

ECONOMICS 7330 – Probability and Statistics, Fall 2023

Homework 5. Due Wednesday October 4.

1. Find the covariance and correlation between $a + bX$ and $c + dY$. (Note: when written like this, it is implicit if I do not explicitly say so that a, b, c , and d are real constants, and X and Y are random variables for which the variances and covariances exist.)

2. Assume that X and Y follows a bivariate normal distribution.

a) Find the covariance of $X - E(X|Y)$ and Y .

b) Find the variance of $X - E(X|y)$ for fixed y (hint: This a linear function of X and y).

c) Demonstrate that $X + Y$ is normally distributed—a super important result. (Hint: use the convolution formula. Also use that $(z - s)^2 + s^2 = 2 * (s - \frac{z}{2})^2 + \frac{z^2}{2}$.)

3. Assume that X is an n -dimensional random variable with covariance matrix Σ and Y is an n -dimensional random variable, independent of X with covariance matrix Ω . Show that the covariance matrix for $X + Y$ is $\Sigma + \Omega$.

4. (24% of final 2005) Assume that Z is a normally distributed random variable with variance 9 and mean 2, and that Z is independent of (X, Y) where (X, Y) is a bivariate normally distributed random variable with mean $\mu' = (0, 0)$ and variance-covariance matrix

$$\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

a) What is the conditional mean of $Y|X$?

b) What is the conditional variance of (X, Z) given Y ?

c) What is the conditional mean of X given (Y, Z) ?

d) What is the distribution of $2X^2 - 2XY + Y^2$?