

**ECONOMICS 7330 – Probability and Statistics, Fall 2023**

Homework 9. Due Wednesday November 8.

1. (Hansen exercise 10.1.) Let  $X$  be distributed Poisson:  $\pi(k) = \frac{\exp(-\theta)\theta^k}{k!}$  for non-negative integer  $k$  and  $\theta > 0$ .
  - (a) Find the log-likelihood function  $l_n(\theta)$ .
  - (b) Find the MLE (ML estimator)  $\hat{\theta}$  for  $\theta$ .
  
2. (Hansen exercise 10.2.) Let  $X$  be distributed as  $N(\mu, \sigma^2)$ . The unknown parameters are  $\mu$  and  $\sigma^2$ .
  - (a) Find the log-likelihood function  $l_n(\mu, \sigma^2)$ .
  - (b) Take the first-order condition with respect to  $\mu$  and show that the solution for  $\hat{\mu}$  does not depend on the solution for  $\hat{\sigma}^2$ .
  - (c) Define the concentrated log-likelihood function  $l_n(\hat{\mu}, \sigma^2)$ . (Notice, that this means that you consider it only as a function of  $\sigma^2$ . You may sometimes encounter people talking about a concentrated (log-) likelihood function.) Take the first-order condition for  $\sigma^2$  and find the MLE  $\hat{\sigma}^2$ .
  
3. (Hansen exercise 10.6) Let  $X$  be Bernoulli  $\pi(X|p) = p^x (1 - p)^{1-x}$ .
  - (a) Calculate the information “matrix” for  $p$  by taking the variance of the score.
  - ( b) Calculate the information for  $p$  by taking the expectation of (minus) the second derivative. Did you obtain the same answer?