

ECONOMICS 7330 – Probability and Statistics, Fall 2024

Homework 4. Due Wednesday September 18.

1. For a random variable X with uniform distribution on the interval $[a, b]$ with density f show

(a) $\int_a^b f(x)dx = 1$.

(b) $E[X] = \frac{1}{2}(b + a)$.

(c) $var[X] = \frac{1}{12}(b - a)^2$.

2. Show that if X and Y are statistically independent, and a, b, c, d are real constants with $a < b$ and $c < d$, then

$$P[a < X < b, c < Y < d] = P[a < X < b]P[c < Y < d].$$

3. Prove that for any random variables X and Y with finite variances (hint: use the law of iterated expectations):

(a) The covariance $cov(X, Y) = cov(X, E[Y|X])$.

(b) X and $Y - E[Y|X]$ are uncorrelated. (This implies they are independent if they are normally distributed. This is sometimes important.)

4. Suppose that Y conditional on X is $N(X, X)$ (that is, Normally distributed with both mean and variance equal to X). If $E[X] = \mu$ and $var(X) = \sigma^2$ what are $E[Y]$ and $var[Y]$? (hint: use the law of iterated expectations.)

5. Consider two random variables X and Y . Assume they both are discrete and that X can take the values 1, 2, and 4 while Y takes the values 0 and 2. The probabilities for (X, Y) are shown in the following table:

	X=1	X=2	X=4
Y=0	3/24	3/24	6/24
Y=2	3/24	5/24	4/24

i) Find the marginal probabilities of X and Y . Mark clearly which are the marginal probabilities of X and which are the marginal probabilities of Y . Explain what the marginal probabilities measure.

ii) Find the means and the variances of X and Y .

- iii) Are the events $X = 1$ and $Y = 2$ independent events?
- iv) Are the random variables X and Y independent?
- v) Find the probability $P(\{X > 1\} \cap \{Y \leq 1\})$
- vi) Find the conditional distribution of X given $Y = 2$.
- vii) Find the random variable $E(X|Y)$.
- viii) Take the mean of the random variable that you derived in vii) and verify that it equals $E(X)$.

6. Find the covariance and correlation between $a + bX$ and $c + dY$, as function of the variances and covariance of X and Y . (Note: when written like this, it is implicit if I do not explicitly say so that a, b, c , and d are real constants, and X and Y are random variables for which the variances and covariances exist.)