

ECONOMICS 7330 – Probability and Statistics, Fall 2026

Homework 3. Due Monday September 14.

1. Prove that for any random variables X and Y with finite variances (hint: use the law of iterated expectations) X and $Y - E[Y|X]$ are uncorrelated. (This implies they are independent if they are normally distributed. This result is important.)

2. Consider two random variables X and Y . Assume they both are discrete and that X can take the values 1, 2, and 4 while Y takes the values 0 and 2. The probabilities for (X, Y) are shown in the following table:

	X=1	X=2	X=4
Y=0	3/24	3/24	6/24
Y=2	3/24	5/24	4/24

- i) Find the marginal probabilities of X and Y . Mark clearly which are the marginal probabilities of X and which are the marginal probabilities of Y . Explain what the marginal probabilities measure.
- ii) Find the means and the variances of X and Y .
- iii) Are the events $X = 1$ and $Y = 2$ independent events?
- iv) Are the random variables X and Y independent?
- v) Find the probability $P(\{X > 1\} \cap \{Y \leq 1\})$
- vi) Find the conditional distribution of X given $Y = 2$.
- vii) Find the random variable $E(X|Y)$.
- viii) Take the mean of the random variable that you derived in vii) and verify that it equals $E(X)$.
- xi) Find the probability distribution for $Z = X + Y$.