ECONOMICS 7330, Fall 2025
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Midterm Exam 1 Monday October 13—5 questions. All sub-questions carry equal weight except if otherwise indicated.

1. (20%)
Let $f(x,y) = e^{-2xy-y-x^2y}$ be the joint density function for some random variables X and Y, where X takes values on the real line and Y between 1 and 2. (It is actually not a joint density because I didn't find the normalizing constant, but solve the question as if it is.) Find the marginal densities $f_X(x)$ and $f_Y(y)$. (Half points for each.)

2. (36%) Consider two random variables X and Y. Assume they both are discrete and that X can take the values 0 and 1, while Y can take the values 0, 1, and 2. The probabilities for (X,Y) are shown in the following table:

	X=0	X=1
Y=0	1/8	2/8
Y=1	1/8	1/8
Y=2	1/8	2/8

i) Find the mean and the variance of Y.
ii) Are the random variables X and Y independent?
iii) Find the conditional distribution of Y given $X = 1$.
iv) Verify for these numbers that $EY = EE(Y|X)$.
v) Find the conditional variance $Var(Y|X = 0)$.
vi) Show the $Var(Y) = EVar(Y|X) + Var(E(Y|X)$. You can choose to prove the identity instead.

3. (24%) X_1 and X_2 are normally distributed random variables with $E(X_1) = 3$, $E(X_2) = 4$ $\text{Var}(X_1) = 4$, and $\text{Var}(X_2) = 4$. The covariance between X_1 and X_2 is 2.
a) Write down the joint density for (X_1, X_2).
b) What is $E(X_1|X_2)$?
c) What is $\text{Var}(X_1|X_2)$?
d) Write down the density for X_1 conditional on X_2.

4. (10%) If X is exponentially distributed with mean 2, what is the distribution of $Y = 1 - e^{-X/2}$? (Remember the support.)

5. (10%) Prove that if the density satisfies $f(x) = f(-x)$ for all $x \in R$ then the distribution function satisfies $F(-x) = 1 - F(x)$.