ECONOMETRICS I, SPRING 2025.

Homework 7. Due Wednesday March 19.

1.
a) Let

$$x_t \ = \alpha_0 \ + \ 5 * u_t \ + \ u_{t-1} \ ,$$

where u_t is white noise.
Find the autocovariances for x_t in terms of σ_u^2 (the variance of u_t).

b) Given the stationary AR(1) process

$$x_t \ = \ 3 \ + \ .5 * x_{t-1} \ + \ u_t$$

where $Eu_t^2 = 3$. Find the variance of x_t, and the first 3 autocovariances and autocorrelations.

2. Given the AR(1) process:

$$e_t \ = \ \ ae_{t-1} \ + \ u_t \ ,$$

with $a = 0.5$. Let E be the vector (e_1, e_2, e_3).
a) Assuming e_0 is a fixed numbert, find the variance matrix $\Omega = var(e)$.
b) Assuming e_t is stationary, find the variance matrix $\Omega = var(e)$.

3. Define the lag polynomials $a(L) = a_0 + a_1 L$ and $b(L) = b_0 + b_1 L + b_2 L^2$. (Notice: in the notes, and in class, it is often assumed $a_0 = 1$ and $b_0 = 1$. This is just for simplification and doesn't matter for any results since you can always re-scale the data and the lag-polynomial such that the first coefficient becomes unity (write a(L) as $a_0\, a'(L)$ where the lag polynomial $a'(L) = 1 + \frac{a_1}{a_0} L$ and similarly for $b(L)$). The constant a_0 will not affect the properties of the lag-polynomial that we care about.)

Assume $a_0 = 1$, $a_1 = -2$, $b_0 = 3$, $b_1 = -.3$, and $b_2 = .5$.
i) If $x_t = 3, x_{t-1} = -3, x_{t-2} = -2$, $x_{t-3} = 9$, and $x_{t-4} = 9$, what is $a(L)x_t$? and $b(L)x_t$? (This should be a number.)
ii) Find the roots of $a(L)$ and $b(L)$.
iii) What is $c(L) = a(L)b(L)$? What are the roots of $c(L)$?
iv) Find the coefficients to the constant (identify), L, and L^2 in the lag-polynomial $b^{-1}(L)$.

4. Computer question (continuation of previous homeworks). In Matlab, regress real per capita U.S. data consumption growth on income growth and the interest rate using the posted dataset. (This is the what you did in homework 1.)

a) Calculate the residuals e. Regress e_t on e_{t-1}? Is there evidence of autocorrelation (Use t-tests.)