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ECONOMICS 7330 – Probability and Statistics, Fall 2024

Homework 3. Due Wednesday September 11.

1. Let $X \sim U[0,1]$ be uniformly distributed on $[0,1]$. Suppose X is truncated to satisfy $X \leq c$ for some $0 \leq c \leq 1$.
(a) Find the density function of the truncated variable X.
(b) Find $E[X|X \leq c]$.

2. (a) Show that an exponential distribution with CDF $1 - \exp(\frac{-x}{\theta})$ and support on the positive real line has mean θ. (The parameter space for θ is also the positive real line).
(b) Show that if X is uniformly distributed on the interval $[0,1]$ then $Y = -\theta \log(X)$ follows an exponential distribution with mean θ. (You need to explicit about the support of the variables.)
(c) Explain why Jensen's inequality implies that $E(Y) > \log(2)$ for $\theta = 1$.

NOTE. We will talk about bivariate distributions on Monday. You should be able to do the following question after class.

3. Let $f(x,y) = (3/16)\, xy^2$; $0 < x < 2$, $0 < y < 2$, be the joint density function for X and Y.
(a) Find the marginal density functions $f_X(x)$ and $f_Y(y)$.
(b) Find the distribution function (CDF) for X.
(c) Are the two random variables independent?