

ECONOMICS 7330 – Probability and Statistics, Fall 2023

Homework 7. Due Wednesday October 18.

This homework has exercises from Bruce Hansen's book.

Most of the problems assume a random sample $\{X_1, X_2, \dots, X_n\}$ from a common distribution F with density f such that $E[X] = \mu$ and $var[X] = \sigma^2$ for a generic random variable $X \sim F$. The sample mean and variances are denoted \bar{X}_n and $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X}_n)^2$, with the bias-corrected variance $s_n^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X}_n)^2$.

1. (Hansen Exercise 6.1) Suppose that another observation X_{n+1} becomes available. Show that

(a) $\bar{X}_{n+1} = (n\bar{X}_n + X_{n+1})/(n+1)$.

(b) $s_n^2 = [(n-1)s_n^2 + \frac{n}{n+1}(X_{n+1} - \bar{X}_n)^2]/n$

(This kind of updating is important in practice when n is very large and new observations regularly enter.)

2. (Hansen Exercise 6.11.) Let p be the unknown probability that a given basketball player makes a free throw attempt. The player takes n random free throws, of which he or she makes X of the attempts.

(a) Find an unbiased estimator \hat{p} of p .

(b) Find $var(\hat{p})$.

3. (Hansen Exercise 6.12.) Suppose we know σ^2 and want our estimator to have a standard deviation smaller than a tolerance How large does n need to be to make this happen?

4. (Hansen Exercise 6.13.) Find the covariance of $\hat{\sigma}^2$ and \bar{X}_n . Under what condition is this zero? (This exercise shows that the zero correlation between the numerator and the denominator of the t ratio does not always hold when the random sample is not from a normal distribution.)

5. (Hansen Exercise 6.14.) Suppose that X_i are i.n.i.d. (independent but not necessarily identically distributed) with $E[X_i] = \mu_i$ and $var[X_i] = \sigma_i^2$.

Find $var[\bar{X}_n]$