

ECONOMICS 7330 – Probability and Statistics, Fall 2026

Homework 5. Due Monday September 28.

1. (Questions 5.2 and 5.3 in Hansen's book). For a standard normal density $\phi(x)$, show that the derivative $\phi'(x) = -x\phi(x)$. Using this, and integration by parts to show that the variance of a standard normal is unity.

2. (24% of final 2005) Assume that Z is a normally distributed random variable with variance 9 and mean 2, and that Z is independent of (X, Y) where (X, Y) is a bivariate normally distributed random variable with mean $\mu' = (0, 0)$ and variance-covariance matrix

$$\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

- a) What is the conditional mean of $Y|X$?
- b) What is the conditional variance of (X, Z) given Y ?
- c) What is the conditional mean of X given (Y, Z) ?
- d) What is the distribution of $2X^2 - 2XY + Y^2$? (This parenthesis was not on the exam, but show that it is chi-square with two degrees of freedom, also I will show on Wednesday how to get from multivariate Normal to chi-square. This is also a super important result.)

3. Consider two vectors X and Y that follows a joint multivariate distribution. Show using the formula for the conditional distribution that $EE(Y|X) = EY$. (This should be easy, but it good to observe anyway.)

4. Assume $X \sim N(0, 9)$, $Y \sim N(2, 9)$, and $Z \sim N(2, 16)$. Further assume that the covariances between X and Y , X and Z , and Y and Z are all 2.

- i) What is $E(X|Y = 2, Z = 3)$? (State the formula you use and then the number.)
- ii) What is the conditional variance $Var(X|Z = 3)$?
- iii) What is $E(X, Z|Y = 3)$? Verify that the numbers are the same as if you did $E(X|Y = 3)$ and $E(Z|Y = 3)$ separately. (And think about why this has to be case.)