ECONOMETRICS I, SPRING 2024.

Homework 9. Due Wednesday April 10.

1. (For White robust standard errors)

a) Let

$$X = \begin{pmatrix} x_{11} & x_{12} \\ \vdots & \vdots \\ x_{N1} & x_{N2} \end{pmatrix} .$$

and $e' = e_1, ..., e_N$. In the White application, each column of x will be N observations of a regressor. Show that if the error terms are not autocorrelated and you set the terms with mean zero to zero (for the White variance estimator, we impose that the off-diagonal terms are 0), then

$$(X'e)(X'e)' = \begin{pmatrix} \Sigma_{i=1}^N x_{1i}^2 e_i^2 & \Sigma_{i=1}^N x_{1i} x_{2i} e_i^2 \\ \Sigma_{i=1}^N x_{1i} x_{2i} e_i^2 & \Sigma_{i=1}^N x_{2i}^2 e_i^2 \end{pmatrix} .$$

(Each of these terms have mean different from 0, if the columns of X are not orthogonal, and if divided by N they will satisfy a Law of Large Numbers under typical conditions. You can set N=3 for example if that makes it easier to see what goes on or even N=2.)

2. Computer question (continuation of previous homeworks). In Matlab, assume real per capita U.S. consumption growth is a linear function of income growth and the interest rate using the posted dataset. (What we have estimated using regression analysis.)

a) Assume that the variance of the residuals are constant (homoskedasticity).

Estimate the coefficients by Maximum Likelihood and verify that the estimates of the slopes a the same as you found using OLS.

Now assume that you are told that the variance of the residuals is proportional to the square of the interest rates.

b) Estimate the relation using Maximum Likelihood.

c) Estimate the variance using the information matrix. (You can evaluate it numerically, we will discuss alternative ways Monday. Extra points for doing it numerically and finding it analytically.)

d) How do the results compare to 2-stage GLS (weighted regression)?