ECONOMICS 7330, Fall 2023
Bent E. Sørensen

Midterm Exam 2, October 30, 5 questions. All sub-questions carry equal weight.

NOTE: We need to be able to follow your calculations, so just giving a number is not considered a full answer (if we really can't follow your reasoning, it is not even a partial answer).

1. (20%) Assume X and Y are independent uniformly distributed random variables on $[0, 1]$. What is the density of $X + Y$. (Hint: you need to pay close attention to the supports of the variables involved.)

2. (20%) You have a sample of i.i.d. observations $x_1, ..., x_n$ with mean zero and variance σ^2. Let $\overline{X}$ be the mean (average) of the observations. Define $\hat{\sigma}^2 = \frac{1}{n}\Sigma_{i=1}^n x_i^2$.

Use the Law of Large Numbers and the Central Limit Theorem to show that

$$\sqrt{n}\frac{\overline{x}}{\sqrt{\hat{\sigma}^2}} \, ,$$

converges in distribution to a standard normal distribution. (You need to point out clearly which rule/law you use for each step in your argument.)

3. (20%) X_1 and X_2 are normally distributed random variables with $E(X_1) = 3$, $E(X_2) = 4$ $\text{Var}(X_1) = 4$, and $\text{Var}(X_2) = 6$. The covariance between X_1 and X_2 is 2.
a) Write down the joint density for (X_1, X_2).
b) Write down the density for X_2 conditional on X_1.

4. (20%) You have a sample of observations $x_1, ..., x_n$. The density can be written as $\frac{1}{\sqrt{2\pi\kappa}}e^{-0.5(x-\mu)^2/\kappa}$ or as $\frac{1}{\sqrt{2\pi}\sigma}e^{-0.5(x-\mu)^2/\sigma^2}$.
a) Find the Maximum Likelihood (ML) estimator for κ.
b) Find the Maximum Likelihood estimator for σ and verify that $\hat{\kappa} = (\hat{\sigma})^2$. (The hat-symbol indicates the ML estimator.)

5. (20%) Consider a sample of N independent log-normally distributed random variables x_i with density $\frac{1}{\sqrt{2\pi}x\sigma}e^{-0.5(\ln(x)-\mu)^2/\sigma^2}$.
a) Find the score vector for the log-likelihood function.
b) Find the ML-estimators for σ^2 and μ.