ECONOMICS 7330 – Probability and Statistics, Fall 2025

Homework 7. Due Wednesday November 5.

1. Assume you have a sample of vector random variables $x_i = (y_i, z_i)'$ mean $\mu = (\mu_y, \mu_z)'$ Assume that $\sqrt{N}(\overline{x} - \mu)$ converges in distribution to $N(0, \Sigma)$ where

$$\Sigma = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} .$$

Find the joint asymptotic distribution of $\sqrt{N}(\frac{\overline{y}}{\overline{z}} - \frac{\mu_y}{\mu_z})$ and $\sqrt{N}(\overline{y}\,\overline{z} - \mu_y\,\mu_z)$ (the main part of the answer is a 2×2 variance matrix).

2. Assume that you have a sample of n observations from a Poisson distribution with probabilities $\pi(k) = \frac{\theta^k\, e^{-\theta}}{k!}$.
a) Write down the log-likelihood function $l_n(\theta)$.
b) Find the Score.
c) Find the ML estimator $\hat{\theta}$.

3. Assume that you have a sample of n observations from an exponential distribution with density $f(x) = \frac{1}{\theta} \exp^{-\frac{x}{\theta}}$. (The mean is θ and the variance is θ^2.)
a) Write down the log-likelihood function $l_n(\theta)$.
b) Find the Score.
c) Find the ML estimator $\hat{\theta}$.
d) Find the Hessian.
e) What is the asymptotic distribution of $\sqrt{N}(\hat{\theta}_N - \theta_0)$? (I need an expression for the asymptotic variance.)
f) Verify the Information Matrix equality.