

ECONOMICS 7330 – Probability and Statistics, Fall 2026

Homework 6. Due Monday October 5.

1. Define the lag polynomials $a(L) = a_0 + a_1 L$ and $b(L) = b_0 + b_1 L + b_2 L^2$. (Notice: in the notes, and in class, it is often assumed $a_0 = 1$ and $b_0 = 1$. This is just for simplification and doesn't matter for any results since you can always re-scale the data and the lag-polynomial such that the first coefficient becomes unity (write $a(L)$ as $a_0 a'(L)$ where the lag polynomial $a'(L) = 1 + \frac{a_1}{a_0} L$ and similarly for $b(L)$). The constant a_0 will not affect the properties of the lag-polynomial that we care about.)

Assume $a_0 = 1$, $a_1 = -2$, $b_0 = 1$, $b_1 = -.3$, and $b_2 = -.5$.

- i) If $x_t = 2, x_{t-1} = -3, x_{t-2} = -2, x_{t-3} = 9$, and $x_{t-4} = 9$, what is $a(L)x_t$? and $b(L)x_t$? (This should be a number.)
 - ii) Find the roots of $a(L)$ and $b(L)$. Are the polynomials invertible (meaning, can be inverted, in class I called that stable which is also used)?
 - iii) What is $c(L) = a(L)b(L)$? What are the roots of $c(L)$? (hint: this should be easy)
 - iv) $c(L)x_t = 8 + u_t$ stable?
 - v) Find the coefficients to the constant, L , and L^2 in the lag-polynomial $a^{-1}(L)$.
- Now define the polynomials $d(x) = 1 + .2x$ and $e(x) = 1 + .5x^2$.
- vi) Find the roots of $f(L) = d(L)e(L)$. Is $f(L)$ invertible?

2. (24% of a previous midterm) Assume that income follows the AR(1) process

$$y_t = 2 + 0.4y_{t-1} + e_t \quad (*)$$

where e_t is white noise with variance 3.

- a) Is this time-series process stable?
- b) Assume that y_0 is a random variable. For what values of the mean $E(y_0)$ and the variance $\text{var}(y_0)$ will the time series y_t ; $t = 0, 1, 2, \dots$ be stationary?
- c) What is $E_1 y_3$ if $y_1 = 5$ and $y_0 = 2$?

3. Assume that income follows the AR(2) process

$$y_t = 3 + 0.3y_{t-1} + y_{t-2} + e_t$$

where e_t is white noise.

a) Is this time-series process stable?

b) What is $E_{t-2}y_{t-1}$ if $y_{t-2} = 5$ and $y_{t-3} = 10$?

c) What is $E_{t-2}y_t$ if $y_{t-2} = 5$ and $y_{t-3} = 10$?

4. Let

$$x_t = \alpha_0 + u_t + 0.5 * u_{t-1} + u_{t-2} ,$$

where u_t is white noise.

Find the auto-covariances for x_t in terms of σ_u^2 (the variance of u_t).

5. Consider the AR(1) model under the assumption that it is stationary:

$$y_t = \mu + ay_{t-1} + e_t$$

a) Find the mean, variance, and first order autocovariance of y_t .

b) Using the results from a), use the formula for the conditional mean of a normal distribution to find $E(y_t|y_{t-1})$ (in the linear model, you can assume normality for this purpose, the formulas won't depend on that). The purpose of this question is not to find the conditional mean, which is obvious in the AR(1), but to remind you that formula for conditional expectations may be useful in the time series setting also.

6. Assume that y_t follows the AR(2) process

$$y_t = 200 + 1y_{t-1} - 0.4y_{t-2} + e_t \quad (*)$$

where e_t is white noise with variance 9.

a) Is this process stable? (You need to show why).

b) Find the mean and variance of y_t for $t=1$ and 2 conditional on all information data $t=0$ and earlier. (That means, treat y_0, y_{-1} etc as constants.)