

**Homework 5. Due Wednesday October 9.**

1. a) Show for  $2 \times 2$  matrices that  $\text{vec}(ABC) = (C' \otimes A) \text{vec} B$ . (It is enough to verify it for the first three elements or so of the resulting vector.)

b) Prove that  $(A \otimes B)(C \otimes D) = (AC \otimes BD)$  using the formula from part a). (Here, I had a brain-freeze in class, so I will give you some hints. I did start right: I think you have to prove that  $(A \otimes B)(C \otimes D) \text{vec}(X) = (AC \otimes BD) \text{vec}(X)$  for arbitrary  $X$ . Next hint (and that is what I for some reason didn't do, even if it the obvious thing to do) is to use the formula from a) two times, and first show that  $(C \otimes D) \text{vec}(X) = \text{vec}(DXC')$  and then use the formula again.

2. Consider the demand and supply model (or whatever the variables may stand for):

$$y1 = \alpha_1 * y2 + \alpha_2 x_1 + u_1,$$

and

$$y2 = \alpha_3 * y1 + \alpha_4 x_4 + \alpha_5 x_5 + u_2.$$

i) Assume you know the  $\Pi$  matrix of the reduced form (this can be estimated consistently), write down and solve the 6 equations for the  $\alpha$ 's. (I wrote down the solution quickly, but I want you to fill in the details.)

2) If instead

$$y2 = \alpha_3 * y1 + \alpha_7 * x_1 + \alpha_4 x_4 + u_2,$$

show that one cannot solve the equation uniquely for  $(\alpha_3, \alpha_7, \alpha_4)$ .

3. Use the program `Econ8331_Sim.m` to estimate a 2SLS estimator for the simultaneous equation model (run the program with, say, 1000 simulations). We removed one line from my program that you have to add. Also, add an OLS estimator of the same equation and show that the results of the OLS estimator are biased.

Change one of the coefficients in the simulation to make the OLS bias worse. Simulate again and show it gets worse.