

ECONOMICS 7330 – Probability and Statistics, Fall 2024

Homework 8. Due Wednesday October 30.

1. Assume you have a sample of vector random variables $x_i = (y_i, z_i)'$ mean $\mu = (\mu_y, \mu_z)'$. Assume that $\sqrt{N}(\bar{x} - \mu)$ converges in distribution to $N(0, \Sigma)$ where

$$\Sigma = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

Find the joint asymptotic distribution of $\sqrt{N}(\frac{\bar{y}}{\bar{z}} - \frac{\mu_y}{\mu_z})$ and $\sqrt{N}(\bar{y}\bar{z} - \mu_y\mu_z)$ (the main part of the answer is a 2×2 variance matrix).

2. Assume that you have a sample of n observations from a Poisson distribution with probabilities $\pi(k) = \frac{\theta^k e^{-\theta}}{k!}$.

a) Write down the log-likelihood function $l_n(\theta)$.

b) Find the Score.

c) Find the ML estimator $\hat{\theta}$.

3. Assume that you have a sample of n observations from an exponential distribution with density $f(x) = \frac{1}{\theta} \exp^{-\frac{x}{\theta}}$. (The mean is θ and the variance is θ^2 .)

a) Write down the log-likelihood function $l_n(\theta)$.

b) Find the Score.

c) Find the ML estimator $\hat{\theta}$.

d) Find the Hessian.

e) What is the asymptotic distribution of $\sqrt{N}(\hat{\theta}_N - \theta_0)$? (I need an expression for the asymptotic variance.)

f) Verify the Information Matrix equality.