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ECONOMICS 7330 – Probability and Statistics, Fall 2024

Homework 9. Due Wednesday November 6.

1. Let X be distributed Poisson: $\pi(k) = \frac{\exp(-\theta)\theta^k}{k!}$ for nonnegative integer k and $\theta > 0$.
(a) Find the log-likelihood function $l_n(\theta)$.
(b) Find the MLE (ML estimator) $\hat{\theta}$ for θ.
(c) Find the asymptotic variance of $\hat{\theta}$. (That is σ^2 where $\sqrt{(N)}(\hat{\theta} - \theta)$ converges in distribution $N(0, \sigma^2)$.)

2. (Hansen exercise 10.2.This we did in class, so do it without looking at your notes, as you will do at the exam.) Let X be distributed as $N(\mu, \sigma^2)$. The unknown parameters are μ and σ^2.
(a) Find the log-likelihood function $l_n(\mu, \sigma^2)$.
(b) Take the first-order condition with respect to μ and show that the solution for $\hat{\mu}$ does not depend on the solution for $\hat{\sigma}^2$.
(c) Define the concentrated log-likelihood function $l_n(\hat{\mu}, \sigma^2)$. (Notice, that this means that you consider it only as a function of σ^2. You may sometimes encounter people talking about a concentrated (log-) likelihood function.) Take the first-order condition for σ^2 and find the MLE $\hat{\sigma}^2$.

3. Let X be Bernoulli $\pi(X|p) = p^x\,(1-p)^{1-x}$.
(a) Calculate the information "matrix" for p by taking the variance of the score and give the formula for the asymptotic variance.

4. Assume that you have a sample of n observations from an exponential distribution with density $f(x) = \frac{1}{\theta}\exp^{-\frac{x}{\theta}}$. (The mean is θ and the variance is θ^2.)
a) Write down the log-likelihood function $l_n(\theta)$.
b) Find the asymptotic limit $l(\theta) = \lim_{n\to\infty} \frac{1}{n} l_n(\theta)$.
c) Show that the value of θ that maximizes $l(\theta)$ is equal to the "true" value that generated the data.