Bent E. Sørensen

ECONOMICS 6331 – Probability and Statistics, Fall 2022

Homework 6. Due Wednesday October 12, 2022.

1. Assume that X and Y follows a bivariate normal distribution.
a) Show that $X - E(X|Y)$ is independent of Y. (Use the law of iterated expectations or just find the covariance.)
b) Find the variance of $X - E(X|y)$ for fixed y (hint: This a linear function of X and y).

c) Demonstrate that $X + Y$ is normally distributed—a super important result. (Hint: use the convolution formula. Also use that $(z - s)^2 + s^2 = 2 * (s - \frac{z}{2})^2 + \frac{z^2}{2}$.)

2. Assume that X is an n-dimensional random variable with covariance matrix Σ and Y is an n-dimensional random variable, independent of X with covariance matrix Ω. Show that the covariance matrix for $X + Y$ is $\Sigma + \Omega$. (If you have problems with the general situation, we will give full point if you show it for 2-dimensional case.)

3. (24% of final 2005) Assume that Z is a normally distributed random variable with variance 9 and mean 2, and that Z is independent of (X, Y) where (X, Y) is a bivariate normally distributed random variable with mean $\mu' = (0, 0)$ and variance-covariance matrix
$$\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$
a) What is the conditional mean of $Y|X$?
b) What is the conditional variance of (X, Z) given Y?
c) What is the conditional mean of X given (Y, Z)?

   d) What is the distribution of $2X^2 - 2XY + Y^2$?