ECONOMICS 7330, Fall 2025

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Midterm Exam 2, November 10, 5 questions. All sub-questions carry equal weight.

1. (20%) Consider a sample of N independent Poisson distributed random variables x_i with probabilities

$$\pi(x) = e^{-\lambda} \frac{\lambda^{x_i}}{x_i!} .$$

a) Find the ML-estimator for λ (a mathematical expression).

b) If the sample is $(N = 3)$ $x_i = 0, 2, 3,$what is the Maximum Likelihood estimate $\hat{\lambda}$?

2. (15%) Assume that y_t follows the AR(1) process

$$y_t = 20 + 0.2\, y_{t-1} + e_t \quad (*)$$

where e_t is white noise with variance 3 and y_0 equals 4.

a) Is this time-series process stable? (As always, "yes" or "no" is not an answer, you need to argue why.)

b) Is it stationary?

c) What are the expected values of y_1 and y_2?

3. (15%) Assume that income follows the stationary AR(2) process

$$y_t = 4 + 0.2\, y_{t-1} + 0.5\, y_{t-2} + e_t \quad (*)$$

where e_t is white noise with variance 3.

a) What is the expected value Ey_t.

b) Assume that you are told that $y_0 = 10$, $y_{-1} = 5$, and $y_{-2} = 5$. Find the conditional expectation $E(y_2|y_0, y_{-1}, y_{-2}, ...)$.

c) Under the same assumptions, what is the conditional variance $Var(y_2|y_0, y_{-1}, y_{-2}, ...)$.

4. (20%) Assume that $(X, Y, Z)'$ is a vector normally distributed random variable with $\mu' = (1, 2, 3)$ and variance-covariance matrix

$$\Sigma = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}.$$

a) What is the conditional mean $E(Y|X, Z)$?

b) What the variance $Var(X, Y|Z)$?

5. (30%) Consider a sample of N independent bivariate normally distributed random variables $W_i = (X_i, Y_i)'$ with density

$$\frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{(x_i-\mu_x)^2+(y_i-\mu_y)^2-2\rho(x_i-\mu_x)(y_i-\mu_y)}{2(1-\rho^2)}}.$$

Here I have set the variance of x and y to unity to get a simpler expression.

a) Find the ML-estimator for $(\mu_x, \mu_y, \rho)'$.

b) Find the score vector.

c) Find the Hessian.