

**ECONOMICS 7330 – Probability and Statistics, Fall 2022**

Homework 4. Wednesday September 14. Due Wednesday September 21.

1. Do exercise 3.4 in Hansen's book.
2. Show that if  $X$  and  $Y$  are statistically independent, and  $a, b, c, d$  are real constants with  $a < b$  and  $c < d$ , then

$$P[a < X < b, c < Y < d] = P[a < X < b]P[c < Y < d].$$

3. Let  $f(x, y) = (3/16)xy^2$ ;  $0 < x < 2$ ,  $0 < y < 2$ , be the joint density function for  $X$  and  $Y$ . Find the marginal density functions  $f_X(x)$  and  $f_Y(y)$ . Find the distribution function (CDF) for  $X$ . Are the two random variables independent?
4. Let  $f(x, y) = 1/6 e^{-x/2-y/3}$  be the joint density function for  $X$  and  $Y$ . Find the marginal density functions  $f_X(x)$  and  $f_Y(y)$ . Are the two random variables independent?
5. Consider two random variables  $X$  and  $Y$ . Assume they both are discrete and that  $X$  can take the values 1, 2, and 4 while  $Y$  takes the values 0 and 2. The probabilities for  $(X, Y)$  are shown in the following table:

X=1	X=2	X=4	
Y=0	3/24	3/24	6/24
Y=2	3/24	5/24	4/24

- i) Find the marginal probabilities of  $X$  and  $Y$ . Mark clearly which are the marginal probabilities of  $X$  and which are the marginal probabilities of  $Y$ . Explain what the marginal probabilities measure.
- ii) Find the means and the variances of  $X$  and  $Y$ .
- iii) Are the events  $X = 1$  and  $Y = 2$  independent events?
- iv) Are the random variables  $X$  and  $Y$  independent?
- v) Find the probability  $P(\{X > 1\} \cap \{Y \leq 1\})$
- vi) Find the conditional distribution of  $X$  given  $Y = 2$ .
- vii) Find the random variable  $E(X|Y)$ .
- viii) Take the mean of the random variable that you derived in vii) and verify that it equals  $E(X)$ .