

**ECONOMICS 7330 – Probability and Statistics, Fall 2022**

Homework 3. Due Wednesday September 14.

1. Show that if the density satisfies  $f(x) = f(-x)$  for all  $x \in \mathbb{R}$  then the distribution function satisfies  $F(-x) = 1 - F(x)$ .
2. The skewness of a distribution (random variable  $X$ ) with mean  $\mu$  and standard deviation  $\sigma$  is skew =  $\frac{E(X-\mu)^3}{\sigma^3}$ .
  - (a) Show that if the density function is symmetric about  $\mu$ , then skew = 0.
  - (b) Calculate skew for a random variable with density  $f(x) = \exp(-x), x \geq 0$ .
3. Let  $X$  be a random variable with mean  $\mu$  and variance  $\sigma^2$ . Show that  $E(X-\mu)^4 \geq \sigma^4$ . (The fourth central moment is called *kurtosis*. This is a commonly used term that you need to know.)
4. Let  $X \sim U[0, 1]$  be uniformly distributed on  $[0, 1]$ . Suppose  $X$  is truncated to satisfy  $X \leq c$  for some  $0 \leq c \leq 1$ .
  - (a) Find the density function of the truncated variable  $X$ .
  - (b) Find  $E[X|X \leq c]$ .
5. (a) Show that if  $X$  is uniformly distributed on the interval  $[0, 1]$  then  $Y = -\theta \log(X)$  follows an exponential distribution with mean  $\theta$ .
  - (b) Explain why Jensen's inequality implies that  $E(Y) > \log(2)$  for  $\theta = 1$ .