

75

50

25

MC

ATC
AVC

AFC

0 1 2 3 4 5 6 7 8 9 10 11 **Output**

Cost Curves for a Firm

- Unit Costs
  - MC = AVC and ATC at minimum AVC and ATC
  - Minimum AVC occurs at a lower output than minimum ATC due to FC

P

100

75

50

25

MC

ATC
AVC

AFC

0 1 2 3 4 5 6 7 8 9 10 11 **Output**
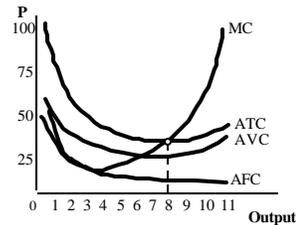
Cost in the Long Run

- Choosing Inputs
  - Assumptions
    - Two Inputs: Labor (L) & capital (K)
    - Wage rate for labor (w) and rental rate for capital (r) are determined in competitive markets

Cost in the Long Run

- Choosing Inputs
  - A Decision Model
    - $C = wL + rK$
    - Isocost: A line showing all combinations of L & K that can be purchased for the same cost
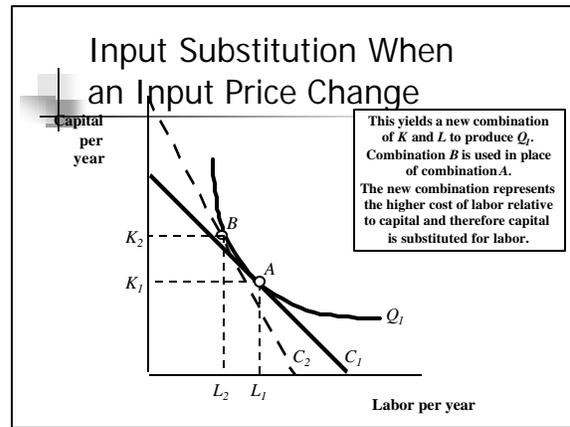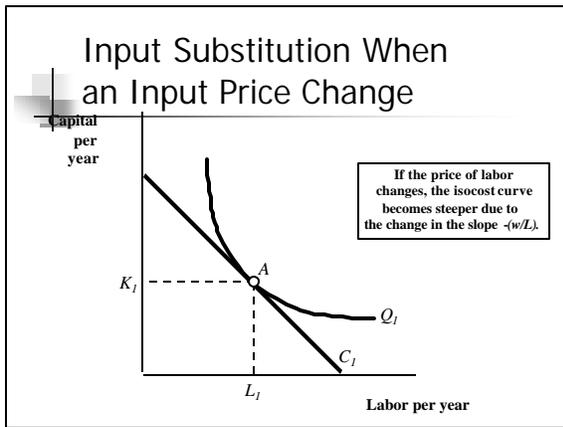
Cost in the Long Run

- Choosing Inputs
  - Rewriting C as linear:
    - $K = C/r - (w/r)L$
    - Slope of the isocost: $\frac{\Delta K}{\Delta L} = -\left(\frac{w}{r}\right)$
      - is the ratio of the wage rate to rental cost of capital.
      - This shows the rate at which capital can be substituted for labor with no change in cost.

Choosing Inputs

- We will address how to minimize cost for a given level of output.
  - We will do so by combining isocosts with isoquants

Producing a Given Output at Minimum Cost

Capital per year

K_2

K_1

K_3

A

Q_1

C_0 C_1 C_2

L_2 L_1 L_3

Labor per year

> Isocost C_2 shows quantity Q_1 can be produced with combination K_2L_2 or K_3L_3. However, both of these are higher cost combinations than K_1L_1.

Producing a Given Output at Minimum Cost

Capital per year

K_1

A

Q_1

C_0 C_1 C_2

L_1

Labor per year

> Isocost C_1 shows quantity Q_1 can be produced with combination K_1L_1. This is the low cost combination because it is tangent to Q_1.

Input Substitution When an Input Price Change

Capital per year

K_1

A

Q_1

C_1

L_1

Labor per year

> If the price of labor changes, the isocost curve becomes steeper due to the change in the slope $-(w/L)$.

Input Substitution When an Input Price Change

Capital per year

K_2

K_1

B

A

Q_1

C_2 C_1

L_2 L_1

Labor per year

> This yields a new combination of K and L to produce Q_1. Combination B is used in place of combination A. The new combination represents the higher cost of labor relative to capital and therefore capital is substituted for labor.

Cost in the Long Run

- Isoquants and Isocosts and the Production Function

$$\text{MRTS} = -\frac{\Delta K}{\Delta L} = \frac{\text{MP}_L}{\text{MP}_K}$$

$$\text{Slope of isocost line} = \frac{\Delta K}{\Delta L} = -\frac{w}{r}$$

Cost in the Long Run

- The minimum cost combination can then be written as:

$$\frac{\text{MP}_L}{w} = \frac{\text{MP}_K}{r}$$

  - Minimum cost for a given output will occur when each dollar of input added to the production process will add an equivalent amount of output.
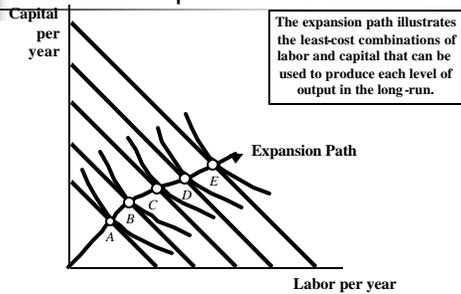
Cost in the Long Run

- Question:
  - If w = $10, r = $2, and $MP_L = MP_K$, which input would the producer use more of? Why?

Long-Run Versus Short-Run Cost Curves

- Cost minimization with Varying Output Levels
  - A firm's expansion path shows the minimum cost combinations of labor and capital at each level of output.
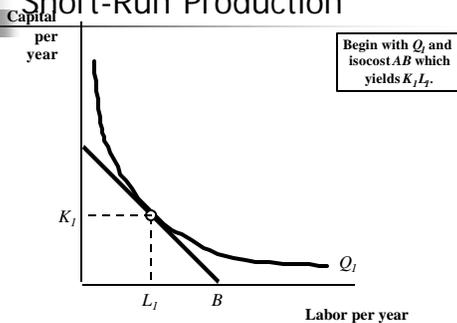
A Firm's Expansion Path

Capital per year

The expansion path illustrates the least-cost combinations of labor and capital that can be used to produce each level of output in the long-run.

Expansion Path
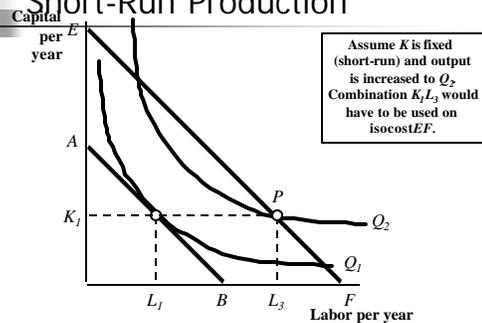
E
D
C
B
A

Labor per year
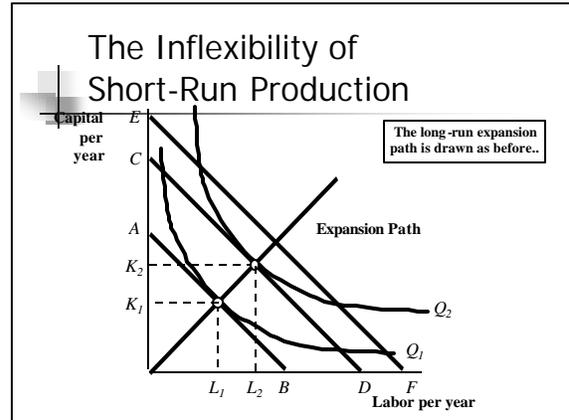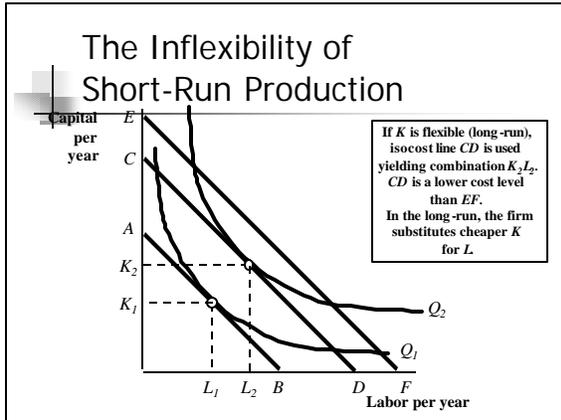
Long-Run Versus Short-Run Cost Curves

- What happens to average costs when both inputs are variable (long run) versus only having one input that is variable (short run)?

The Inflexibility of Short-Run Production

Capital per year

Begin with Q_1 and isocost AB which yields K_1L_1.

K_1

Q_1

L_1 B

Labor per year

The Inflexibility of Short-Run Production

Capital per year

E

Assume K is fixed (short-run) and output is increased to Q_2. Combination K_1L_3 would have to be used on isocost EF.

A

P

K_1 Q_2

Q_1

L_1 B L_3 F

Labor per year

The Inflexibility of Short-Run Production

Capital per year

E

C

A

K_2

K_1

L_1 L_2 B   D F

Labor per year

> If *K* is flexible (long-run), isocost line *CD* is used yielding combination K_2L_2. *CD* is a lower cost level than *EF*.
> In the long-run, the firm substitutes cheaper *K* for *L*

Q_2

Q_1

The Inflexibility of Short-Run Production

Capital per year

E

C

A

K_2

K_1

L_1 L_2 B   D F

Labor per year

> The long-run expansion path is drawn as before..

Expansion Path

Q_2

Q_1

Long-Run Versus Short-Run Cost Curves

- Long-Run Average Cost (LAC)
  - Constant Returns to Scale
    - If input is doubled, output will double and average cost is constant at all levels of output.

Long-Run Versus Short-Run Cost Curves

- Long-Run Average Cost (LAC)
  - Increasing Returns to Scale
    - If input is doubled, output will more than double and average cost decreases at all levels of output.
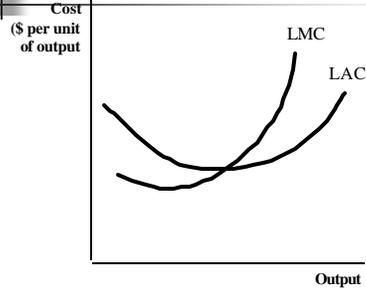
Long-Run Versus Short-Run Cost Curves

- Long-Run Average Cost (LAC)
  - Decreasing Returns to Scale
    - If input is doubled, the increase in output is less than twice as large and average cost increases with output.

Long-Run Versus Short-Run Cost Curves

- Long-Run Average Cost (LAC)
  - In the long-run:
    - Firms experience increasing and decreasing returns to scale and therefor long-run average cost is "U" shaped.

Long-Run Versus Short-Run Cost Curves

- Long-Run Average Cost (LAC)
  - Long-run marginal cost leads long-run average cost:
    - If LMC < LAC, LAC will fall
    - If LMC > LAC, LAC will rise
    - Therefore, LMC = LAC at the minimum of LAC

Long-Run Average and Marginal Cost

Cost
($ per unit of output)

LMC

LAC

Output

Long-Run Versus Short-Run Cost Curves

- Question
  - What is the relationship between long-run average cost and long-run marginal cost when long-run average cost is constant?

Long-Run Versus Short-Run Cost Curves

- Economies and Diseconomies of Scale
  - Economies of Scale
    - Increase in output is greater than the increase in inputs.
  - Diseconomies of Scale
    - Increase in output is less than the increase in inputs.

Long-Run Versus Short-Run Cost Curves

- Measuring Economies of Scale

$$E_c = \text{Cost} - \text{Output Elasticity}$$
$$= \%\,\Delta \text{ in cost from a 1\% increase in output}$$

Long-Run Versus Short-Run Cost Curves

- Measuring Economies of Scale

$$E_c = (\Delta C / C) / (\Delta Q / Q)$$

$$E_c = (\Delta C / \Delta Q) / (C / Q) = \text{MC/AC}$$

## Long-Run Versus Short-Run Cost Curves

- Therefore, the following is true:
  - $E_C < 1$: MC < AC
    - Average cost indicate decreasing economies of scale
  - $E_C = 1$: MC = AC
    - Average cost indicate constant economies of scale
  - $E_C > 1$: MC > AC
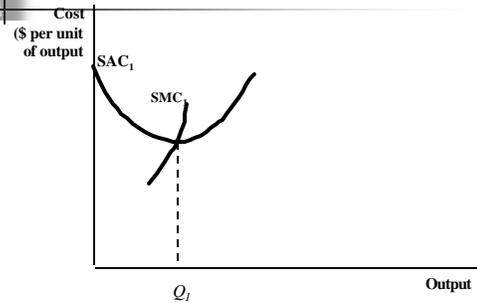   - Average cost indicate increasing diseconomies of scale

Long-Run Versus Short-Run Cost Curves

- The Relationship Between Short-Run and Long-Run Cost
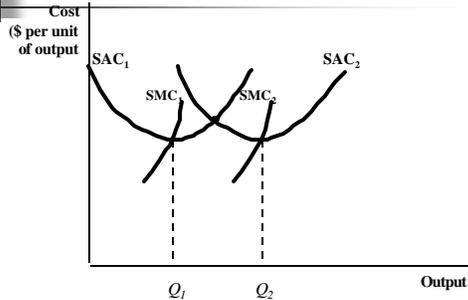 - We will use short and long-run cost to determine the optimal plant size

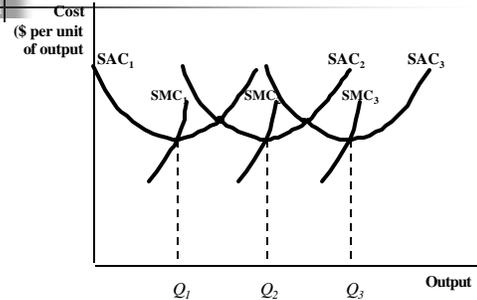## Long-Run Cost with Constant Returns to Scale
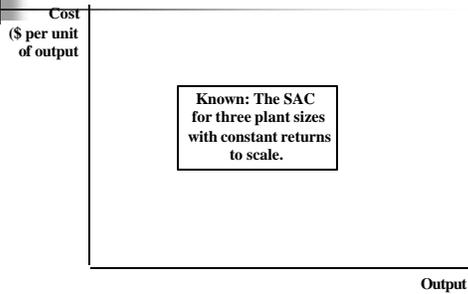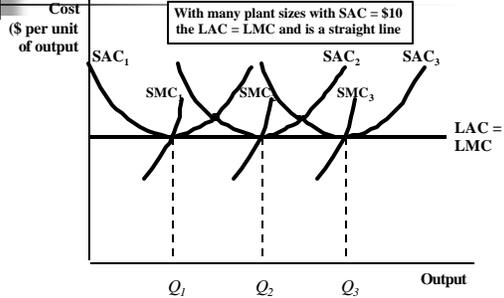

Long-Run Cost with Constant Returns to Scale

Long-Run Cost with Constant Returns to Scale
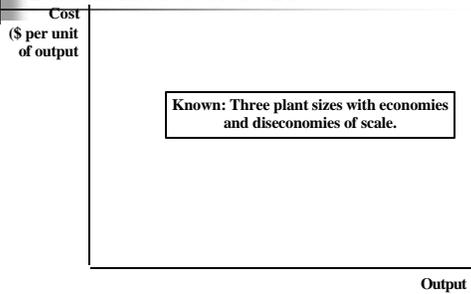

Long-Run Cost with Constant Returns to Scale

Long-Run Cost with Constant Returns to Scale

Cost
($ per unit of output)

With many plant sizes with SAC = $10 the LAC = LMC and is a straight line

SAC₁ → SAC_1

SMC_1 SMC_2 SMC_3

SAC_2 SAC_3

LAC = LMC

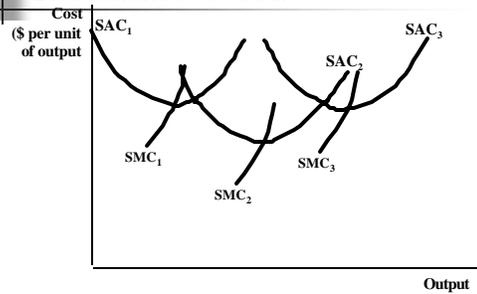Q_1 Q_2 Q_3 Output

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Long-Run Cost with Constant Returns to Scale

- Observation
  - The optimal plant size will depend on the anticipated output (e.g. Q_1 choose SAC_1, etc).
  - The long-run average cost curve is the *envelope* of the firm's short-run average cost curves.

- Question
  - What would happen to average cost if an output level other than that shown is chosen?
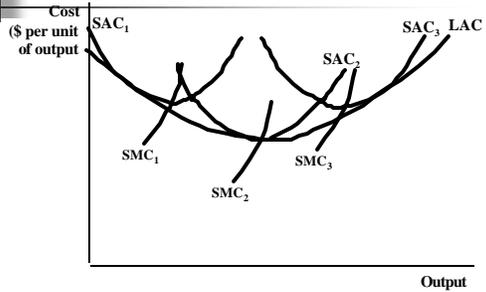

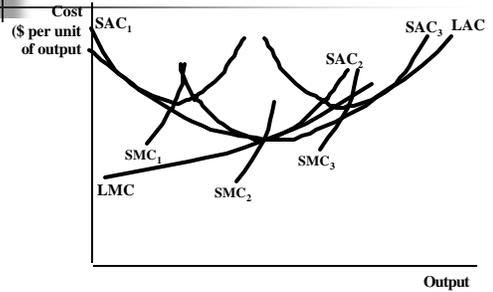## Long-Run Cost with Economies and Diseconomies of Scale

Cost
($ per unit of output)

Known: Three plant sizes with economies and diseconomies of scale.

Output

Long-Run Cost with Economies and Diseconomies of Scale

Cost
($ per unit of output)

SAC_1 SAC_3

SAC_2

SMC_1 SMC_3

SMC_2

Output

Long-Run Cost with Economies and Diseconomies of Scale

Cost
($ per unit of output)

SAC_1 SAC_3 LAC

SAC_2

SMC_1 SMC_3

SMC_2

Output

Long-Run Cost with Economies and Diseconomies of Scale

Cost
($ per unit of output)

SAC_1 SAC_3 LAC

SAC_2

SMC_1 SMC_3

LMC SMC_2

Output

Long-Run Cost with Economies and Diseconomies of Scale

Cost
($ per unit of output)

SAC₁ ... SAC₃ LAC

SAC₂

$10
$8

A
B

SMC₁ ... SMC₃

LMC ... SMC₂

If the output is Q_1 a manager would chose the small plant SAC₁ and SAC $8. Point *B* is on the LAC because it is a least cost plant for a given output.

Q_1

Output

Long-Run Versus Short-Run Cost Curves

- What is the firms' long-run cost curve?
  - Firms can change scale to change output in the long-run.
  - The long-run cost curve is the dark blue portion of the SAC curve which represents the minimum cost for any level of output.

Long-Run Cost with Economies and Diseconomies of Scale

- Observations
  - The LAC does not include the minimum points of small and large size plants? Why not?
  - LMC is not the envelope of the short-run marginal cost. Why not?

Production with Two Outputs-- Economies of Scope

- Examples:
  - Chicken farm--poultry and eggs
  - Automobile company--cars and trucks
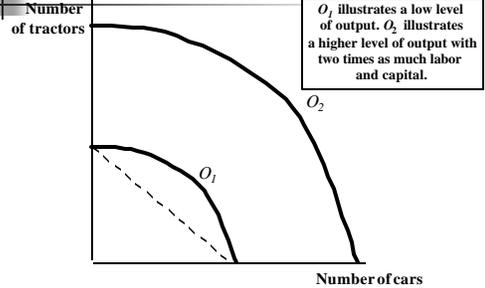 - University--Teaching and research

Production with Two Outputs-- Economies of Scope

- Economies of scope exist when the joint output of a single firm is greater than the output that could be achieved by two different firms each producing a single output.

- What are the advantages of joint production?
  - Consider an automobile company producing cars and tractors

Production with Two Outputs-- Economies of Scope

- Advantages

  1) Both use capital and labor.

  2) The firms share management resources.

  3) Both use the same labor skills and type   of machinery.

Production with Two Outputs-- Economies of Scope

- Production:
  - Firms must choose how much of each to produce.
  - The alternative quantities can be illustrated using product transformation curves.

Product Transformation Curve

Number of tractors

O_1 illustrates a low level of output. O_2 illustrates a higher level of output with two times as much labor and capital.

O_2

O_1

Number of cars

Production with Two Outputs-- Economies of Scope

- Observations
  - Product transformation curves are negatively sloped
  - Constant returns exist in this example
  - Since the production transformation curve is concave is joint production desirable?

Production with Two Outputs-- Economies of Scope

- Observations
  - There is no direct relationship between economies of scope and economies of scale.
    - May experience economies of scope and diseconomies of scale
    - May have economies of scale and not have economies of scope

Production with Two Outputs-- Economies of Scope

- The *degree of economies of scope* measures the savings in cost can be written:

$$SC = \frac{C(Q_1) + C(Q_2) - C(Q_1, Q_2)}{C(Q_1, Q_2)}$$

  - $C(Q_1)$ is the cost of producing Q_1
  - $C(Q_2)$ is the cost of producing Q_2
  - $C(Q_1 Q_2)$ is the joint cost of producing both products

Production with Two Outputs-- Economies of Scope

- Interpretation:
  - If SC > 0 -- Economies of scope
  - If SC < 0 -- Diseconomies of scope

Example: Economies of Scope in the Trucking Industry

- Issues
  - Truckload versus less than truck load
  - Direct versus indirect routing
  - Length of haul

Example: Economies of Scope in the Trucking Industry

- Questions:
  - Economies of Scale
    - Are large-scale, direct hauls cheaper and more profitable than individual hauls by small trucks?
    - Are there cost advantages from operating both direct and indirect hauls?

Example: Economies of Scope in the Trucking Industry

- Empirical Findings
  - An analysis of 105 trucking firms examined four distinct outputs.
    - Short hauls with partial loads
    - Intermediate hauls with partial loads
    - Long hauls with partial loads
    - Hauls with total loads

Example: Economies of Scope in the Trucking Industry

- Empirical Findings
  - Results
    - SC = 1.576 for reasonably large firm
    - SC = 0.104 for very large firms
  - Interpretation
    - Combining partial loads at an intermediate location lowers cost management difficulties with very large firms.

Summary

- Managers, investors, and economists must take into account the opportunity cost associated with the use of the firm's resources.

- Firms are faced with both fixed and variable costs in the short-run.

Summary

- When there is a single variable input, as in the short run, the presence of diminishing returns determines the shape of the cost curves.

- In the long run, all inputs to the production process are variable.

Summary

- The firm's expansion path describes how its cost-minimizing input choices vary as the scale or output of its operation increases.

- The long-run average cost curve is the envelope of the short-run average cost curves.

Summary

- A firm enjoys economies of scale when it can double its output at less than twice the cost.

- Economies of scope arise when the firm can produce any combination of the two outputs more cheaply than could two independent firms that each produced a single product.

Summary

- A firm's average cost of production can fall over time if the firm "learns" how to produce more effectively.

- Cost functions relate the cost of production to the level of output of the firm.