ECONOMETRICS I, SPRING 2025.

Homework 8. Due Wednesday March 26.

1. A common model in economics is one where a typical agent's log wage is a sum of a random walk and independent white noise.

Define y_t such that

$$y_t = y_{t-1} + u_t,$$

where u_t is white noise with variance σ_u^2, and define

$$w_t = y_t + e_t,$$

where e_t is white noise with variance σ_e^2. (The innovations u and e are independent.)

Econometricians has estimated this model by matching the moments of empirical wages to the theoretical moments of this model. w_t is not stationary (it is not stable so it cannot be stationary), so econometricians instead finds the moments of Δw_t.
Find the variance, and the autocovariances of order one and two for Δw_t.

2. Assume that an agent's wage income follows the AR(1) process

$$y_t = \mu + \beta y_{t-1} + e_t \quad (*)$$

where e_t is white noise with variance σ_e^2 and $\beta < 1$.

Assume the agent's wage was 100$ period 0.
a) What is the agent's expected wages in period t (for any $t > 0$)?
b) If the discount rate $\delta = \frac{1}{1+r}$ is 0.9 percent, what is the discounted (conditional) expected value of all future income ($\Sigma_{t=0}^{\infty} \delta^t E_0 y_t$)? (Hint: use the formula for geometric sums in δ times β. It takes a few more steps to get the expression for the mean term, but make sure you at least get the stochastic term right.)

3. Computer question using Matlab (continuation of previous homeworks).
a) In Matlab, regress real per capita U.S. data income growth on it own lag and a constant. (An AR(1) model.) Using a t-test, can you reject that income growth is white noise?
b) Also estimate and AR(2) mode for income growth. Can you reject the AR(2) in favor of an AR(1)?

4. In Matlab. Simulate an AR(1) model a thousand times for different values of the coefficient a to the lag (you can include a constant or not, but estimate the same model as you simulate). Let $T = 40$
Use values of a equal to 0.5, 0.9, and 0.99. Show that as a gets larger $\hat{a}$ gets more biased towards zero.