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ECONOMICS 7330 – Probability and Statistics, Fall 2025

Homework 5. Due Wednesday October 22.

1. Assume $X \sim N(0,9)$, $Y \sim N(2,9)$, and $Z \sim N(2,16)$. Further assume that the covariances between X and Y, X and Z, and Y and Z are all 2.
i) What is $E(X|Y=2, Z=3)$? (State the formula you use and then the number.)
ii) What is the conditional variance $Var(X|Z=3)$?
iii) What is $E(X,Z|Y=3)$? Verify that the numbers are the same as if you did $E(X|Y=3)$ anf $E(Z|Y=3)$ separately. (And think about what this has to be case.)

2. Let $X_1, ..., X_N$ be a sample of i.i.d. random variables with $\mathrm{E}\log(X_i) = 0$ and $\mathrm{Var}\log(X_i) = 1$. Let
$$Z_N = (X_1 * .... * X_N)^{\frac{1}{N}} .$$
Show that Z_n converges in probability to 1.
Let
$$Y_N = (X_1 * .... * X_N)^{\frac{1}{\sqrt{N}}} .$$
Show that Y_N converges in distribution to a log-normal distribution.

3. Let Y_N be a χ^2-distributed random variable with N degrees of freedom and let $Z_N = (Y_N - N)/\sqrt{2N}$. Show that Z_N converges in distribution to a $N(0,1)$ variable. (You can use without showing that the variance of a χ^2 distribution with k degrees of freedom is $2k$, although it follows easily from $E(x^2)^2 = 3$ and Ex^2=1 for a standard normal.)

4. Assume $\sqrt{n}(\hat{\theta} - \theta)$ converges in distribution to $N(0, \Sigma)$,
with
$$\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$
and
$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$
Use the delta rule (aka delta method) to find the asymptotic variance of
a) $\hat{\theta}_1^3$.
b) $\frac{\hat{\theta}_2}{1+2\hat{\theta}_1^2}$.