

Homework 9. Due Wednesday April 19.

1. (For White robust standard errors)

a) Let

$$X = \begin{pmatrix} x_{11} & x_{12} \\ \vdots & \vdots \\ x_{N1} & x_{N2} \end{pmatrix}.$$

and $e' = e_1, \dots, e_N$. In the White application, each column of x will be N observations of a regressor. Show that if the error terms are not autocorrelated and not correlated with X and you set the terms with mean zero to zero (for the White variance estimator, we impose that the off-diagonal terms are 0), then

$$(X'e)(X'e)' = \begin{pmatrix} \sum_{i=1}^N x_{1i}^2 e_i^2 & \sum_{i=1}^N x_{1i} x_{2i} e_i^2 \\ \sum_{i=1}^N x_{1i} x_{2i} e_i^2 & \sum_{i=1}^N x_{2i}^2 e_i^2 \end{pmatrix}.$$

(Each of these terms have mean different from 0, if the columns of X are not orthogonal, and if divided by N they will satisfy a Law of Large Numbers under typical conditions. I did this part quickly in class, partly because it is better that you verify it yourself.)

2. Assume that random variables y_i for $i=1, \dots, 20$ are independent with $E(y_i) = \alpha + \beta x_i$, $Var(y_i) = \sigma^2 x_i^2$, where $x_i = i$ and $\sigma^2 = 2$.

a) If you estimate α and β by OLS, what is the variance of $\hat{\beta}$?

b) If you estimate α and β by GLS, what is the variance of $\hat{\beta}$?

3. Computer question (continuation of previous homeworks). In Matlab, regress real per capita U.S. data consumption growth on income growth and the interest rate using the posted dataset. (This is the what you did in homework 1.)

a) You are told that income growth is not exogenous to consumption growth, but lagged income growth is. Suggest a suitable IV estimator. (Just words here.)

b) In Matlab, estimate the coefficients using your suggested IV estimator.

c) Calculate the standard errors of the coefficients and compare to the estimated standard errors from an OLS regression.

d) Based on the IV estimation, test if the coefficient to income growth is zero.

(NOTE: I ask you to use the lagged variable here, because you already have it. It is, or was, rather common to lagged variables used as instruments without much discussion and that is very often not a good idea. So do not take the setup of this problem as a suggestion for doing good empirical economics.)