Bent E. Sørensen

ECONOMICS 7330 – Probability and Statistics, Fall 2022

Homework 5. Wednesday September 29. Due Wednesday October 5.

1. Let the joint probability function for X and Y be defined by

$$f(x, y) = \frac{x + y}{32} \, , \; x = 1, 2 \, ; \; y = 1, 2, 3, 4 \, .$$

Find
a) $f_X(x)$, the marginal probability function for X.
b) $f_Y(y)$, the marginal probability function for Y.
c) $P(X < Y)$.
d) $P(Y = 3X)$.
e) $P(X + Y = 4)$.
f) $P(X \leq 4 - Y)$.
g) Are X and Y independent or dependent?

2. Prove that for any random variables X and Y with finite variances (hint: use the law of iterated expectations):
(a) The covariance $cov(X, Y) = cov(X, E[Y|X])$.
(b) X and $Y - E[Y|X]$ are uncorrelated. (This implies they are independent if they are normally distributed. This is sometimes important.)

3. Suppose that Y conditional on X is $N(X, X)$ (that is, Normally distributed with both mean and variance equal to X). If $E[X] = \mu$ and $var(X) = \sigma^2$ what are $E[Y]$ and $var[Y]$? (hint: use the law of iterated expectations.)

4. Find the covariance and correlation between $a + bX$ and $c + dY$. (Note: when written like this, it is implicit if I do not explicitly say so that $a, b, c,$ and d are real constants, and X and Y are random variables for which the variances and covariances exist.)