

Homework 2. Due Wednesday January 29.

1. Frisch-Waugh with 2 regressors. Assume you regress

$$y_i = \beta_1 x_{1i} + \beta_2 x_{2i} + \text{error} .$$

In vector notation:

$$Y = \beta_1 X_1 + \beta_2 X_2 + \text{error} .$$

Now

$$(X'X) = \begin{pmatrix} X_1'X_1 & X_1'X_2 \\ X_1'X_2 & X_2'X_2 \end{pmatrix} .$$

Find the vector of parameters $\beta = (\beta_1, \beta_2)'$, writing in terms of the inner products (like $X_1'X_2$, $X_1'Y$, $X_2'Y$ etc.). It is not hard to invert $X'X$, because the inner products are scalars. If you find it easier, you can assume without loss of generality that the units are chosen such that $X_1'X_1 = 1$ and $X_2'X_2 = 1$. (“Without loss of generality,” because we can change the units of the regressors without changing the results.)

Now regress

$$X_2 = X_1\xi + \text{error} ,$$

and find the fitted value $P_1X_2 = X_1 * \hat{\xi}$ (remember that P_1X_2 is proportional to X_1) and the residual $M_1X_2 = X_2 - P_1X_2$.

Finally, regress

$$Y = (M_1X_2)\beta_2 ,$$

and verify that the $\hat{\beta}_2$ that you get from this second regression is the same as the $\hat{\beta}_2$ from the original regression.

2. Computer question (continuation of homework 1). In Matlab, regress real per capita U.S. data consumption growth on income growth and the interest rate using the posted dataset. (This is the what you did in homework 1.)

- a) Regress income growth on the interest rate and take the residual which we can call M_rY .
- b) Regress consumption growth on M_rY and compare the estimated coefficient to the one from the first regression.
- c) Regress consumption growth on the interest rate and take the residual which we can call M_rC
- d) Regress M_rC on M_rY and compare the estimated coefficient to the one from the first and second regressions.
- e) Draw a series of iid mean zero terms and add to income growth, getting Y^* and regress consumption on Y^* (income with measurement error) and the interest rate. Verify that the estimated coefficient to income gets closer to zero when there is measurement error. What happens to the

coefficient to the interest rate? Repeat the regression with a much higher variance of the measurement error (“larger” measurement errors). What happens?